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Theta Concept: The Reality of Doppler and Lorentz Factors

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Abstract: *Despite being known for over 125 years, Lorentz Factor has been poorly understood and used liberally without understanding the applicable situation. Currently there is no explanation for the directional symmetry of the Lorentz Factor (V = -V). It is also considered that in low velocity approximation Relativistic approach yields Classical concepts. A new simple high school math based concept, named here as Theta Concept, clears all the confusion and explains the directional independence of Lorentz Factor in a physically, mathematically and logically meaningful way. The forgotten angle between the lines (vectors), line of motion and line of observation, is highlighted. Without this angle, we cannot calculate anything meaningful. It is this angle that separates the Relativistic and Classical approaches, not the high/low velocities. It is pointed that we limited ourselves for centuries to the few angles of 0, 90 and 180°. We never even threw a ball from moving train other than 0 or 180°. A new equation is provided to go beyond such tiny limit. Now we can throw the ball in any angle we wish and calculate it correctly.*

Keywords – *Classical Relativity, Doppler Effect, Lorentz Factor, Special Relativity, Theta Concept*

I. Introduction

Lorentz Factor (LF)[1] has been known for nearly 125 years, and has been an integral part of Special Relativity [2], the most popular theory for several generations, for about 111 years. Still LF has not been well understood and remains extremely difficult to fully grasp for most people. It is generally considered incompatible with, or very different from, classical concepts such as **Doppler Effect** (DE) [3], except during low velocity approximations. Although a link was available almost same time as the LF was born, it slipped away and later got twisted too much. A new concept linking classical and relativistic approaches is detailed here which clarifies the whole picture with several advantages: i) Very simple, even most high school students can understand; ii) Physically, mathematically and logically meaningful; iii) Unexplained directional independence becomes obvious and meaningful in special situations; iv) Clearly defines the applicability or the applicability becomes obvious and v) Bridges Classical and Relativistic concepts while avoiding conflicts and inconsistencies.

II. DISCUSSION

Since early 2006, I have been trying to find the relation between Doppler Effect (DE) and Lorentz Factor (LF). It was assumed that there must be a relation since both involve same Light and Motion. Though many times it was frustrating and looked like there was no relation, eventually a correlation has been found after nine years of non-continuous analysis. Before describing the details fully, it will be easier if the same process of realizing this solution is followed. In this article, abbreviation **“sqrt” is used to denote “square root”**.

At first it appeared that there was no correlation between DE and LF. So, the problem was put aside and a research was carried out to find a single equation describing DE factors. DE has (C-v) or (C+v) factors for increasing or reducing distance between source and receiver, both in sound and light **without any square term**. By trial and error, and using high school math learned over three decades back, it appeared that these factors can be unified. It was recognized simply as the **Theta problem**. If the light wave path and motion are in the same direction, the angle is zero which is a common convention. If the wave path and motion are in opposite directions, the angle is 180° which is also a common convention as shown in Fig. 1. In such case, the various velocity factors used in DE can be obtained from a single equation, $\mathrm{sqrt}(C^2 + v^2 - 2*C*v*\mathrm{Cos}(\theta))$ ➔ $(C\text{-}v_r)$ for $\theta = 0°$ for receiver moving away or $(C+v_r)$ for $\theta = 180°$ for receiver moving closer; similarly, $(C\text{-}v_s)$ for source moving towards or $(C+v_s)$ for source moving away, as used in sound DE. It is the **Law of Cosines**, part of Trigonometry [4]. The DE is a 1-D problem within resolution limits (wave path overlap is needed, i.e., θ must be 0 or 180°). If the motion is beyond resolution limits (beyond 1-D), no DE will be observed. It should be noted that the same factors can be alternately derived using **vectors**.

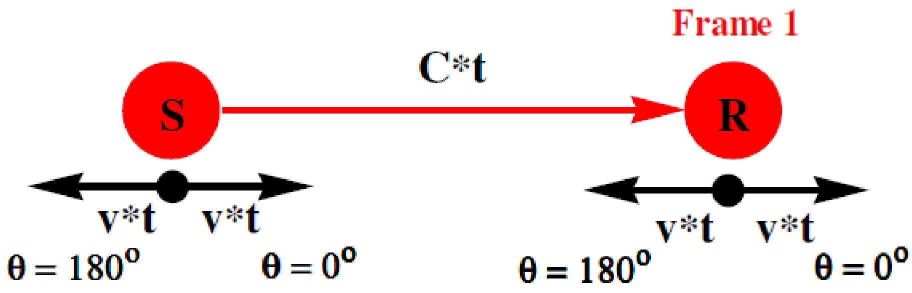

Figure 1. Diagram showing criteria used to arrive common equation for Doppler Effect velocity factors

This simplification used the special cases of theta being 0 or 180° in the Law of Cosines. Immediately it was obvious that it would be worthwhile to check the other special cases of theta being ±90°. Interestingly, both these gave the same result of sqrt(C^2 + v^2) **with all squared terms**, which can be recognized as a result from Pythagoras Theorem. It should be noted that Pythagoras theorem is only a special case of Law of Cosines. Since such direction independent nature is observed in LF, this case was further studied and found that LF can be derived easily from this same Law of Cosines equation in a physically meaningful and simple way. However, it also revealed the conditions for the use of LF [5].

Here is a simple derivation of LF. Consider an event observed by a stationary observer positioned at C*t distance and by a moving observer after a distance of v*t' at C*t' (see Fig. 2). It should be noted that this uses the constancy of speed of light principle of Special Relativity, even though LF had been derived with different principle earlier (see below for older literature). Here t & t' are time taken, not clock times. Equation (1) below is based on Law of Cosines [6].

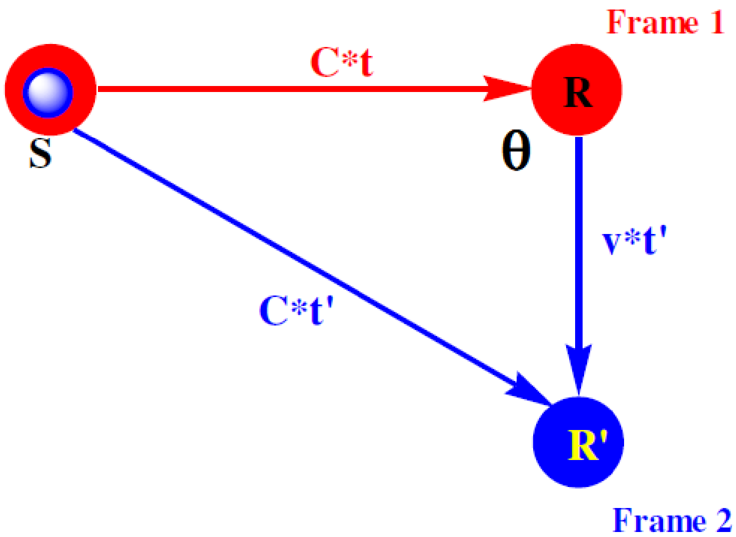

Figure 2. Receiver motion relative to the light path

$(C*t')^2 = (C*t)^2 + (v*t')^2 - 2*(C*t)*(v*t')*Cos(\theta)$ **(1)**

When (and **only when**) **θ = ±90°, Cos(θ) = 0**

$(C*t')^2 = (C*t)^2 + (v*t')^2$

$(C*t)^2 = (C*t')^2 - (v*t')^2$

$(C*t)^2 = t'^2(C^2 - v^2) \rightarrow t'^2(C^2 - v^2) = (C*t)^2$

$t'^2 = (C*t)^2/(C^2 - v^2)$

$t'^2 = t^2/(1 - (v/C)^2)$

$t' = t / \text{sqrt}(1 - (v/C)^2) \rightarrow$ Lorentz Transformation for time dilation.

$t'/t = 1/\text{sqrt}(1 - (v/C)^2) \rightarrow \gamma$, Lorentz Factor.

The same derivation shown here is often used in light clocks used to physically explain the time dilation [7]. The angle being 90° between the original photon direction and the moving direction is unrecognized. Though the photon clock can be oriented at any angle, ranging from 0 to 180° relative to motion, only 90° is used for the explanation or derivation. No other angle can give the LF, without bringing a fill-gap correction called length contraction which has no clear angular dependency [8]. Similar derivation is used in Michelson-Morley experiment [9] to calculate path distance in vertical direction using simple classical approach itself, where also the angle is 90°. Here it was found that θ = 0° and θ = 90° gave different results, but still the importance of theta was not recognized. One of the oldest similar looking equation arrived by Heaviside [10] had Sin(θ) term which gave LF only when θ = 90°. In Joules-Bernoulli equation, only E and B fields at right angle to motion (θ = 90°) are related by LF, but not the parallel fields [11].

Generally, when moving charges in a wire were considered against a resting charge, the motion was at right angle (θ = 90°) to the interaction and not in the same direction. Similarly, several approaches use Pythagoras theorem in deriving or explaining Relativistic effects such as time dilation, but without specifying that it involves θ = 90° and hence limited to use in only such situations. Another common use of $x^2 + y^2 + z^2 = r^2$ to derive LF also involves Pythagoras theorem and θ = 90° condition. This condition is same in all physically explained cases where even when two observers are traveling in parallel (in same or opposing directions, or in collinear motions), their observation light paths are always perpendicular to their travel paths (θ = 90°).

Thus, in all these cases, the θ = 90° was inseparable. However, it is not clear whether such association was understood earlier. When this prior known γ (later known as LF) was taken to electromagnetic waves (likely by Lorentz), the association was completely lost. Here a new physically meaningful derivation is shown and the association has become obvious and inseparable again. The above method can be used to derive γ values for any angle (full derivation is shown later). Since different theta values will give different results, γ is not a universal value but theta dependent. Here writing γ(90) or γ_{90} will be more accurate and avoid wrong uses.

The conditions of use for this γ(90) are:

1. To use two frames: source & receiver must have a relative velocity (V_r).
2. To use γ(90), LF: the motion must be at right angle to the observation line (θ = 90°).

Interestingly, above condition 2 can also be arrived by much simpler symmetry consideration as below:

a) When an object is alone, +V = -V in all directions (infinite or spherical symmetry).

b) When a reference object is added, +V = -V only in the plane normal to the line connecting them.

It can also be considered that the real world situation and the equation describing it must have similar symmetry criteria to be compatible, similar to unit of measure compatibility. One having symmetry and the other having asymmetry should suggest incompatibility and inappropriate use.

In another view, +V = -V actually applies to the vectors, which can be converted to scalar component as v*Cos(θ) = v*Cos(180-θ). This holds true only when θ = 90° (or 270°).

This situation is not much different from when someone tells of a place where all directions are equal. It is pretty easy to narrow the answer. In earth, only two such places exist; called as North and South Poles.

This clearly explains why LF is direction independent within the 90° (Fig. 3). The θ = 90° condition does not define a single line, rather it defines a plane normal to the observer with shortest distance of C*t. Objects moving in that plane from the shortest distance point will have ±90° (but without another out-of-plane reference, ± becomes meaningless). Identically, this will also apply to observer motion.

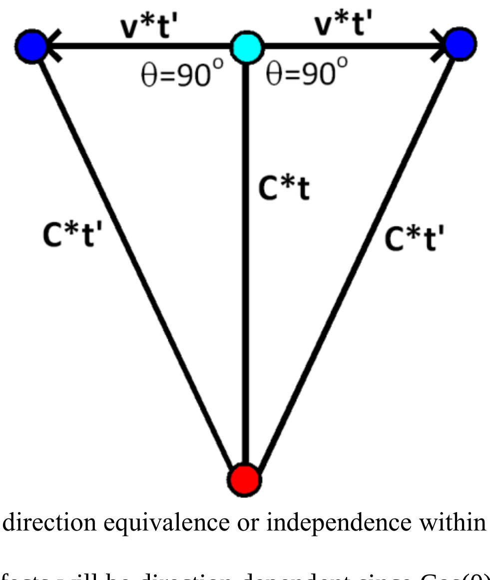

Figure 3. Diagram showing the direction equivalence or independence within θ = 90° (± is meaningless here)

However, for DE, the effects will be direction dependent since Cos(0) = -Cos(180). Thus theta concept explains all simultaneously in both physically and mathematically meaningful way. Theta value of 90° will very well approximate the rotating objects such as moon, stars, galaxies, satellites and GPS, at sometimes. Particles in accelerators can also be approximated to be observed or affected at θ = 90°. It is possible that Relativistic contributions to DE may be observed when an objects moves with θ = 90° but still stays within resolution limit such as distant orbiting objects in transverse Doppler effect. Such resolution limit problem may make these two concepts overlap in some cases and help explain observed variations or agreements.

At this stage, in hindsight, it is easy to see that an important variable in dealing with two velocities, the Theta has not been paid any attention. Now it may look obvious that when dealing two velocities, their relative angle is as important as the scalar part of the velocities themselves. Without this theta value, we cannot arrive at any useful conclusion and we cannot use scalar operations. When we use the velocities without theta, essentially we are using speed (scalar value), not vectors. However, unknowingly θ = 0° and θ = 180° have been used by classical approaches whereas θ = 90° has been unknowingly used often in relativistic approaches, but not always, which leads to some confusion. This clarification will avoid all confusions and conflicts. Without θ = 90°, no where else we can see such sum (or difference) of squares in an equation as seen in LF. This should have been the straight clue, in hindsight at least.

The derivation for the more broadly applicable γ(θ) is as follows:

$(C*t')^2 = (C*t)^2 + (V*t')^2 – 2*(C*t)*(V*t')*Cos(\theta)$ Earlier equation (1)

$(C^2 - V^2)*t'^2 + 2*C*V*t*Cos(\theta)*t' - (C*t)^2 = 0$ (dividing all by C^2 gives next step)

$(1 - \beta^2)*t'^2 + 2*\beta*t*Cos(\theta)*t' - t^2 = 0$ (Quadratic equation where β = V/C)

Quadratic equation solution: $aX^2 + bX + c = 0 \Rightarrow X = [-b \pm sqrt(b^2 - 4ac)]/2a$;

$t' = [-2*\beta*t*Cos(\theta) \pm sqrt((2*\beta*t*Cos(\theta))^2 + 4(1 - \beta^2)*t^2)]/2(1 - \beta^2)$

$t' = [-\beta*t*Cos(\theta) \pm sqrt((\beta*t*Cos(\theta))^2 + (1 - \beta^2)*t^2)]/(1 - \beta^2)$

$\gamma(\theta) = t'/t = [-\beta*Cos(\theta) \pm sqrt\{(\beta*Cos(\theta))^2 + 1 - \beta^2\}]/(1 - \beta^2)$

$$\boldsymbol{\gamma(\theta) = t'/t = [-\beta*Cos(\theta) \pm sqrt\{1 - \beta^2 + \beta^2*Cos^2(\theta)\}]/(1 - \beta^2)} \quad \textbf{(2)}$$

Though it appears that this will lead to two results, only one will be positive. Since negative time is not known at present, negative result may be omitted. The equation (2) is capable of providing answers to both Classical and Relativistic approaches when their appropriate angle is used.

Thus, the **Theta Concept** using **Law of Cosines** bridges the **Doppler Effect** and **Lorentz Factor**, although different domains are assigned for them. Normally DE and LF have been thought of as classical and modern physics respectively, as exclusive of other. Though they rule different domains of theta and are often mutually exclusive in individual instances, in Science they can coexist and not always mutually exclusive. The Law of Cosines can also be used to determine the relative velocity V_r of two objects when their velocities relative to a third common object are known, using the following equation (3) [12] (when they are in same time frame, time factor from the distance values cancels out):

$$V_r = sqrt\{V_a^2 + V_b^2 - 2*V_a*V_b*cos(\theta)\} \quad (3)$$

(If $V_a = V_b$ & θ = 0°, then $V_r = 0$)

It should be remembered that this velocity is derived from distance correlations assuming a common time frame whereas LF equation is arrived using a dual time frame (and for θ = 90°). In some cases more detailed equations can be arrived by appropriate distance or coordinate considerations. If the lines do not intersect at any point, D^2/t^2 factor will also appear inside the V_r formula based on same trigonometry, where D is the closest distance of the two velocity vectors and the V_r will appear to change with time as shown in (4).

$$V_r = sqrt\{D^2/t^2 + V_a^2 + V_b^2 - 2*V_a*V_b*cos(\theta)\} \quad (4)$$

In many cases, LF has been used without considering theta values. This new Theta Concept makes it necessary to re-evaluate the cases where theta was not 90° but LF was used to explain the data. For example, LF has been used to explain the Michelson-Morley experiments and Kennedy-Thorndike Experiments [13] which violate the conditions pointed here, and hence LF is not applicable. However, as mentioned earlier, the resolution limit problem may allow the continued use in some cases.

The relative velocity depends only on the two objects; the V_r is relative to each other. Even when a third object is used as a temporary reference, its contributions will cancel out. But, the group velocity (V_g) or total velocity (V_t) will need a third object as a permanent reference. The V_g is relative to the third object which may have its own velocity and may reside away from both the velocity vectors, adding more complication as it extends the two object problem to a three object problem. As a result, group velocity problems have not been addressed so far. However, V_a+V_b and V_a-V_b are commonly used for total velocity, relative to a "stationary" observer which are obviously for θ = 0° and θ = 180° respectively between the velocity vectors (with signs being opposite of what is seen for V_r). This is used in such examples as ball being thrown from a moving train or a fly flying on a moving ship, only forward or backward but not vertical or other angles. When the ball or fly is on vertical path, vector addition will give $V_g = V_t/2 = sqrt(V_a^2 + V_b^2)/2$, which is somewhat similar to LF with all square terms. This is never considered even in simple classical cases. Since V_g and V_t are not the main interest here, details are not discussed any further.

III. Conclusion

This Theta Concept clarifies that Law of Cosines links the Classical and Relativistic approaches. It also clarifies the observed direction independent nature of LF equation within θ = 90°. It is noted that V_r and V_g are different relation between two velocities. It is emphasized that when dealing with two vectors, the relative theta is important in determining the relative vector (vector subtraction) or group vector (vector addition). Without theta, we cannot use the common scalar operations of addition and subtraction in derivations as done often.

It is clear that the Classical and Relativistic approaches address only limited angular motions (just four: 0, 180° & ±90° at Cos(θ) extremes of ±1 and 0 respectively) leaving out all other angular motions. Thus, we have restricted our calculations to just a few angles for centuries, as far back as Galilean times or earlier. But, we never realized that we had always forgotten the angle between the lines (or vectors), line of motion and line of observation, in all methods of calculations, be it Classical or Relativistic. The significance of the missed angle cannot be overstated which is the sole source of conflict between Classical and Relativistic approaches and nothing to do with low/high velocities [14]. This angle is also very important in studying the orbiting objects, be it an elementary particle or a star.

The Theta Concept clarifies that LF cannot be used in common twin paradox (where theta is 0 or 180°). Also, neither LF nor DE can be used for circular motions (where theta varies from 0 to 360°), except when the object moves at a few limited angles relative to the line of observation.

Without knowing the theta, we can only get results with severe errors and inconsistencies in any approach. It is not possible to expect meaningful results by simply ignoring theta or randomly assuming a few limited values, assuming that our data quality is not that terrible. Though we have been doing this omission for centuries, it has to be corrected at some point and sooner will be better. A simple way is shown here.

References

[1] G. F. FitzGerald, The Ether and the Earth's Atmosphere, *Science*, *13*, 1889, 390; S. G. Brush, Note on the History of the FitzGerald-Lorentz Contraction, *Isis*, *58* , 1967, 230–232 (Source: http://philsci-archive.pitt.edu/987/1/Michelson.pdf).

[2] A. Einstein, Elektrodynamik bewegter Korper, *Annalen der Physik, 17*, 1905, 891–921 (Source: http://www.fourmilab.ch/etexts/einstein/specrel/www/ from The Principle of Relativity, 1923, Methuen and Company, Ltd., London).

[3] Fizeau, Des effets du mouvement sur le ton des vibrations sonores et sur la longeur d'onde des rayons de lumière, *Annales de Chimie et de Physique, 19*, 1870, 211-221.

[4] Law of Cosines is known in the present form for over 200 years. In an earlier form, prior to the invention of cosines and modern Algebra, it was known for over 2 millennia from Euclid's Elements (Source: https://en.wikipedia.org/wiki/Law_of_cosines). It should also be noted that this is an inappropriate use, since Law of Cosines is intended for triangles and not for direction angles where the angle relation is different. Still, this convention yielded correct answers in all cases, but caution is needed in such uses.

[5] All equations come with conditions of use, limits and limitations. Equations cannot be randomly used without these considerations. Similarly, all measurements involve errors, calibrations and standards. These are science foundations.

[6] Equation (1) can also be derived readily from vector operations. It is not shown here since vector model is also used to prove Law of Cosines, they are inter-related. It should be noted that even though often C or V or both are claimed as vectors, almost never vector operations are used in LF derivations, instead simple scalar operations without angle are used. Though vector model can work even better, since vector operations are not as common, the much simpler Law of Cosines is maintained here.

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[8] Length Contraction has several problems: i) it was simply made up to force desired result on a calculation; ii) the LF used here is derived from θ = 90° condition and cannot be used liberally at other angles; iii) no angular dependency is included in the equation, other than to simply state a condition of direction; iv) such out of place use also ignores the fact that except in θ = 90°, there will be asymmetry in all other angles, similar to DE. If there is length contraction in θ, then obviously there can be length elongation in the opposite (180-θ) direction. But no thought was given on these angular differences and the consequences. The need for length contraction simply means that LF does not work for θ ≠ 90°.

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[11] For example, L.C. Tai, Electromagnetic theory (Jones and Bartlett, 2006, Ch. 10.21, p. 402–403 ff. ISBN 0-7637-3827-1).

[12] It should be noted that sometimes v*Cos(θ) is used. This approximation can be derived from Law of Cosines when $V_a >> V_b$ by using Binomial or Taylor series expansion and limiting to first order.
$V_r = sqrt\{V_a^2 + V_b^2 - 2*V_a*V_b*\cos(\theta)\}$; if $V_b << V_a$, V_b^2 can be ignored →
$V_r = sqrt\{V_a^2 - 2*V_a*V_b*\cos(\theta)\}$
$Vr = V_a*sqrt(1 - 2V_b*Cos(\theta)/V_a)$; Taylor series expansion and limiting to first order →
$Vr = V_a*(1 - V_b*Cos(\theta)/V_a) = V_a - V_b*Cos(\theta)$
So, when V_b is small, $V_r \approx V_a - V_b*Cos(\theta)$.

[13] R.J. Kennedy and E.M. Thorndike, Experimental Establishment of the Relativity of Time, *Phys. Rev. 42*, 1932, 400–418 (Source: http://www.edu-observatory.org/physics-faq/Relativity/SR/experiments.html).

[14] At low velocities both Classical and Relativistic effects become low and their difference also becomes low; C/(C-V) ≈ C/sqrt(C^2-V^2) ≈ C/(C+V) ≈ others ≈ 1 for angles 0, 90, 180 and θ° respectively. All the correction or distortion factors must converge on 1 when V=0; C/(C-V) = C/sqrt(C^2-V^2) = C/(C+V) = others = 1, with no meaning of angle at that point. This does not mean that one approximates the other, rather it means that all start to lose their differences and importance without which they cannot converge.