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On Different Representations of the FLRW Metrics

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Abstract
We present different types of representations of FLRW metrics. The form of FLRW metric depends on the solution from which it was obtained as a limit.

I. Introduction:

It has been a conventional wisdom in Cosmology that the Friedmann (1922, 1924), Lemaitre (1927, 1931), Robertson (1929, 1933), and Walker (1935) (FLRW) models describe the large scale features of our observed universe. An exact solution of the Einstein field equations is known as cosmological if it may reproduce a FLRW metric when its arbitrary constants or functions assume certain limiting values.

II. Different Forms of The FLRW metrics:

There are several representations of the FLRW metrics in literature. The form of a FLRW metric depends on the solution from which it was obtained as a limit. One of the standard representations of the FLRW metrics reads as

$$ds^2 = dt^2 - \left(1+\frac{1}{4}kr^2\right)^{-2} R^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] \qquad (1)$$

$$= dt^2 - \left(1+\frac{1}{4}kr^2\right)^{-2} R^2(t)\left[dx^2 + dy^2 + dz^2\right], \qquad (2)$$

where $r^2 = x^2 + y^2 + z^2$, $R(t)$ being the scale factor is an arbitrary function to be evaluated from the Einstein equations, and k being the curvature index be an arbitrary constant. The above metric automatically defines a pure perfect fluid energy momentum tensor, and the only limitations on *R(t)* may come from an equation of state. For $k = 0$, the spatial sections t = constant are flat. This case is known as the flat FLRW model, its group B_3 may be Bianchi type I or VII_o. For $k < 0$, the spatial sections t = constant have a constant negative curvature. This case is known as the open FLRW model, its B_3 may be of Bianchi type V or VII_h. For $k > 0$, the spatial sections t = constant have a constant positive curvatrue. This is known as the closed FLRW model, its B_3 is of Bianchi type IX. The form (1) and (2) results from Barnes- Stephani family. From the Szekeres-Srafron family the FLRW limit results in the form

$$ds^{\to 2} = dt^2 - R^2(t)\left[\frac{dr^2}{\left(1-kr^2\right)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right], \qquad (3)$$

or in the form
(4)

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{\left(1-kr^2\right)} + r^2\left(dx^2 + dy^2\right)/1 + \frac{1}{4}\left(x^2 + y^2\right)\right].$$

Equations (1), (2), (3) and (4) cover all the signs of k in one formula. Another form is

$$ds^2 = dt^2 - R^2(t)\left[dr^2 + f^2(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right], \qquad (5)$$

where

$$f(r)=\begin{cases}\sin r & \text{for } k>0,\\ r & \text{for } k=0,\\ \sinh r & \text{for } k<0.\end{cases} \quad (6)$$

The three cases of (6) may be given in on equation as

$$ds^2=dt^2-R^2(t)\left[dr^2+k^{-1}\sin^2\left(k^{\frac{1}{2}}r\right)\left(d\theta^2+\sin^2\theta d\varphi^2\right)\right]. \quad (7)$$

The range of r may be finite or infinite, depending on the sing of k and on the coordinates used.

The other form of the FLRW metric results from the solutions with two commuting Killing vector fields as

$$ds^2=dt^2-R^2(t)\left[dx^2+f_{,x}^2dy^2+f(x)dz^2\right], \quad (8)$$

where

$$f(x)=\begin{cases}\sin\left(k^{\frac{1}{2}}x\right) & \text{for } k>0,\\ x & \text{for } k=0,\\ \sinh\left[(-k)^{\frac{1}{2}}x\right] & \text{for } k<0.\end{cases} \quad (9)$$

Another form of the FLRW metrics results as a limit of plane symmetric solutions as

$$ds^2=dt^2-R^2(t)\left[dx^2+e^{2cx}\left(dy^2+dz^2\right)\right], \quad (10)$$

where c being a constant. For C = 0 we have flat FLRW metric, for $C\neq 0$ one obtains the open FLRW metric. The closed FLRW metric is in compatible with plane symmetry.

The FLRW models result from the Goode and Wainwright (1982b) representation of the Szekeres models. One of the G-W forms is

$$ds^2=dt^2-S^2\left[e^{2v}\left(dx^2+dy^2\right)+W^2f^2v_{,z}^2dz^2\right], \quad (11)$$

where

$$W^2=\left(\in -kf^2\right)^{-1}, \quad (12)$$

$$e^v=f(z)\left[a(z)\left(x^2+y^2\right)+2b(z)x+2c(z)y+d(z)\right]^{-1}, \quad (13)$$

$\in$ and *k* are arbitrary constants, and *S(t), f(z), a(z), b(z), c(z), d(z)* are arbitrary functions subject to

$$ad-b^2-c^2=c/4. \quad (14)$$

The constants $\in$ and *k* may be scaled by coordinate transformation and repraparametrisations of the functions so that each one of them is either 0 or +1 or -1. But without any condition on $\in$ and *k*, the slices *t* = constant of the metric are spaces of constant curvature equal to k/s^2, while the t -coordinate lines are shearfree geodesics with the expansion scalar depending on t. This is a property of the FLRW spacetimes.

The other G-W form is

$$ds^2=dt^2-S^2\left[e^{2v}\left(dx^2+dy^2\right)+A^2dz^2\right], \quad (15)$$

where

$$\begin{cases}e^v=\left[1+\dfrac{1}{4}k\left(x^2+y^2\right)\right]^{-1}\\ A=e^v\left\{a(z)\left[1-\frac{1}{4}k\left(x^2+y^2\right)\right]+b(z)x+c(z)y\right\},\end{cases} \quad (16)$$

where $S(t), a(z)$ and $c(z)$ are arbitrary functions and k as an arbitrary constant.

III. Concluding Remarks:

We have presented different representations of the FLRW metrics which are given in literature.

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