

Modelling The Effect Of Production Cost In A Decentralized Channel Using Game Theory

Katule, Innocent Adinebo^a and Ezimadu, Peter Emeke^b

Department Of Mathematics, Delta State University, Abraka

Abstract

Some advertising models tend not to consider production cost by either using price margins or categorically being silent by excluding it from the model, probably to avoid landing at complex results. This research work considered the incorporation of the cost of production into a version of advertising model which does not dwell on production cost. It considered an advertising channel model setting involving a manufacturer and a retailer with the manufacturer as the Stackelberg game leader and the retailer, as the follower. The work considered two model settings: a setting that does not consider the manufacturer production cost and a setting involving the production cost. The work used the Stackelberg game theory to establish the players' optimal prices and advertising efforts for both settings. It compares the nature of the players' efforts and payoffs for both settings and show that; the retail price is inversely proportional to both advertising efforts; the retail advertising efforts for both cost and no-cost scenarios reduce with the manufacturer's wholesale price; as the manufacturer's effort increases, the retailer's payoff for both the no-production cost scenario and the production cost scenario increase continuously; and increase in retailer's advertising expenditure leads to continuous increase in the manufacturer's payoffs for both scenarios.

Keywords: *Stackelberg game, Advertising models, Supply chain, Decentralized channel, Pricing*

Date of Submission: 18-03-2024

Date of Acceptance: 28-03-2024

I. Introduction

For businesses, advertising is essential brand makers have been connecting with a large number of consumers on a national scale for the past few decades (Sridhar et al. 2016). This can assist brand manufacturers in driving price premiums (Sridhar et al. 2016), building brand knowledge and preference (Li et al. 2002), and raising perceived quality (Woodside and Taylor 1978). For instance, Apple Inc., a multinational technology firm based in the United States, spends about half of its budget on brand promotion to promote its benefits and features and strengthen the company's reputation. The era of the sharing economy has arrived for business operations. More than ever, businesses and individuals are thinking about sharing, including pooling their advertising funds to more efficiently and affordably boost the sales of their own goods (Karray and Sigué 2017). In dual-channel supply chains, a new cooperative advertising strategy involves brand manufacturers running nationwide advertisements for their online merchants, with the online retailers splitting the cost of the advertising. This can assist online retailers in using the manufacturer's well-known brand and reputation to spread information, foster confidence, and generate demand. This lessens consumer skepticism or mistrust regarding the workings of e-commerce, its opaque effects and procedures, and the caliber of many things sold on the internet (Grabner-Kraeuter 2002).

Local and national (global) advertising are two distinct categories of advertising, according to Huang & Li (2001). While local advertising aims to stimulate consumers' quick purchase behavior, national or global advertising primarily focuses on persuading future consumers to choose a specific brand and creating a brand preference. Numerous studies have been carried out in the joint area of pricing and advertising decisions. He et al. (2009) developed a new contract scheme in a two-tier supply chain when demand was influenced by both retail price and advertising costs. They suggested that achieving a win-win situation would be probable in the proposed situation, which included a combination of return policy and revenue sharing contract that embodied sales rebate and penalty. Szmerekovsky and Zhang (2009) studied pricing decisions in a manufacturer-retailer supply chain when demand was stochastic and influenced by both retail price and advertisement. The model was then analyzed via three games, including Stackelberg, Nash, and cooperative games. The results showed that local advertising costs in a two-tier distribution channel were insufficient to support decision-making. Alternatively, the manufacturer's national marketing combined with a wholesale price break for the store might produce the intended effects. Using the same methodology, Xie and Wei (2009) and Xie and Neyret (2009) examined the effects of altering price and advertising functions in terms of Stackelberg, Nash, and cooperative games. In order to ascertain the values of inventory and advertising choice factors in both cooperative and non-cooperative scenarios, Chen (2015) looked at the effects of advertisement with return policy for a sales problem

in a manufacturer-retailer supply chain. Seyed et al(2011) used four game scenarios; cooperative game, Nash equilibrium, Stackelberg manufacturer game, and Stackelberg retailer game, to examine the function of coordination in a two-echelon supply chain with advertising and price-dependent demand. The authors came to the conclusion that, in a cooperative setting, commercials would strategically influence important choices. The model underwent application of a non-linear demand function. Dridi and Ben (2015) took into account a supply chain with rival stores and a demand that was influenced by price as well as national advertising spending.

In explanation of the behavior of an advertising model based on problem related to a system of ordinary differential equations, Ungureanu (2011) used two real parameters to develop an advertising model of two first order ordinary differential equations. The work showed that a degenerate bifurcation phenomenon occurs for two values of the parameters. Considering online advertising Asedemir et al (2012) examined pricing model. They studied input-based cost per thousand impressions and performance-based cost per click-through which are two very popular pricing models. They showed that four factors provide insight on preferable online-advertising pricing model. Another advertising situation based on internet setting was considered by Wu and Tan (2023). They developed decision models on profit maximization on an internet platform, and analysed the optimal level advertising volume, product price and promotion strategies. They showed that the platform is capable of influencing advertising volume, product price and product marked demand.

Cosgun et al (2013) constructed a model on advertising time reservation and considered an extension on the model for some real life situations. They discussed how these real life situations taken by a television network, and proposed the adoption of mixed integer linear programming method for solving these problems. In a study of the advertising strategies employed by firms when they are faced with consumers who have limited attention Liu (2022) considered a situation under competition in which a firms advertisement can affect may engage less in advertising in comparison to monopolist. Cheng et al (2023) incorporated advertising frequency and company's cost of managing a relationship with its customers into product demand function. They developed a maximization model to aid analyzing joint marketing policy. The work constructed an algorithm for optimal decision taking.

The importance of production cost in profit or payoff functions cannot be overemphasized. Some works do not categorically consider production cost (Xie and Wei, 2009; Ezimadu, 2019a;). This probably due to the possibility of obtaining problems which may be poss difficulty. Further, there are others which bypass the use of production cost by considering price margin instead of actual prices (He et al 2011; He et al 2014; Ezimadu and Nwozo, 2018; Ezimadu 2022). Thus we note that there appears to be three schools of thought on inclusion of production cost: those who do not consider it necessary to use production cost in profit functions; those who do not consider appropriate to lump it in price margin; and those who think it is to categorically include it in models (He et al, 2009; Ezimadu 2019b); It is a fact that problems with fewer number of variables are less complex, while those with more variables are more complex, so that the incorporation of production cost possess more complex problem. This work aligns with the school of thought that consider it necessary to incorporate the production cost. Thus we consider an extension of Xie and Wei (2009) with the incorporation of production cost.

In this work, we develop two models in which channel members' cooperative advertising efforts and retail prices drive customer demand. Our focus is limited to the conventional scenario of a bilateral monopoly model, wherein a single manufacturer sells goods through a single retailer. By concentrating on static models, we can create analytical answers and insights for important variables such as the wholesale and retail prices set by the manufacturer, the amount each channel member spends on advertising, and the retailer is the follower. The work is organized as follows: the basic structure of the game-theoretic model and its assumptions are presented in the following section. Next, two models - one based on a cooperative game, in which the retailer is the follower and the manufacturer is the leader - are described. The primary findings of these two models are examined and contrasted, with production cost equal to zero, as well as greater than zero. Ultimately, the conclusion provides a summary of the results and suggests further lines of inquiry.

II. The Market Structure

Considering a single-manufacturer-single-retailer channel, where the retailer sells only the brand of the manufacturer. The advertising expenditure such as (manufacturer's wholesale price and the retailer's retail price) together with the manufacturer's participation rate are the decision variables for the channel members. α_m and α_r shall denote the variables for the manufacturer's national advertising expenditure and the retailer's local advertising expenditure respectively. The consumer demand $C(p_r, \alpha_r, \alpha_m)$ depends on the retail price and the advertising levels α_r and α_m as:

$$C(p_r, \alpha_r, \alpha_m) = D(p_r)f(\alpha_r, \alpha_m), \tag{1}$$

where $D(p_r)$ reflects the impact of the retail price on the demand and $f(\alpha_r, \alpha_m)$ reflects the impact of the advertising expenditures on the demand. (Jeuland and Shugan, 1988; Weng, 1995) opined that $D(p_r)$ is

linearly decreasing with respect to p_r while (Kuehn, 1962, Thompson and Teng, 1984, Jorgensen and Zaccour, 1999, 2003; Yue et al., 2006) reviewed that using a multiplication effect by price and advertising, we assume:

$$D(p_r) = 1 - \theta p_r, \tag{2}$$

where θ is a positive constant. To be noted is the fact that the maximum value for $D(p_r)$ is normalized to be 1, for simplicity. Also, to ensure $D(p_r) > 0$, we restrict $p_r < \frac{1}{\theta}$.

We shall model the advertising effects on consumer demand as follows:

$$f(\alpha_r, \alpha_m) = \beta_r \sqrt{\alpha_r} + \beta_m \sqrt{\alpha_m}, \tag{3}$$

where β_r and β_m are positive constants reflecting the efficiency of each type of advertising in generating sales. Equation (3) captures both types of advertising which often are not substitutes. The demand in (3) is an increasing and concave function with respect to α_r and α_m with a property that is in line with the commonly observed advertising saturation effect viz, more spending in advertising, gives rise to constant diminishing returns. Simon and Arndt (1980) argued that diminishing returns characterize the shape of the advertising-sales response function.

From (1) – (3), we have

$$C(p_r, \alpha_r, \alpha_m) = (1 - \theta p_r)(\beta_r \sqrt{\alpha_r} + \beta_m \sqrt{\alpha_m}). \tag{4}$$

We let p_m denote the manufacturer's wholesale price to the retailer.

In this research work, we wish to extend the work of Xie and Wei (2009), which modelled a manufacturer and retailer payoff respectively by:

$$\pi_m = p_m(1 - \theta p_r)(\beta_r \sqrt{\alpha_r} + \beta_m \sqrt{\alpha_m}) - t\alpha_r - \alpha_m \tag{5}$$

and

$$\pi_r = (p_r - p_m)(1 - \theta p_r)(\beta_r \sqrt{\alpha_r} + \beta_m \sqrt{\alpha_m}) - (1 - t)\alpha_r \tag{6}$$

respectively.

In this formulation, they did not consider the cost of production. By incorporating the cost of production into the above model in equations (5) and (6), together with the fact that this work considers a situation where subsidy is not provided by the manufacturer. We have that the manufacturer and retailer payoffs are as given.

$$\pi_m = (p_m - p_c)(1 - \theta p_r)(\beta_r \sqrt{\alpha_r} + \beta_m \sqrt{\alpha_m}) - \alpha_m \tag{7}$$

and

$$\pi_r = (p_r - p_m)(1 - \theta p_r)(\beta_r \sqrt{\alpha_r} + \beta_m \sqrt{\alpha_m}) - \alpha_r \tag{8}$$

Respectively, where p_c is the cost of production, α_m and p_m are the manufacturer's decision variables where $0 < p_c < p_m < p_r < \frac{1}{\theta}$

α_r and α_m may take any non-negative real values.

III. The Players' Control Strategies

The decision process shall be modelled as a sequential non-cooperative game with the manufacturer as the leader and the retailer as the follower. The Stackelberg equilibrium is the solution of the leader-follower game. The Stackelberg equilibrium is determined by backward substitution. The retailer's optimal problem is first solved, after the manufacturer's decision variables α_m , p_m and t have been declared.

The Retailer's Strategies

To determine the nature (form) of the retailer's strategies, we maximize (8)

subject to

$$0 < p_r < \frac{1}{\theta} \text{ and } \alpha_r > 0$$

Expanding (8), gives:

$$\pi_r = p_r \beta_r \sqrt{\alpha_r} + p_r \beta_m \sqrt{\alpha_m} - \theta p_r^2 \beta_r \sqrt{\alpha_r} - \theta p_r^2 \beta_m \sqrt{\alpha_m} - p_m \beta_r \sqrt{\alpha_r} - p_m \beta_m \sqrt{\alpha_m} + \theta p_m p_r \beta_r \sqrt{\alpha_r} + \theta p_m p_r \beta_m \sqrt{\alpha_m} - \alpha_r. \tag{9}$$

We note that (9) is concave on α_r and p_r . Thus, carrying out the stated maximization, we have that.

$$\frac{\partial \pi_r}{\partial p_r} = \beta_r \sqrt{\alpha_r} + \beta_m \sqrt{\alpha_m} - 2\theta p_r \beta_r \sqrt{\alpha_r} - 2\theta p_r \beta_m \sqrt{\alpha_m} + \theta p_m \beta_r \sqrt{\alpha_r} + \theta p_r \beta_m \sqrt{\alpha_m} = 0$$

$$\Rightarrow 1 - 2\theta p_r + \theta p_m = 0,$$

$$\Rightarrow p_r = \frac{1 + \theta p_m}{2\theta}. \tag{10}$$

From (10) we see the retailer's best response for setting a retail price (p_r) is a linearly increasing function of the manufacturer's wholesale price (p_m), but does not depend on either the manufacturer's advertising expenditure α_m or the participation rate t for subsidy for the retailer. A look at (10), clearly shows the retailer's best strategy for level of advertising (α_r) decreases as the manufacturer's wholesale price (p_m)

increases and increases, no doubt as the manufacturer's participation rate increases. Local advertising does not depend on the manufacturer's advertising expenditure α_r .

Also,

$$\frac{\partial \pi_r}{\partial \alpha_r} = \frac{p_r \beta_r}{2\sqrt{\alpha_r}} - \frac{\theta p_r^2 \beta_r}{2\sqrt{\alpha_r}} - \frac{p_m \beta_r}{2\sqrt{\alpha_r}} - \frac{p_m \beta_r}{2\sqrt{\alpha_r}} + \frac{\theta p_m p_r \beta_r}{2\sqrt{\alpha_r}} - 1 = 0$$

$$\Rightarrow \frac{p_r \beta_r - \theta p_r^2 \beta_r - p_m \beta_r + \theta p_m p_r \beta_r}{2\sqrt{\alpha_r}} = 1$$

$$\Rightarrow \alpha_r = \frac{\beta_r^2 (p_r - p_m)^2 (1 - \theta p_r)^2}{4} \tag{11}$$

Substituting (10) into (11), we have

$$\alpha_r = \frac{(1 - \theta p_m)^4 \beta_r^2}{64 \theta^2} \tag{12}$$

Equation (12) demonstrates that as the manufacturer's wholesale price p_m rises, the retailer's best response for local advertising level α_r lowers. The ideal level of local advertising is independent of the cost of advertising incurred by the manufacturer α_m . Next, maximizing is used to determine the ideal values of α_r and p_m the ideal issue for the manufacturer.

Manufacturer's Strategies with Production Cost

To find the optimal values for β_r and p_m we would maximize the manufacturer's profit function.

Thus, substituting (10) and (11) into (7), we have

$$\pi_m = \frac{p_m^2 \beta_r^2 p_r^2 \theta^2}{4} + \frac{p_c p_r p_m^2 \beta_r^2 \theta^2}{4} - \frac{p_c p_m \beta_r^2 p_r^2 \theta^2}{4} - \frac{p_r \beta_r^2 \theta^2 p_m^3}{4} + \frac{p_c \theta \beta_r^2 p_r^2}{4} - \frac{p_m \theta \beta_r^2 p_r^2}{4} - \frac{p_c \theta p_m^2 \beta_r^2}{4} + \frac{\theta \beta_r^2 p_m^3}{4} - \frac{p_m^2 \beta_r^2}{4} + \frac{\sqrt{\alpha_m} \beta_m p_c p_m \theta}{2} - \frac{\sqrt{\alpha_m} \beta_m \theta p_m^2}{2} + \frac{p_m p_r \beta_r^2}{4} + \frac{p_c p_m \beta_r^2}{4} - \frac{p_c p_r \beta_r^2}{4} + \frac{\sqrt{\alpha_m} \beta_m p_m}{2} - \frac{\sqrt{\alpha_m} \beta_m p_c}{2} - \alpha_m \tag{13}$$

Maximizing (13) with respect to p_m

$$\frac{\partial \pi_m}{\partial p_m} = \frac{p_c p_m p_r \beta_r^2 \theta^2}{2} + \frac{p_m \beta_r^2 p_r^2 \theta^2}{2} - \frac{p_c \beta_r^2 p_r^2 \theta^2}{4} - \frac{3 p_r p_m^2 \beta_r^2 \theta^2}{4} - \frac{p_c p_m \theta \beta_r^2}{2} - \frac{\theta \beta_r^2 p_r^2}{4} + \frac{3 \theta p_m^2 \beta_r^2}{4} + \frac{\sqrt{\alpha_m} \beta_m p_c \theta}{2} - \sqrt{\alpha_m} \beta_m p_m \theta + \frac{p_r \beta_r^2}{4} + \frac{p_c \beta_r^2}{4} - \frac{p_m \beta_r^2}{2} + \frac{\sqrt{\alpha_m} \beta_m}{2} = 0 \tag{14}$$

Solving (14) for p_m , gives;

$$p_m = \frac{2 \left(\sqrt{(3 p_r \beta_r^2 \theta^2 - 3 \theta \beta_r^2) \left(-\frac{\theta \beta_r^2 p_r^2}{4} + \frac{p_r \beta_r^2}{4} + \frac{\sqrt{\alpha_m} \beta_m}{2} \right)} + \left(\frac{\beta_r^2 p_r^2 \theta^2}{2} - \sqrt{\alpha_m} \beta_m \theta - \frac{\beta_r^2}{2} \right)^2 - \frac{\beta_r^2 p_r^2 \theta^2}{2} + \sqrt{\alpha_m} \beta_m \theta + \frac{\beta_r^2}{2} \right)}{-3 p_r \beta_r^2 \theta^2 + 3 \theta \beta_r^2} \tag{15}$$

or

$$p_m = \frac{2 \left(-\sqrt{(3 p_r \beta_r^2 \theta^2 - 3 \theta \beta_r^2) \left(-\frac{\theta \beta_r^2 p_r^2}{4} + \frac{p_r \beta_r^2}{4} + \frac{\sqrt{\alpha_m} \beta_m}{2} \right)} + \left(\frac{\beta_r^2 p_r^2 \theta^2}{2} - \sqrt{\alpha_m} \beta_m \theta - \frac{\beta_r^2}{2} \right)^2 - \frac{\beta_r^2 p_r^2 \theta^2}{2} + \sqrt{\alpha_m} \beta_m \theta + \frac{\beta_r^2}{2} \right)}{-3 p_r \beta_r^2 \theta^2 + 3 \theta \beta_r^2} \tag{16}$$

Maximizing (7) with respect to α_m

$$\frac{\partial \pi_m}{\partial \alpha_m} = \frac{1}{2} \frac{p_m \beta_m}{\sqrt{\alpha_m}} - \frac{1}{4} \frac{(\theta p_m + 1) p_m \beta_m}{\sqrt{\alpha_m}} - \frac{1}{2} \frac{p_c \beta_m}{\sqrt{\alpha_m}} + \frac{1}{4} \frac{(\theta p_m + 1) p_c \beta_m}{\sqrt{\alpha_m}} - 1 = 0 \tag{17}$$

$$\Rightarrow \alpha_m = \frac{(p_m - p_c)^2 (1 - \theta p_m)^2 \beta_m^2}{16} \tag{18}$$

Equation (18) reveals that the manufacturer's optimal response for national advertising level α_m increases as the manufacturer's wholesale price p_m increases and the cost of production increases.

Manufacturer's Strategies without Production Cost

Considering a situation without production cost and subsidy, (7) becomes

$$\pi_m = p_m (1 - \theta p_r) (\beta_r \sqrt{\alpha_r} + \beta_m \sqrt{\alpha_m}) - \alpha_m \tag{19}$$

Substituting (10) and (11) into (19), gives

$$\pi_m = \frac{p_m^2 \beta_r^2 p_r^2 \theta^2}{4} - \frac{p_r \beta_r^2 \theta^2 p_m^3}{4} - \frac{p_m \theta \beta_r^2 p_r^2}{4} + \frac{\theta \beta_r^2 p_m^3}{4} - \frac{p_m^2 \beta_r^2}{4} - \frac{\sqrt{\alpha_m} \beta_m \theta p_m^2}{2} + \frac{p_m p_r \beta_r^2}{4} + \frac{\sqrt{\alpha_m} \beta_m p_m}{2} - \alpha_m$$

(20)

Maximizing (20) with respect to p_m gives;

$$\frac{\partial \pi_m}{\partial p_m} = \frac{p_m \beta_r^2 p_r^2 \theta^2}{2} - \frac{3 p_r p_m^2 \beta_r^2 \theta^2}{4} - \frac{\theta \beta_r^2 p_r^2}{4} + \frac{3 \theta p_m^2 \beta_r^2}{4} - \sqrt{\alpha_m} \beta_m p_m \theta + \frac{p_r \beta_r^2}{4} - \frac{p_m \beta_r^2}{2} + \frac{\sqrt{\alpha_m} \beta_m}{2}.$$

(21)

Solving (21)

$$p_m = \frac{2 \left(\sqrt{(3 p_r \beta_r^2 \theta^2 - 3 \theta \beta_r^2)} \left(-\frac{\theta \beta_r^2 p_r^2}{4} + \frac{p_r \beta_r^2}{4} + \frac{\sqrt{\alpha_m} \beta_m}{2} \right) + \left(\frac{\beta_r^2 p_r^2 \theta^2}{2} - \sqrt{\alpha_m} \beta_m \theta - \frac{\beta_r^2}{2} \right)^2 - \frac{\beta_r^2 p_r^2 \theta^2}{2} + \sqrt{\alpha_m} \beta_m \theta + \frac{\beta_r^2}{2} \right)}{-3 p_r \beta_r^2 \theta^2 + 3 \theta \beta_r^2}$$

(22)

Or

$$p_m = \frac{2 \left(-\sqrt{(3 p_r \beta_r^2 \theta^2 - 3 \theta \beta_r^2)} \left(-\frac{\theta \beta_r^2 p_r^2}{4} + \frac{p_r \beta_r^2}{4} + \frac{\sqrt{\alpha_m} \beta_m}{2} \right) + \left(\frac{\beta_r^2 p_r^2 \theta^2}{2} - \sqrt{\alpha_m} \beta_m \theta - \frac{\beta_r^2}{2} \right)^2 - \frac{\beta_r^2 p_r^2 \theta^2}{2} + \sqrt{\alpha_m} \beta_m \theta + \frac{\beta_r^2}{2} \right)}{-3 p_r \beta_r^2 \theta^2 + 3 \theta \beta_r^2}$$

(23)

Maximizing (20) with respect to α_m gives

$$\frac{d \pi_m}{d \alpha_m} = \frac{1}{2} \frac{p_m \beta_m}{\sqrt{\alpha_m}} - \frac{1}{2} \frac{p_m \beta_m \theta p_r}{\sqrt{\alpha_m}} - 1 = 0$$

(24)

$$\Rightarrow \alpha_m = \frac{(1 - \theta p_m)^2 p_m^2 \beta_m^2}{16}.$$

(25)

IV. Results And Discussion

The advertising effectiveness β_r and β_m for the retailer and the manufacturer respectively, cannot be equal to zero. Thus $\beta_r \neq 0$ and $\beta_m \neq 0$. Also, because the retailer is much closer to the consumers than the manufacturer, he is conventionally expected to be more efficient in advertising, thus, $\beta_r > \beta_m$. In this research, we used $\beta_r = 0.4$ and $\beta_m = 0.2$.

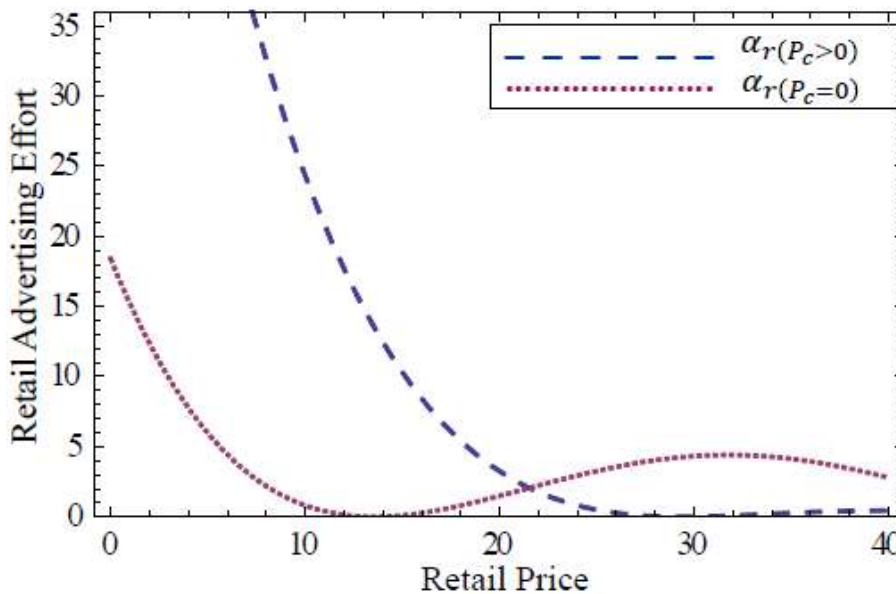


Figure 1 Illustration of the Effect of Retail Price on the Retail Advertising Effort

Figure 1 shows the trend of the retail advertising effort with the retail price. We observe that as the retail price increases, both advertising efforts reduce, eventually becoming zero, and exhibit increase thereafter, with the no-cost scenario effort being larger than the cost scenario effort. We observe a similar trend in Figure 2

where the manufacturer’s advertising effort for the no-cost scenario is larger than his effort for the cost scenario at optimal price level.

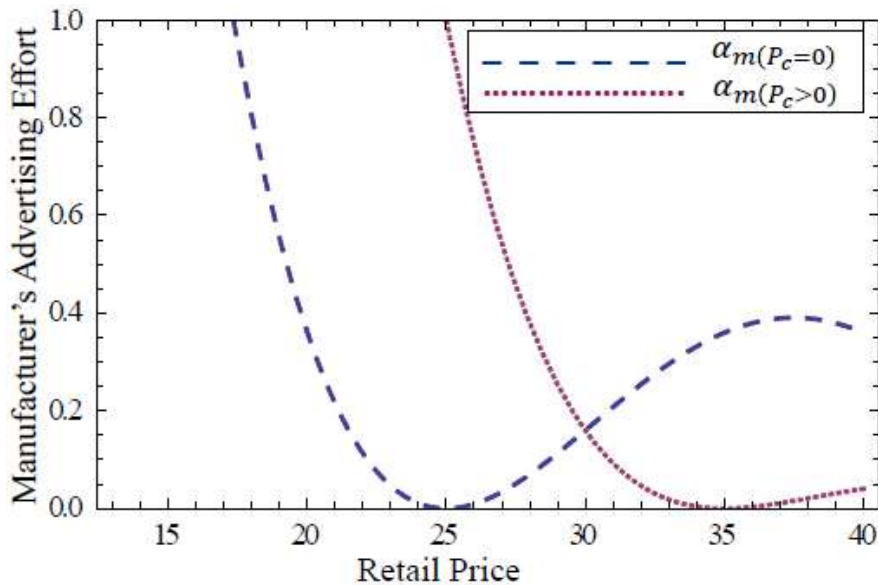


Figure 2 Illustration of the Effect of Retail Price on the Manufacturer’s Advertising Effort

Figure 2 shows that as the retail price increases, the efforts for both no-cost scenario and the production cost scenario decrease rapidly for lower retail price levels, with the no-cost scenario effort being smaller than the cost scenario effort and then exhibits increase for high retail price vales, with the no-cost scenario effort being larger than the cost scenario effort at optimal price levels.

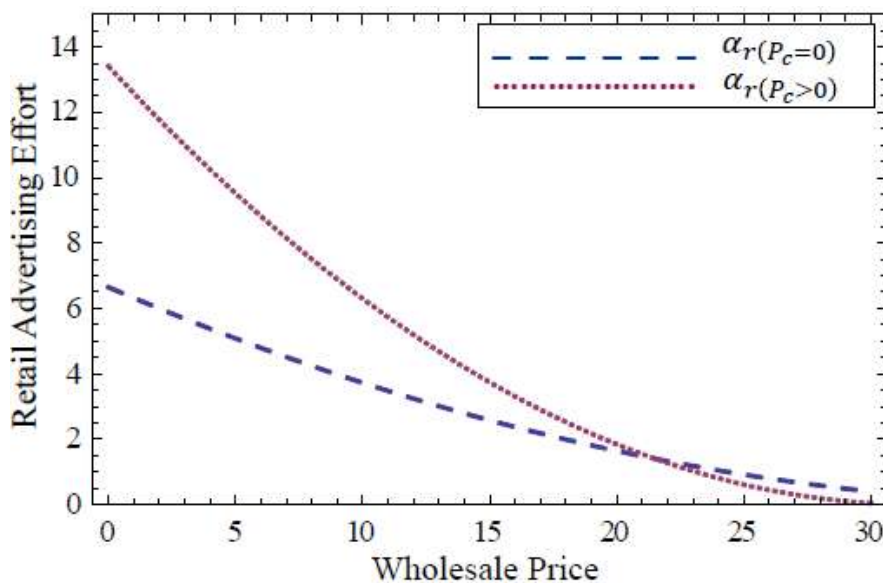


Figure 3 Illustration of the Effect of Wholesale Price on the Retail Advertising Effort

From Figure 3 we note that the retail advertising efforts for both cost and no-cost scenarios reduce with the manufacturer’s wholesale price. Clearly, the effort for production cost scenario reduces more rapidly than the no-cost scenario effort. This is possible because with non-zero production cost, the wholesale price must be comparatively larger than that of zero cost. As such, the retail price becomes comparatively larger. By extension the retailer is forced to reduce his advertising effort due to financial constraint. Thus, force the retail effort constrained by production cost to reduce more rapidly than the no-cost effort.

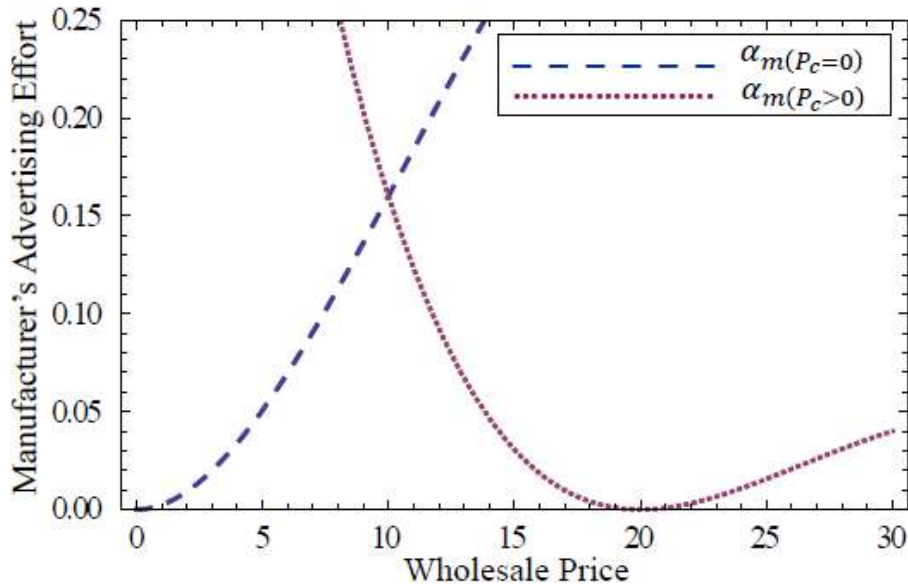


Figure 4 Illustration of the Effect of Wholesale Price on the Manufacturer’s Advertising Effort

Unlike the case in Figure 3, the plots in Figure 4 show that while the manufacturer’s effort for production cost scenario reduces, his effort for the no-cost scenario increases with increasing wholesale price. The implication is that as the wholesale price increases, a manufacturer who is not cumbered by production cost tends to increase effort because of availability of extra fund which could have been used for production. On the other hand a manufacturer who spends on production tends to comparatively reduce effort with increasing wholesale price because of non-availability of extra fund.

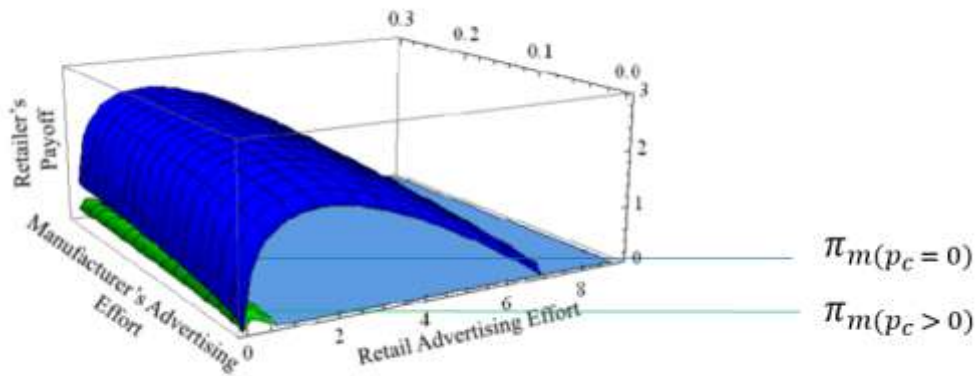


Figure 5 Illustration of the Effect of the Advertising Efforts on the Retailer’s Payoff

Figure 5 shows that as the manufacturer’s effort increases, the retailer’s payoff for both the no-production cost scenario and the production cost scenario increase continuously. We note that the advertising effort is the manufacturer’s expenditure meant to increase product patronage, and eventually boost payoff. Such expenditure while increasing the cost burden on the manufacturer, reduces on the retailer’s cost burden, leading to larger payoff. On the contrary, the retailer’s payoffs for both the no-production cost scenario and the production cost scenario exhibit increase for low values of the retail effort, but reduce with large values, eventually becoming zero. Clearly, the marginal returns on the advertising effort diminishes continuously, getting to a saturation point where it is zero, eventually becoming negative afterwards.

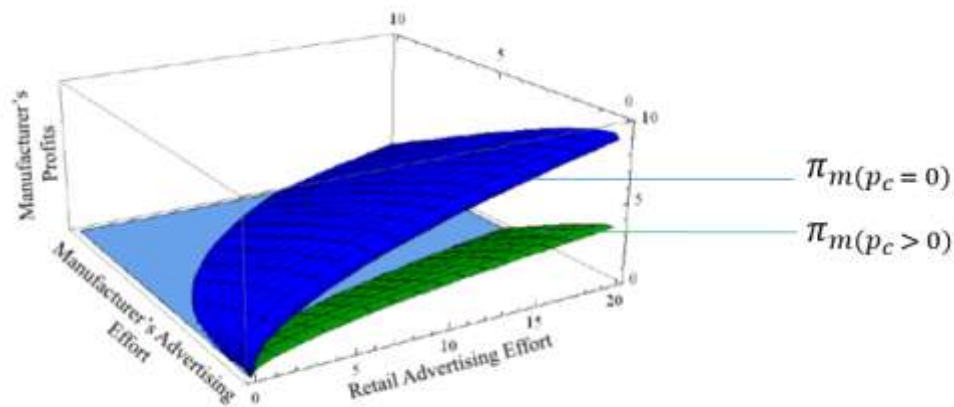


Figure 6 Illustration of the Effect of the Advertising Efforts on the Manufacturer's Payoff

Considering Figure 6 we observe increase that in retailer's advertising expenditure leads to continuous increase in the manufacturer's payoffs for both scenarios. This is because increasing the retailer's effort with the aim of increasing patronage is a cost burden on the retailer whereas it is not on the manufacturer. Thus, such an increase will eventually increase will be more beneficial to the manufacturer than to the retailer. On the contrary, increasing the manufacturer's advertising effort leads to continuous reduction in the manufacturer's payoff for both scenarios. This is because spending on manufacturer's advertising without manufacturer wholesale price increase in the retail price is an additional reduction in the manufacturer's revenue.

V. Managerial Implications

From the figure 1, we observe that for a situation where there is no production cost, both players are very much willing to increase advertising spending with increasing retail price. Such willingness is understandably possible since a no-cost situation expectedly reduces pressure on the manufacturer to increase wholesale price which also imply reduced retail price. Figure 2 shows that it would be misleading to implement price the manufacturer's effort without recourse to the optimal retail price level. From figure 3, we note that at optimal level the no-cost scenario advertising effort is larger than the cost scenario for both players. This implies that budgeting with no-cost scenarios advertising efforts will certainly result in inaccuracies, and is detrimental. Thus modelling with production cost scenario is more realistic in advertising. Figure 5, shows that the retailer should not continuously increase effort in the hope of a very large payoff.

VI. Conclusion

This work considered advertising in a manufacturer-retailer supply channel in which the manufacturer is the Stackelberg leader and the retailer is the follower. It considered a situation where the production cost is incorporated into the model and a situation where it is not and obtained a closed-form solution of the prices, advertising efforts and payoffs for both situations. The work observed that the optimal values are all larger for almost all the cases without production cost. This is clearly misleading. Thus, implementation of the model without production cost will lead to results which could be detrimental. The work observed; as retail prices increase, both advertising efforts decrease to zero, then rise thereafter, with this work is based on manufacturer-retailer supply channel. This appears to be too restrictive because manufacturer usually engage the services of a middle-man, which distributes his services to the retailers. Thus an extension can consider a manufacturer-distributor-retailer channel.

References

- Berger, P. D. (1972). Vertical Cooperative Advertising Ventures. *Journal of Marketing Research*, 9, 309–312. <https://journals.sagepub.com/doi/full/10.1177/002224377200900310>
- Chen, T. H. (2015). Effects of the pricing and cooperative advertising policies in a two-echelon dual-channel supply chain. *Computers & Industrial Engineering*, 87, 250–259.
- Chutani, A., & Sethi, S. P. (2012). Cooperative advertising in a dynamic retail market oligopoly. *Dynamic Games and Applications*, 2(4), 347–375. <https://link.springer.com/article/10.1007/s13235-012-0053-8>
- Dant, R. P., & Berger, P. D. (1996). Modeling cooperative advertising decisions in franchising. *The Journal of Operational Research Society*, 47(9), 1120–1136. <https://www.jstor.org/stable/3010372?refreqid=excelsior%3Ad7b12ee00a370aa67ba9c31b87e3b026>

- Dridi, D., & Ben Youssef, S. (2015). A game theoretic framework for competing/cooperating retailers under price and advertising dependent demand. Munich Personal RePEc Archive. Available at: <https://mpra.ub.uni-muenchen.de/62705/>
- Ezimidu PE (2019a) Modelling Subsidy as a Cooperate Adversity Coordination Mechanism, Nigeria Journal of Basic and Applied Science 27(2). 127 – 135, 2019
- Ezimidu, P. E. (2019b). A game-theoretic cooperative advertising model: The feasibility of the distributor's involvement in a manufacturer-distributor-retailer channel. *Future Trends in Science & Technology Journal*, 4(2), 416–421. <http://www.fstjjournal.com/digital%20library/42%20article%2019.php>
- Ezimidu, P. E. (2022). A Game-Theoretic Credit Period and Promotion Model in a Supplier-Retailer Channel, *Asian Research Journal of Mathematics* 18 (11), 351-361
- PE Ezimidu, CR Nwozo (2018). Modeling dynamic cooperative advertising in a decentralized channel, *Yugoslav Journal of Operations Research* 28 (4), 539-566
- Grabner-Kraeuter, S. (2002). The role of consumers' trust in online-shopping. *Journal of Business Ethics*, 39(1–2), 43–50.
- He, X., Prasad, A., & Sethi, S. P. (2009). Cooperative advertising and pricing in a dynamic stochastic supply chain: Feedback Stackelberg strategies. *Production and Operations Management*, 18(1), 78–94. <https://onlinelibrary.wiley.com/doi/10.1111/j.1937-5956.2009.01006.x>
- He, X., Prasad, A. and Sethi, S. P.(2009). “Cooperative advertising and pricing in a dynamic stochastic supply chain: Feedback Stackelberg strategies”, *Production and Operations Management*, 18 (1) 78-94.
- He, X., Krishnamoorthy, A., Prasad, A. and Sethi, S. P. (2011), “Retail competition and cooperativ advertising”, *Operations Research Letters*, 39 (1) 11-16.
- He, Y., Liu, Z. and Usman, K. (2014). “Coordination of cooperative advertising in a two-period fashion and textiles supply chain”, *Mathematical Problems in Engineering*, <http://dx.doi.org/10.1155/2014/356726>, Article ID 356726 10pp.
- Huang, Z., Li, S. X., & Mahajan, V. (2002). An analysis of manufacturer-retailer supply chain coordination in cooperative advertising. *Decision Science*, 33(3), 469–494. <https://onlinelibrary.wiley.com/doi/10.1111/j.1540-5915.2002.tb01652.x>
- Jorgensen, S., Sigue, S. P., & Zaccour, G. (2000). Dynamic cooperative advertising in a channel. *Journal of Retailing*, 76(1), 71–92. <https://www.sciencedirect.com/science/article/pii/S002243599900024X>
- Karray, S., Martín-Herrán, G., & Sigué, S. P. (2017). Cooperative advertising for competing manufacturers: The impact of long-term promotional effects. *International Journal of Production Economics*, 184, 21–32.
- Li, S. X., Huang, Z., Zhu, J., & Chau, P. Y. (2002). Cooperative advertising, game theory and manufacturer-retailer supply chains. *Omega*, 30(5), 347–357.
- Nagler, M. (2006). An exploratory analysis of the determinants of cooperative advertising participation rates. *Marketing Letters*, 17(2), 91–102.
- Rabbani, M., & Shahraki, S. D. (2021). A game-theoretic approach with cooperative advertising and quality level in two-level supply chain with considering complementary products and uncertainty in demand. *Journal of Quality Engineering and Production Optimization*, 6(1), 165–188. http://jqepo.shahed.ac.ir/article_3170.html
- Seyed Esfahani, M. M., Biazaran, M., & Gharakhani, M. (2011). A game theoretic approach to coordinate pricing and vertical co-op advertising in manufacturer-retailer supply chains. *European Journal of Operational Research*, 211, 263–273.
- Sridhar, S., Germann, F., Kang, C., & Grewal, R. (2016). Relating online, regional, and national advertising to firm value. *Journal of Marketing*, 80(4), 39–55.
- Szmerekovsky, J. G., & Zhang, J. (2009). Pricing and two-tier advertising with one manufacturer and one retailer. *European Journal of Operational Research*, 192(3), 904–917.
- Woodside, A. G., & Taylor, J. L. (1978). Consumer purchase intentions and perceptions of product quality and national advertising. *Journal of Advertising*, 7(1), 48–51.
- Xie, J., & Wei, J. (2009). Coordination advertising and pricing in a manufacturer-retailer channel. *European Journal of Operational Research*, 197, 785–791. <https://www.sciencedirect.com/science/article/pii/S0377221708004992>
- Xie, J., & Neyret, A. (2009). Co-op advertising and pricing models in manufacturer-retailer supply chains. *Computers & Industrial Engineering*, 56(4), 1375–1385.
- Yu, C., Ren, Y., & Archibald, T. W. (2021). Differential game model of cooperative advertising in a supply chain with deteriorating items, competing retailers and reference price effects. *Enterprise Information Systems*. <https://doi.org/10.1080/17517575.2021.1941273>
- Yue, J., Austin, J., Wang, M., & Huang, Z. (2006). Coordination of cooperative advertising in a two-level supply chain when manufacturer offers discount. *European Journal of Operational Research*, 168, 65–85. <https://www.sciencedirect.com/science/article/pii/S0377221704003443>

Zhang, J., Li, J., Lu, L., & Dai, R. (2017). Supply chain performance for deteriorating items with cooperative advertising. *Journal of Systems Science and Systems Engineering*, 26, 23–49. <https://doi.org/10.1007/s11518-015-5279-8>