

A Study of Speed of Sound in Water

Anuj Kumar², P.P.Pathak² and N.Dass¹

¹Physics Department, College of Engineering Roorkee, Roorkee, zip-247667

²Physics Department, Gurukul Kangari Vishwavidyalaya, Haridwar

Abstract: In the present paper, two-state theory of water is applied to study the temperature dependence of speed of sound. The computed results are found in good agreement with experimental data. The theory also predicts a thermal maximum in speed of sound near 76^oC which matches with the experimental value.

Keyword: Water, speed of sound, Temperature and Two-state theory

I. Introduction

The study of speed of sound in liquids is an important property from theoretical as well from practical point of view due to the fact that it can provide some thermo-dynamical properties of liquids. The study of sound speed in water becomes still more important due to fact that it exhibits a maximum near 76^oC.

Some empirical relations [1-4] are available in literature to study the speed of sound in water as a function of temperature but no model or theory is available so far. Though, a model [5-6] is available to study the pressure dependence of speed of sound in liquids. Therefore, the aim of present paper is to study the temperature dependence of speed of sound in water based on two-state theory.

According to two-state theory[7], water is considered as a mixture of two species- one specie is called open-packed specie resembling with ice structure and the second is called close-packed specie resembling with un-hydrogen bonded water molecules. Both the species are in equilibrium at all temperatures. Mole fraction of open-packed specie will decrease with the rise in temperature due the braking of bonds while the mole fraction of close-packed specie will increase.

Some physical properties of water studied successfully on two-state theory are (i) ultrasonic absorption [8], (ii) NMR shift [9], refractive index [10], dielectric constant [11] and viscosity [12]

II. Theory

To study the temperature dependence of speed of sound in water based on two-state theory, we write the relation as

$$V = X_0 V_0 + X_C V_C \quad (1)$$

Where X_0 and V_0 represent the mole fraction and speed of sound of open-packed specie, respectively, whereas X_C and V_C are the same quantities for close-packed specie. Further, to obtain the values of V_0 and V_C , we write the relations as

$$V_0 = A + (B/T) \quad (2)$$

$$V_C = C + (D/T) \quad (3)$$

Where A, B, C and D are temperature independent parameters and T is in K. Thus, eq. (1) can be expressed as

$$V = X_0 [A + (B/T)] + (1 - X_0) [C + (D/T)] \quad (4)$$

$$\text{Where } X_0 + X_C = 1 \quad (5)$$

It is clear that once the values of X_0 and T become known, one can compute the values of speed of sound in water by obtaining the values of A, B, C and D by least square fitting.

But, Davis and Litovitz [7] have reported the values of X_0 in the temperature range of 0- 100 ^oC at an interval of 10 ^oC. But, the values of X_0 are required at the interval of 5 ^oC to study the temperature dependence of speed of sound in water. Therefore, an empirical relation is developed to study the temperature dependence of X_0 as

$$X_0 = 0.637 - 0.6668[1 - \exp(-6.8328 \times 10^{-3} t)] \quad (6)$$

Where t is in ^oC. The standard deviation obtained is 0.0016.

III. Calculations and Discussions

Knowing the values of X_0 and T, eq(4) is used to obtain the values of A, B, C and D by least square fit. The values of A, B, C and D along with the values of V, V_0 and V_C are reported in Table 2. A comparison between the computed values and the experimental data is also reported in Fig 1. It is evident from Table 2 and Fig 1 that the computed values of speed of sound in water exhibits a thermal maximum near 75 ^oC whereas the

experimental value of thermal maximum is 76 °C. Thus, it may be said that a very good agreement exists between the computed values and the experimental data of speed of sound in water as the standard deviation obtained is 0.75 m/s.

Further, it is evident from Table 2 that the value of V_0 is increasing whereas the value of V_C is decreasing with rise in temperature and this might has given a maximum.

Table 1 Comparison of values of X_0 with temperature in water

Temp. (°C)	X_0 [7]	X_0 Calc.	Temp. (°C)	X_0 [7]	X_0 Calc.
0	0.637	0.637	55	----	0.428
5	----	0.615	60	0.411	0.143
10	0.595	0.593	65	---	0.398
15	----	0.572	70	0.382	0.383
20	0.554	0.552	75	---	0.370
25	---	0.532	80	0.355	0.359
30	0.515	0.513	85	----	0.343
35	----	0.495	90	0.331	0.331
40	0.478	0.477	95	----	0.319
45	----	0.460	100	0.309	0.307
50	0.443	0.444	---	----	----

Table 2. Comparison of speed of sound (m/s) in water along with the values of V_0 and V_C

TEMP. (°C)	V_0	V_C	V(Calc.)	V (Exptl.)
0	1051.88	2017.80	1402.55	1402.74
5	1072.69	1991.52	1426.47	1426.60
10	1092.76	1966.18	1446.53	1447.59
15	1112.13	1941.71	1467.22	1466.25
20	1130.85	1918.08	1481.98	1482.66
25	1148.93	1895.24	1498.23	1497.00
30	1166.42	1873.15	1509.21	1509.44
35	1183.34	1851.78	1520.93	1520.12
40	1199.72	1831.10	1529.32	1529.18
45	1215.59	1811.06	1537.17	1536.72
50	1230.96	1791.65	1543.29	1542.87
55	1245.86	1772.82	1547.31	1547.70
60	1260.32	1754.57	1551.45	1551.30
65	1274.35	1736.85	1552.80	1553.76
70	1287.97	1719.65	1554.77	1555.12
75	1301.20	1702.94	1554.32	1555.45
80	1314.05	1686.71	1554.43	1554.81
85	1326.55	1670.93	1552.82	1553.25
90	1338.70	1655.58	1550.71	1550.79
95	1352.45	1640.65	1548.12	1547.50
100	1362.02	1626.13	1544.53	1543.41

A= 2.2087e+03 B= -3.1581e+05
 C= 5.5685e+02 D= 3.9884e+05

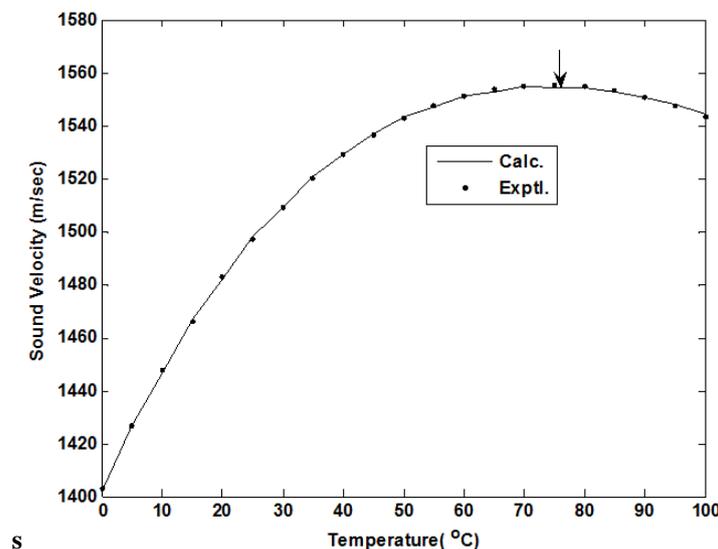


Fig. 1: Variation of sound velocity with temperature

References

- [1] G.W. Willard, Temperature coefficient of ultrasonic velocity in solutions, *J. Acous. Soc. Am.* 19, 235 (1947)
- [2] R.A. McConell and W.F. Mruk, Microacoustic interferometer using 30-MC Pulses, *J. Acous. Soc. Am.* 27, 672 (1955)
- [3] M. Greenspan and E. Tschiegg, Tables of speed of sound in water, *J. Acous. Soc. Am.* 31, 75 (1959)
- [4] A.J. Barlow and E. Yazgan, *Brit. J. Appl. Phys.* 18, 645 (1967)
- [5] R. Kumar, P. Kuchhal and N. Dass, The pressure and temperature dependence of the velocity of sound in liquid metals, *J. Phys.: Condens. Matter* 8, 10891 (1996)
- [6] A. Kumar, P. Kuchhal, N. Dass and P.P. Pathak, Anomalous behavior in sound velocity of water, *Phys Chem Liquids* 49, 453 (2011)
- [7] C.M. Davis and T.A. Litovitz, Two-state theory of the structure of water, *J. Chem. Phys.* 42, 2563 (1965)
- [8] L. Hall, The origin of ultrasonic absorption in water, *Phys. Rev.* 73, 775 (1948)
- [9] N. Muller, Concerning structural models for water and chemical shift data, *J. Chem. Phys.* 43, 2555 (1965)
- [10] S.K. Mitra, N. Dass and N.C. Varshneya, Temperature dependence of the refractive index of water, *J. Chem. Phys.* 57, 1798, 1972.
- [11] N. Dass, Temperature dependence of dielectric constant in light and heavy water, *Phys. Chem. Liquids* 15, 323 (1986)
- [12] C.H. Cho, J. Urquidi and G.W. Robinson, Molecular-level description of temperature and pressure effects on the viscosity of water, *J. Chem. Phys.* 111, 10171 (1999).