

Varying Gravitational Constant under Special Case

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Abstract: In this paper I worked on the special case for studying the behavior of the universal gravitational constant by merging both Newtonian and relativistic mechanics. By that certain case I found that the value of the gravitational constant i.e. $6.67 \times 10^{-11} \text{Nm}^2/\text{Kg}^2$ is not actually a constant but vary with some conditions and reaches to zero at the speed of light.

Keyword: Force, Lorentz transformation, Photon, Special relativity, speed of light, etc.

I. Introduction

In this paper we have mainly focused on the special case designed for the analysis of the universal gravitational constant in different frames of reference. For considering gravitational constant we used the formula for gravitational force between two bodies in accordance with Newton's gravitational law. The primary key to our work began as the introduction of Einstein's special theory of relativity in Newtonian mechanics.

II. Theory And Formulation

We all know, by Newton's gravitational law, the force between the two bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between their center of masses.

i.e.
$$F = \frac{Gm_1m_2}{R^2}$$

Where G is the gravitational constant. By the results obtained in special theory of relativity, as the object moves faster, its mass increases. This is shown by the Lorentz factor, as

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

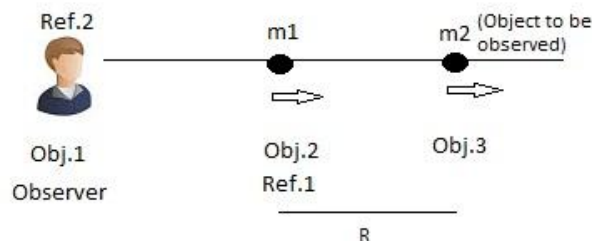
Where m_0 is the rest mass of that object (i.e. mass when its velocity is zero).

Also with increasing velocity the length of the object decreases in the direction of velocity.

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

Now considering an ideal case, in the universe we have only three objects, which are collinear. Let the object-1 be the observer in all the cases we are discussing.

III. Diagram



As shown in above figure, we have obj.1, obj.2 and obj.3. Obj.2 and obj.3 are moving with constant and same velocity towards right in reference of obj.1

When we think about the observation by the observer with respect to itself we can consider obj.2 and obj.3 as a whole system such that the distance between them 'R' and the whole system is moving with velocity 'v' towards right.

Case1: When observer thinks about the force of gravitational attraction between obj.2 and obj.3 while considering ref.1, the relative velocity of obj.2 and obj.3 is zero. So then we consider their masses at rest. Let it be as m_1 for obj.2 and m_2 for obj.3. for observer.

$$F = \frac{Gm_1m_2}{R^2}$$

Case2: But the same observer when thinks while keeping itself as a reference i.e. considering reference-2, the obj.2 and obj.3 both are moving with velocity ‘v’ away from it. As we have stated earlier by special theory of relativity the distance between obj.2 and obj.3 will shrink and their respective masses will increase such that,

$$m_1' = \frac{m_1}{\sqrt{1-\frac{v^2}{c^2}}} \text{ and } m_2' = \frac{m_2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$R' = R \sqrt{1 - \frac{v^2}{c^2}}$$

Therefore the force of gravitation observed by the observer in this case will be,

$$F' = \frac{G' m_1' m_2'}{R'^2}$$

i.e.

$$F' = \frac{G' m_1 m_2}{R^2 \left(1 - \frac{v^2}{c^2}\right)^2}$$

$$F' = \frac{G' m_1 m_2}{R^2 \left(1 - 2\frac{v^2}{c^2} + \frac{v^4}{c^4}\right)}$$

-2

From these observations we are getting two different magnitudes of forces. But the forces between obj.2 and obj.3 must be same in all the cases. Therefore equating equation 1 and equation 2,

$$F = F'$$

$$G' = G \left(1 - 2\frac{v^2}{c^2} + \frac{v^4}{c^4}\right)$$

So the variable for this adjustment must be ‘G’. i.e. universal gravitational constant.

Here we get the relation between gravitational constant in both the cases as

$$G' = G \left(1 - 2\frac{v^2}{c^2} + \frac{v^4}{c^4}\right)$$

This simply indicates that the value of G decreases more rapidly as the objects keeps some velocity with reference of observer itself. Simply saying, with more and more velocity the value of G decreases more rapidly than the increase in mass and decrease in length happens.

IV. Conclusion

From above conditions and special cases, the value of G which we generally consider constant is not actually constant but decreases very rapidly such that when the object approaches to the speed of light, the G tends to zero. Which also indicates that the particle nature of light i.e. Photon do not attract each other through the force of gravity since G tends to 0. So finally we conclude that, the value of gravitational constant is not an actual constant but a variable.

Reference

- [1] Relativity: The Special And The General Theory
- [2] The Feynman Lectures On Physics