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Optimal Tarzan Jump from Rigid Rod

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Abstract : *We study the optimal Tarzan jump from massive rigid rod. Since a part of the initial potential energy of the rod can be changed to the kinetic energy of Tarzan, he can jump further than that from massless rope.We find that his flying distance can be arbitrarily large depending on mass and density distribution of the rod. This study will be suitable for students of introductory calculus-based physics class.*

Keywords: *Classical Mechanics, Physics Education, Tarzan's Dilemma*

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I. Introduction

It has been discussed a problem called ``Tarzan's Dilemma" in an educational point of view for students of introductory physics course [1-5].Tarzan, holding onto a rope, starts swinging due to gravity and releases the rope at a point. The problem is to find the releasing point to jump furthest and to find its largest distance.This problem can be solved by making use of equations for projectile motion and the energy conservation law of Tarzan.

In the previous works of Refs.[1-5], Tarzan swings with massless rope. However if the rope is massive (we assume it to be a rigid rod), the initial potential energy of the rod can be changed to the kinetic energy of Tarzan, and he can jump further than that from massless rope.

In this paper, we discuss the optimal Tarzan jump from rigid rod with mass M. First we solve the problem with the rod of uniform density, and find that Tarzan can reach further as expected. Next we discuss function form of density distribution of the rod. We find that if the rod is much heavier than Tarzan, and if most of mass of the rod is located near the axis of swing, Tarzan can reach a place arbitrarily far away.

Calculations and qualitative discussions in this paper are suitable for students of introductory calculus-based physics class.

II. Tarzan Jump

A rigid rod of lengthℓ and mass$M$ with uniform density $\rho = M/\ell$ is fixed at a point of height$H$ above the ground.Tarzan of mass m starts swinging due to gravity from point A with an angle θ_A as shown in Figure 1. He passes the lowest point B and launches from point C at time $t = 0$ with launching angle θ_C and speed v_C. Finally he lands on the ground at point D at $t = t_D$. Here we calculate the distance L, the horizontal distance between point B to D, and the problem is to find the conditions to maximize L.

After releasing the rod at point C, the position of Tarzan of $0 \leq t \leq t_D$ is represented by

$$x(t) = \ell \sin\theta_C + v_C t \cos\theta_C, \tag{2.1}$$

$$y(t) = H - \ell\cos\theta_C + v_C t \sin\theta_C - \frac{1}{2}gt^2, \tag{2.2}$$

where$g$is gravitational acceleration. Since Tarzan lands on the ground at $t = t_D$, $x(t_D) = L$ and $y(t_D) = 0$are satisfied. The energy conservation law between point A and C is given by

$$\begin{aligned} &mg(H - \ell\cos\theta_A) + Mg\left(H - \frac{1}{2}\ell\cos\theta_A\right) \\ &= \frac{1}{2}mv_C^2 + mg(H - \ell\cos\theta_C) + \frac{1}{2}I\omega_C^2 + Mg\left(H - \frac{1}{2}\ell\cos\theta_C\right), \end{aligned} \tag{2.3}$$

where$I = M\ell^2/3$and $\omega_C = v_C/\ell$are the moment of inertia and angular velocity at point C, respectively. Since we are assuming the rod to have uniform density, the potential energy of the rod is concentrated at the center of gravity, $\ell/2$ distant from the axis of rotation. From Eqs.(2.1), (2.2) and (2.3) at $t = t_D$, one obtains the launching speed

$$v_C^2 = 2\mu g\ell\Delta_2, \tag{2.4}$$

and the distance

$$L = \ell\sin\theta_C + \mu\ell\Delta_2\sin 2\theta_C + 2\mu\ell\cos\theta_C\sqrt{\Delta_2\left(\Delta_2\sin^2\theta_C + \frac{1}{\mu}\Delta_1\right)}, \tag{2.5}$$

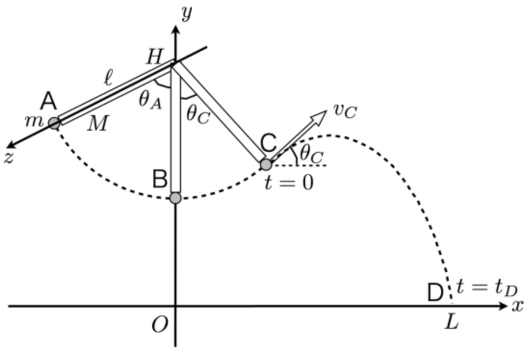

Figure1: Schematic diagram of Tarzan Jump with rigid rod.

where

$$\Delta_1 = \frac{H}{\ell} - \cos\theta_C\,, \qquad \Delta_2 = \cos\theta_C - \cos\theta_A\,, \tag{2.6}$$

are defined as with Ref. [2], which corresponds to the vertical height of point C and the difference in height between point A and C, respectively. In Eqs. (2.4) and (2.5), we have introduced a new dimensionless parameter

$$\mu = \frac{m + \frac{1}{2}M}{m + \frac{1}{3}M}, \tag{2.7}$$

in order to represent the effect of the rigid rod. The factor 1/2 in numerator and 1/3 in denominator comes from the position of the center of gravity and moment of inertia of the rod, respectively. The parameter μhas a value of $1 \le \mu \le 3/2$, corresponding to massless ($M = 0$) and extremely heavy ($M \gg m$) rod.

Figure 2 shows the distance L as a function of the launching angle θ_C for two limiting cases$\mu = 1$(black solid curve) and 1.5 (red dashed curve) in the case of $\theta_A = 90°$, $\ell = 5.0$ m, and $H = 10$ m. The case of $\mu = 1.5$ always gives larger distance L than that of $\mu = 1$ case independent of the launching angle for$0 \le \theta_C < 90°$, because a part of the initial potential energy of the rod changes to the kinetic energy of Tarzan. In particular for $\theta_C = 0$, one can see that $L(\mu = 1.5)/L(\mu = 1)|_{\theta_C = 0} = \sqrt{1.5/1} \simeq 1.22$. Although the optimal launching angle $\theta_{C\max}$ seems to be around 33° for both cases, it actually depends on μ. We discuss the μ dependence of$\theta_{C\max}$ in the followings.

As written above, the form of μ in Eq. (2.7) comes from the assumption that the rod has uniform density. Here we discuss the possibility of different form of μ by extending the density distribution of the rod, and discuss the μ dependence of$\theta_{C\max}$.

Let us consider the density function $\rho(z)$, where $z$axis measures from the rotational axis ($z = 0$) to Tarzan ($z = \ell$) along the rod (See Figure 1). The mass of the rod is calculated by

$$M = \int_0^\ell \rho(z)\,dz, \tag{2.8}$$

and we parametrize the center of gravity $z_G$as

$$z_G = \frac{1}{M}\int_0^\ell \rho(z) z dz = a\ell, \tag{2.9}$$

and the moment of inertia I as

$$I = \int_0^\ell \rho(z) z^2 dz = bM\ell^2. \tag{2.10}$$

In the previous case of uniform density $\rho = M/\ell$, Eqs. (2.9) and (2.10) gives $a = 1/2$ and $b = 1/3$, respectively. In general, μ has the form

$$\mu = \frac{m + aM}{m + bM}. \tag{2.11}$$

When $a > b$, μ has the maximal value $\mu_{\max} = a/b$ for $M \gg m$. The problem is to find $\rho(z)$ to improve the distance L. As practices for students, we consider two example functions; $\rho(z) \propto z^n$ and $(\ell - z)^n$for $n \ge 0$.

First we consider the case of $\rho(z) \propto z^n$. In this case the rod is denser near Tarzan. One obtains

$$\rho(z) = \frac{n+1}{\ell^{n+1}} M z^n, \tag{2.12}$$

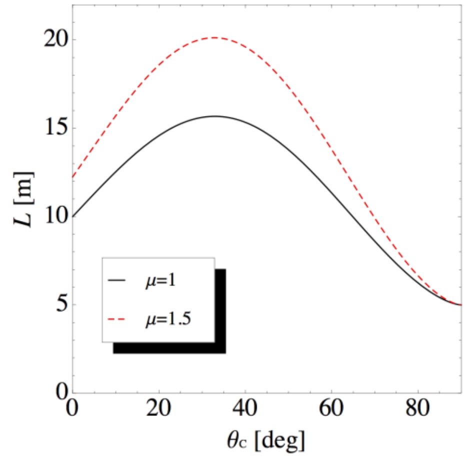

Figure2:The distance L vs. launching angle θ_C in the case of$\theta_A = 90°$, $\ell = 5.0$ m, and $H = 10$ m.

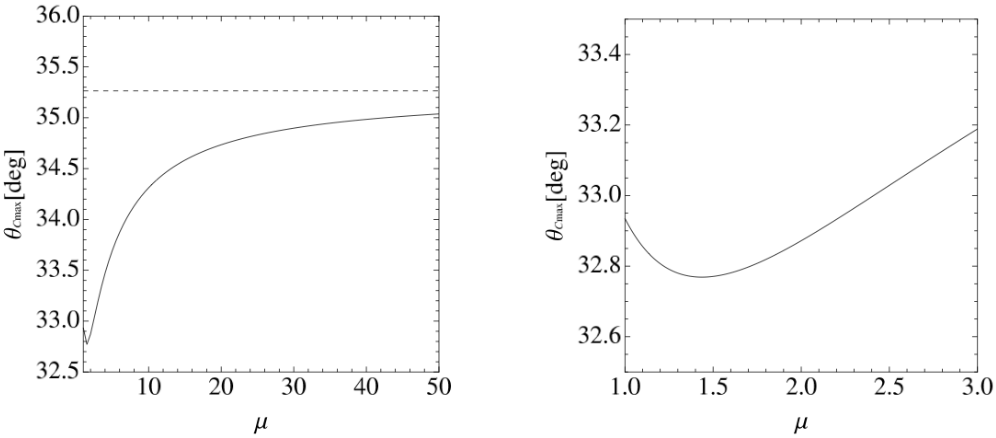

Figure3: The relation between the optimal launching angle$\theta_{C\max}$and μ (left) and its close-upview near the minimum (right). For both panels,$\theta_A = 90°$, $\ell = 5.0$ m, and $H = 10$ m. Dashed line in the left panelindicates the asymptotic line $\theta_{C\max} = 35.26°$.

from Eq. (2.8), and

$$a = \frac{n+1}{n+2}, \quad b = \frac{n+1}{n+3}, \tag{2.13}$$

fromEqs. (2.9) and (2.10). Therefore μ has the maximal value

$$\mu_{\max} = \frac{n+3}{n+2}. \tag{2.14}$$

In this case, $\mu_{\max}$ has the largest value $3/2$ for $n = 0$, which corresponds to the case of uniform density. On the other hand, for the case of $\rho(z) \propto (\ell - z)^n$ (the rod is denser near the rotational axis), one obtains

$$\rho(z) = \frac{n+1}{\ell^{n+1}} M(\ell - z)^n, \tag{2.15}$$

and

$$a = \frac{1}{n+2}, \quad b = \frac{2}{(n+2)(n+3)}, \tag{2.16}$$

in a similar fashion. As a result, we obtain

$$\mu_{\max} = \frac{n+3}{2}. \tag{2.17}$$

This tells us thatμ can be arbitrarily large for large$n$.
From these two examples, one finds that the flying distance can be improved if *i)* the rod is extremely heavy, and *ii)* the heavier end of the rod is fixed at the rotational axis.

Figure 3 shows the relation between the optimal launching angle$\theta_{C\max}$ and μin the case of$\theta_A = 90°$, $\ell = 5.0$ m, and $H = 10$ m. The right panel is a close-up view near the minimum of the left panel. For $\mu \gg 1$, Eq. (2.5) has the approximate form

$$L \simeq 2\mu\ell\Delta_2 \sin 2\theta_C, \tag{2.18}$$

and its maximum condition gives an equation of the asymptotic line,

$$3\cos^3\theta_{C\max} - 2\cos\theta_A \cos^2\theta_{C\max} - 2\cos\theta_{C\max} + \cos\theta_A = 0. \quad (2.19)$$

For the case of $\theta_A = 90°$, Eq. (2.19) gives

$$\theta_{C\max} = \cos^{-1}\sqrt{\frac{2}{3}} = 35.26°, \quad (2.20)$$

which is the asymptotic line of Figure 3 (left).

III. Conclusions

We discussed the optimal Tarzan jump from a massive rigid rod. Since a part of the initial potential energy of the rod can be changed to the kinetic energy of Tarzan, he can reach further than the case of massless rope. Moreover, if the rod has a non-uniform density, the flying distance can be arbitrarily large depending on its density function. In particular, we found that if the rod is heavier than Tarzan and the heavier end of the rod is fixed at the rotational axis, the flying distance can be improved. Calculations and qualitative discussions in this paper are suitable for students of introductory calculus-based physics class.

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