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Validity of Application of Rayleigh's Limit of Resolution in Presence of Defocus Wavefront Errors

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Abstract : *In the field of image science, Rayleigh criterion of resolution has been and is still being extensively used in the assessment of performance of optical imaging systems including telescopes. This is in spite of the fact, that Rayleigh's criterion is applicable in incoherent illumination and it loses its validity in the presence of atmospheric turbulence, various types of wavefront degrading factors, image motion etc. In the present paper, we have discussed the validity of application of Rayleigh's criterion to the resolution of Binary Stars of unequal brightness due to defocus wavefront errors.*

Keywords: *Point Spread Function, Rayleigh's Criterion of Resolution*

I. INTRODUCTION

The technique of apodization helps the astronomers in detecting the presence of a very weak satellite star in the vicinity of a very intense primary star viz. SIRIUS and its companion star [1]. Subsequently, it was found that though the detectability of the faint star of the binary system was almost a certainty, the optical system had to pay a heavy price in terms of its resolving power [2]. A suitable compromise between detection and resolution had to be found out in each and every application of the technique of apodization. However, the above situation holds good for imaging of stars by telescopes in a clear night sky. In a turbulent night sky, there are multiples of wavefront degrading factors which ultimately make the applicability of Rayleigh's criterion for resolution invalid. Rayleigh criterion states that the two point sources are just resolved if the maximum of one irradiance pattern coincides with the first minimum of the other [3]. This means that two closely spaced points can be considered as just resolved if we are able to distinguish the resultant Point Spread Function (PSF) in the image as being due to two objects instead of one. A point-source object can be represented mathematically by a delta function. A small bright point object produces an image field that is much more spread out. This image field is known as the "Point Spread Function, PSF" [4]. It may be pointed out that Rayleigh proposed his criterion to be used for line objects in spectroscopy. But it can be equally applicable to point objects as well [5]. In its original form, the Rayleigh criterion is applicable to two equally bright points under incoherent illumination [6]. Several researchers modified the Rayleigh criterion to suit the case of object points of unequal intensities and various imaging situations [7].

In the present study, we have made an analytical study of resolution of binary stars in the presence of apodization by a cascaded system of two complimentary filters and wave-front defocus errors [8].

II. Expression For Image Intensity Distribution In The Binary Star

The resultant intensity distribution in the image of a binary star has been shown to be [9] as follows

$$I(z) = |G(z-B)|^2 + \alpha|G(z+B)|^2 + 2\sqrt{\alpha}\,\gamma(z_0)|G(z-B)||G(z+B)| \quad (1)$$

Where,

$2B(=Z_o)$: is the actual separation between the object points;

α : is the ratio of the intensities of the object points;

$\gamma(Z_o)$: is the real part of the complex degree of coherence of illumination of the object point;

$G(z-B)$ & $G(z+B)$: are the complex amplitude impulse response functions of the optical imaging system corresponding to the object points each of which is situated at a distance of $Z_0/2$ on either side of the optical axis.

Now, in the present study, we are concerned only with Rayleigh's criterion of resolution, which is valid only for incoherent illumination. Therefore, by putting $\gamma = 0$ in eqn. (1), we get,

$$I(z) = |G(z-B)|^2 + \alpha|G(z+B)|^2 \quad (2)$$

Further, for Rayleigh's criterion of resolution, we need to study the intensity distribution in the image of the primary star only. Thus, putting $\alpha = 0$ in the eqn. (2) above, we obtain the intensity impulse function distribution in the image of the primary star, i.e.

$$I(z) = |G(z)|^2 \qquad (3)$$

The amplitude response function G (z) is given by [9],

$$G(z) = 2\int_0^1 f(r) J_0(zr) r dr \qquad (4)$$

where $J_0(zr)$ is the Bessel function of the first kind and order is zero and $f(r)$ is the pupil function of the exit pupil of the image forming telescope. For our present study, we have considered an exit pupil which is apodized with two cascaded complimentary parabolic amplitude filters. [10],

i.e. $$f(r) = (\beta r^2)(1 - \beta r^2) \qquad (5)$$

where β is the apodization parameter.

Further, if the optical system suffers from defocus wave-front errors, the resultant pupil function will assume the form,

$$f(r) = (\beta r^2)(1 - \beta r^2)\, e^{-iyr^2/2} \qquad (6)$$

Where y is the amount of defocusing in units of 2π.

We, thus, finally obtain,

$$G(z) = 2\int_0^1 \{(\beta r^2)(1 - \beta r^2) e^{\frac{-iyr^2}{2}} J_0(zr) r dr \qquad (7)$$

By substituting the value of $G(z)$ from the Eq. (7) in Eq. (3), we get the desired expression for the intensity impulse response function in the diffraction image of the primary star formed by a telescopes objective, i.e.

$$I(z) = \left| 2\int_0^1 \{(\beta r^2)(1 - \beta r^2) e^{\frac{-iyr^2}{2}} J_0(zr) r dr \right|^2 \qquad (8)$$

III. RESULTS AND DISCUSSION

We have used Eq. (8) to evaluate $I(z)$ or the Point Spread Function (PSF) of the system. The results obtained by us for various values of the apodization parameter β and the defocus amount y have been graphically presented in figures 1 through 9.

III.1 EFFECT OF VARYING Β

In the table, we have shown the variation of central intensity with β and y. It is found from the table that for a particular value of β, the amount of defocus error has practically no significant effect. However, for a particular value of y, the value of the central intensity increases, as the value of the apodization parameter β increases, up to a value of β= 0.8. Increasing β thereafter causes decrease of the central intensity. This behaviour can be explained from the consideration of the pupil transmission curves of the cascaded filter [11]. It was observed there that the cascaded filter behaves as a super-resolving one in the range of 0< β ≤ 0.75 and as an apodising one in the range 0.75 ≤ β≤ 1.0. The over-all effect of this situation totally destroys the purpose of applying the technique of apodization, thereby causing invalidity of Rayleigh's criterion of resolution for application to imaging systems in presence of defocus errors.

Table - 1
Variation of central intensity with β and y

y/β	0.2	0.4	0.6	0.8	1.0
0.0	0.2160	0.6170	0.7900	1.0000	0.7910
2π/10	0.2160	0.6170	0.9300	1.0000	0.7980
4 π/10	0.2160	0.6170	0.9295	1.0000	0.8001
6 π/10	0.2160	0.6180	0.9292	1.0000	0.8038
8 π/10	0.2160	0.6179	0.9289	1.0000	0.8090
10 π/10	0.2170	0.6183	0.9283	1.0000	0.8160
12 π/10	0.2174	0.6189	0.9277	1.0000	0.8247
14 π/10	0.2182	0.6196	0.9270	1.0000	0.8353
16 π/10	0.2193	0.6207	0.9263	1.0000	0.8482
18 π/10	0.2207	0.6222	0.9255	1.0000	0.8636
20 π/10	0.2224	0.6242	0.9249	1.0000	0.8819

III.2 EFFECT OF VARYING y

The effect of varying the amount of defocus can be understood from the figures 1 through 9. For a particular value of y, as the value of β is increased, the central intensity decreased as stated earlier. The lowering of the central intensity is accompanied with the broadening of the central disk. Further, a part of the decreased energy goes into the diffracted maxima to increase the heights of the side-lobes. This has to happen to satisfy the law of conservation of energy. The over-all effect of this situation totally destroys the purpose of applying the technique of apodization, thereby causing invalidity of Rayleigh's criterion of resolution for application to imaging systems in presence of defocus errors.

The intensity distribution I(Z) has been computed as a function of the dimensionless coordinate Z for various defocus parameter y and apodization parameter β.

Before drawing the conclusion, it may be mentioned that the suitable criterion of resolution in such cases should be the determination of "Full – width at Half Maximum (FWHM)" which is very popular in Radiological Imaging and other areas of adaptive optics.

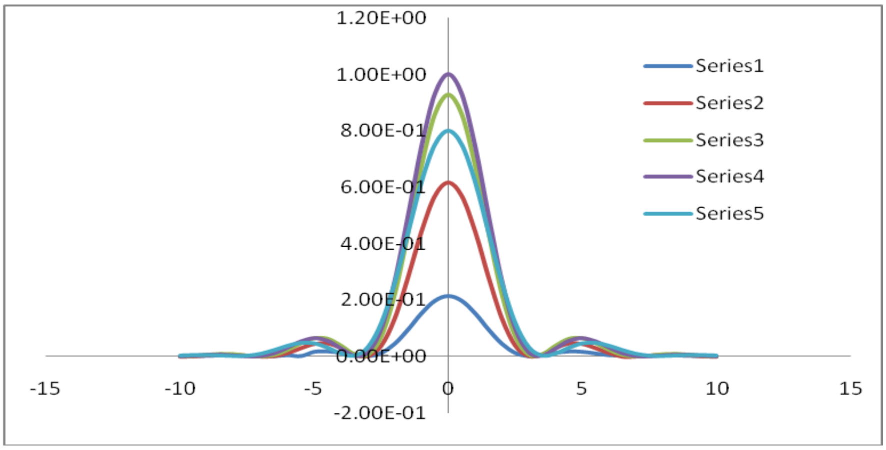

Fig. 1. Transverse variation of Intensity Point Spread Function (PSF) for various value of β: y = 4π/10

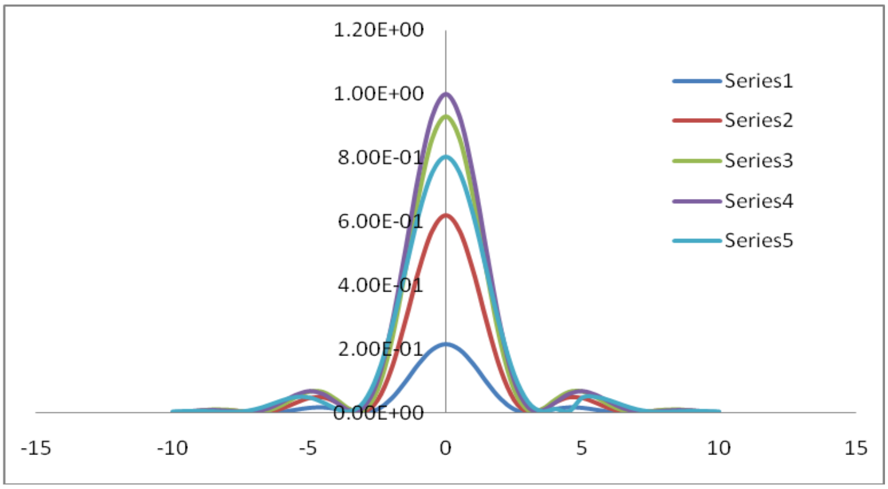

Fig. 2. Transverse variation of Intensity Point Spread Function (PSF)
for various value of β: y = 6π/10

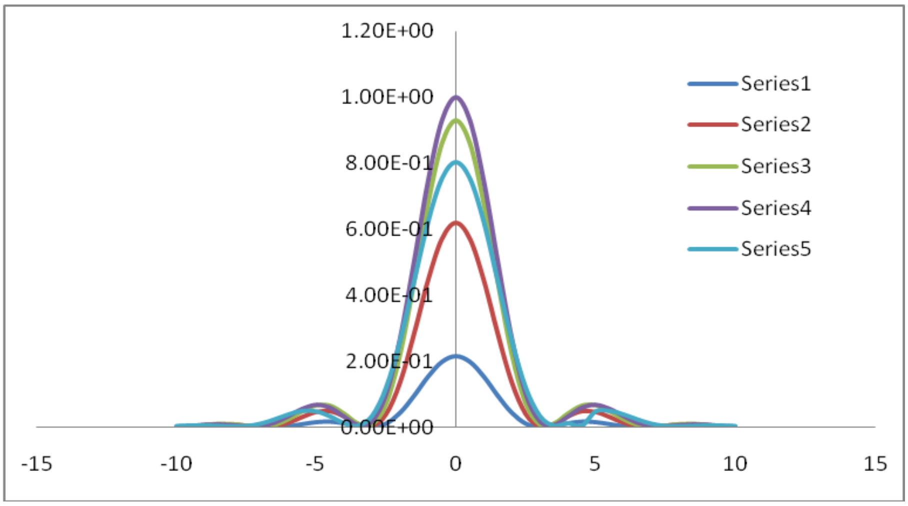

Fig. 3. Transverse variation of Intensity Point Spread Function (PSF) for various value of β: y = 8π/10

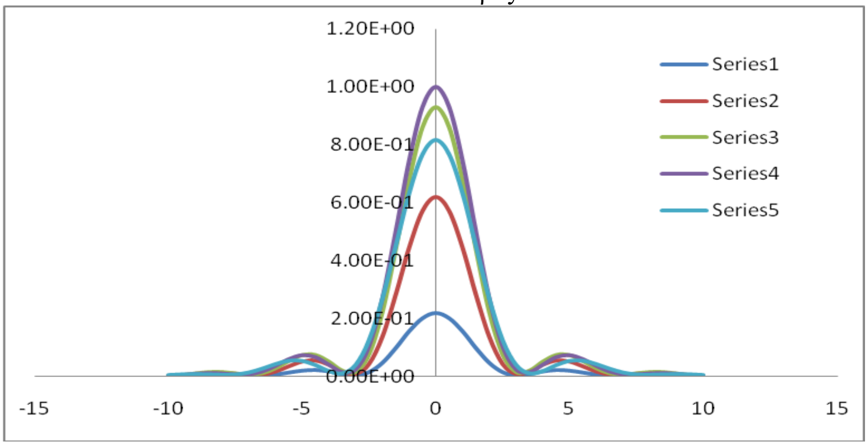

Fig. 4. Transverse variation of Intensity Point Spread Function (PSF) for various value of β: y = 10π/10

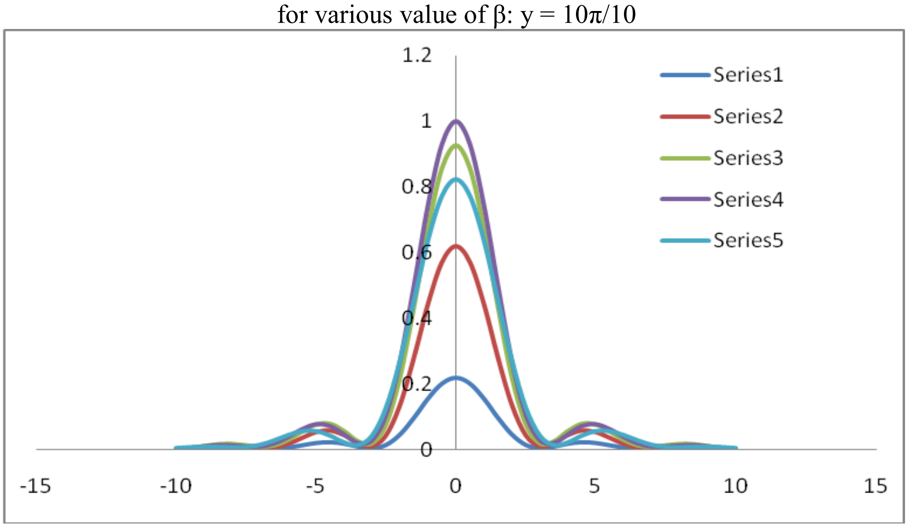

Fig. 5. Transverse variation of Intensity Point Spread Function (PSF) for various value of β: y = 12π/10

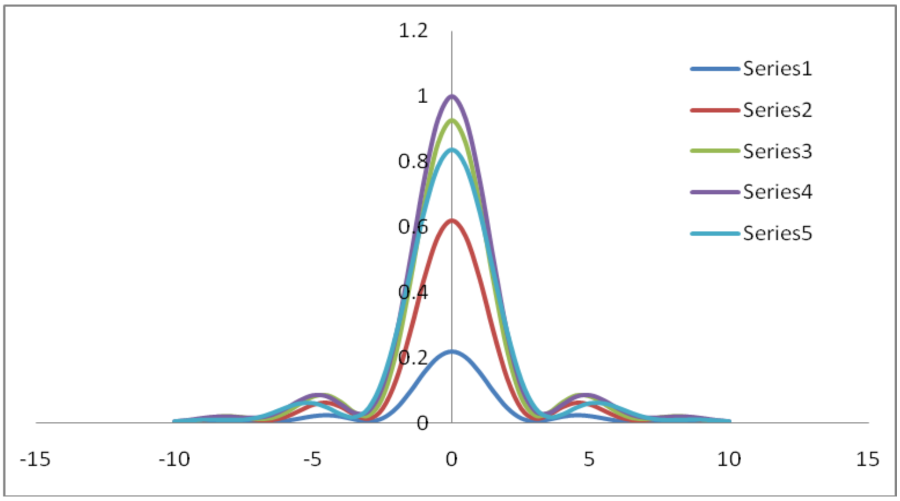

Fig. 6. Transverse variation of Intensity Point Spread Function (PSF) for various value of β: y = 14π/10

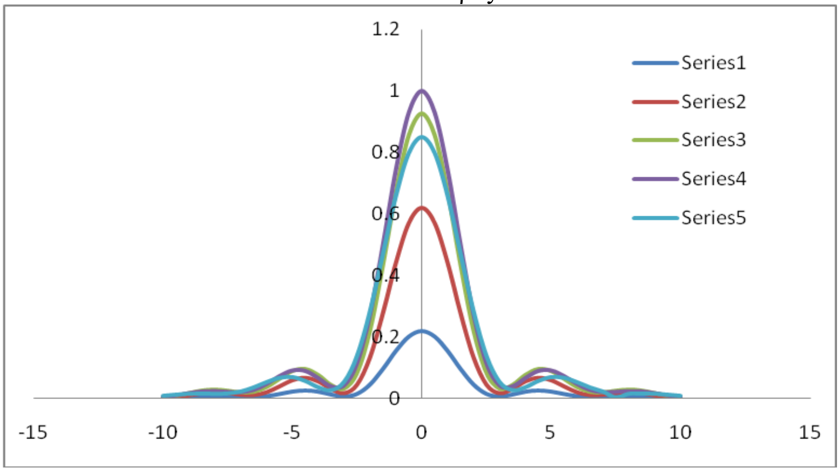

Fig. 7. Transverse variation of Intensity Point Spread Function (PSF) for various value of β: y = 16π/10

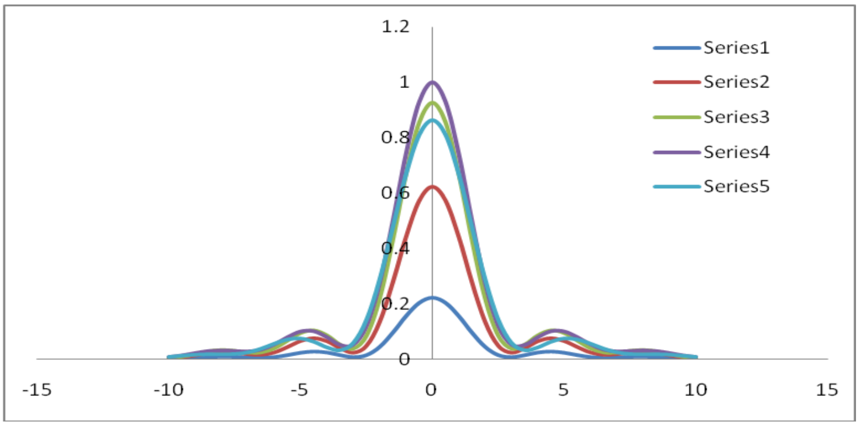

Fig. 8. Transverse variation of Intensity Point Spread Function (PSF) for various value of β: y = 18π/10

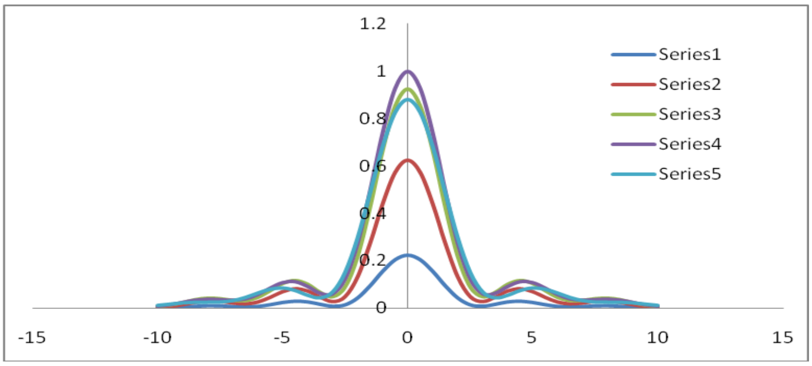

Fig. 9. Transverse variation of Intensity Point Spread Function (PSF)
for various value of β: y = 20π/10

IV. Conclusion

So we can draw the conclusion that the resolution of binary stars of unequal brightness by applying the technique of apodization through a cascaded system of two complimentary filters has the over-all effect of totally destroying the very purpose of apodization, thereby causing invalidity of Rayleigh's criterion of resolution for application to imaging systems in presence of defocus errors under incoherent illumination.

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