

## Design of 32 States, Rate 4/5, 32 QAM, TCM Code for Fading Channel

Raj Kumar Goswami

(ECE Department, GVP College of Engineering for Women, India)

---

**Abstract:** If we look at the communication system per se then we find that the coder and modulator are two different sections and function independently. Therefore, if the only coding rate is varied as per the design by keeping other parameters same then this directly affects the bandwidth (BW) of the system. For example, if the coding rate is 1/2 the bandwidth reduced to one half and if rate is 1/3 then BW reduces to one third and likewise. However, there is a method by which the BW can be preserved and this can be done if coding and modulation are combined, which means that they do not function independently and whenever the coding rate is changed then the modulation is also changed accordingly. Let us take an example when no coding is employed and the modulation scheme is BPSK. If now the only coding is introduced having the rate as 1/2 then the BW reduces to half but if at the same time modulation scheme is also changed from BPSK to QPSK then the BW can be preserved. This scheme wherein coding and modulation have been combined in order to preserve the bandwidth is known as Trellis Coded Modulation. This was invented in 1970s and presently being utilized in most of the contemporary communication systems. Traditionally, it is convolutional coding scheme that is combined with the modulation techniques. By taking above concept into consideration a novel TCM scheme of rate 4/5 has been designed for fading channel, which utilizes 32 states and integrated with 32 QAM. The performance of this scheme has been compared with the uncoded 16-Ary QAM and a coding gain of about 3dB has been observed.

**Key Word:** Convolutional Code, TCM, QAM.

---

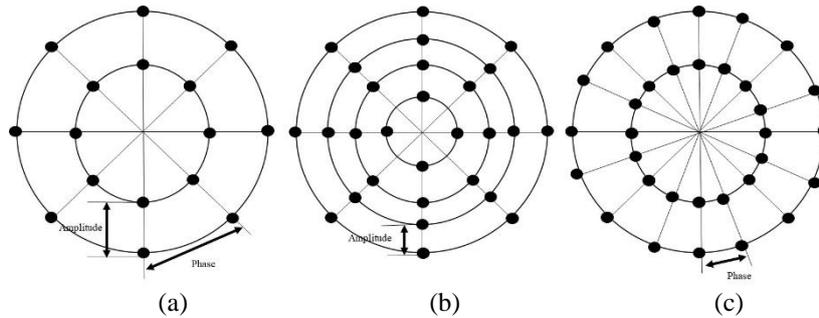
Date of Submission: 06-09-2021

Date of Acceptance: 22-09-2021

---

### I. Introduction

There has been a continuous effort, being put in by the researchers in designing the efficient coding schemes so as to achieve the Nyquist limit. However, there is a dilemma between the code rate and the bandwidth [1]. Better codes are achieved by adding more redundant bits but this results in expansion of bandwidth and when this expanded bandwidth is absorbed through the change in modulation techniques then the Euclidean distance between the constellation points decreases. This decrease in distance results in increase in errors. The use of Trellis Coded Modulation (TCM) [2] is therefore to improve the system performance without increasing the transmit power and channel bandwidth [3] For example, when no coding is employed and the modulation utilized is 16 QAM, then by taking the specific BW and power into consideration, the distance between the Constellation points is taken as  $d$ . Search for better and better codes are being undertaken by the researchers. Zhuqing Yue et al has also presented a three-dimension 9QAM–TCM scheme which combines a novel probabilistic shaping method with three-dimension QAM mapping[4]. In this paper TCM code having rate 4/5 32 states and employing 32 QAM [5] has been presented. It is to be noted that when the coding having rate 4/5 is implemented then because of TCM, the modulation has to be changed to 32 QAM from 16 QAM in order to preserve BW. But then the distance between the constellation points decreases as shown in Fig 1. As can be seen from the Fig 1(b) and (c) that in both the constellation the distance between the signal points decreases. In first case shown in Fig 1(b) the amplitude between the signal points decreases and in the other case in Fig 1 (c) the distance between the phase decreases. It is to be noted that any configuration of constellation is utilized, the distance between the signal points will be reduced. As mentioned before this decrease in distance will result in introduction of errors. In such cases the design of coding scheme should be such that it caters for correction of these errors and also offers coding gain over and above the uncoded one.



**Fig. 1** Reduction in Euclidean distance after change in modulation scheme from 16 QAM to 32 QAM

It has been observed that it is possible to combine the coding scheme and modulation schemes into one block, wherein both are dependent on each other i.e., when the coding technique is changed then along with modulation scheme is also changed in order to preserve BW. This combination of Coding and Modulation schemes is known as Trellis Coded Modulation (TCM). As mentioned previously, coding techniques employed in such a scheme is Convolutional coding [6]. Such TCM schemes have been developed which not only preserve the BW but also provide coding gain over the uncoded one. This has been the main reason for undertaking maximum research in this direction.

Now we will look into the design aspect of TCM. The main aim in designing the coding scheme is to increase the minimum free distance by taking into the consideration the relation between trellis and modulation. It means that the output of modulator is not directly mapped to input provided to the modulator from the encoder but mapped to the symbols selected from the partition of signal constellation. This selection of symbol from the partition is the heart of coding scheme and has an impact on the performance of TCM scheme.

The detection at the receiver is performed using soft decision, because hard decision results in irreversible loss of information leading to the loss of Signal to Noise Ratio (SNR). The criteria used in soft decision detection in respect of fading channel is maximum likelihood, in which Euclidean distance plays the major role in selecting the optimum sequence. Which implies that the one of the code sequences which has the least Euclidean distance when compared with the received sequence will be selected as the decoded sequence. Further if interleaver is also combined with TCM, the coding can be further improved [7], provided suitable criteria is utilized in designing.

The organization of paper is as follows. System model is explained in section II. The design of rate 4/5, 32 states, 32 QAM TCM coding scheme in respect of fading channel has been presented in section III. In section IV code construction has been elaborated. In section V performance analysis has been presented and in section VI, the results have been discussed.

## II. System Model

The block diagram of general communication system [8] has been shown in Fig. 2. Since our focus on the TCM we will discuss the part of the Block Diagram involving Channel Encoder, Modulator, Fading Channel, Demodulator and Channel Decoder [3]. Considering the output of Source Encoder as the input bits to the Channel Encoder, the first block of TCM Scheme. As now we know that the TCM is combination of Coding and modulation, the combination of these two blocks we will call as TCM Encoder. The output of the TCM Encoder is the output of QAM modulator, which produces a signal sequence  $s_1, s_2, \dots, s_n$ , where  $s_1, s_2$  etc. are vectors selected from the appropriate QAM signal set.

To make the coding system more robust to the burst errors an interleaver can also be introduced which will spread the burst errors so that these will be corrected by the TCM coding scheme. Further in order to eliminate the Inter Symbol Interference pulse shaping can also be implemented prior undertaking the modulation. The output of the modulator when passed through the fading channel, the errors get introduced due to multiple reflections from the objects, Gaussian Noise gets added to the signal and finally the distorted signals are received at the receiver.

At the receiver these signals are detected, demodulated by the demodulator i.e.,  $r_1, r_2, \dots, r_n$  and then passed on to the decoder, which undertakes the Maximum Likelihood decoding.

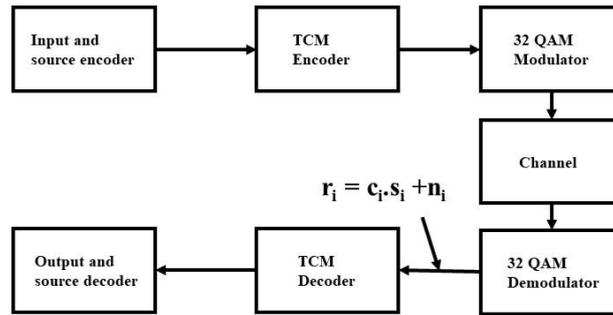


Fig. 2 Communication block diagram.

A simplified diagram of the system shown in Fig. 2 has been depicted in Fig. 3 for the purpose of analysis. It can be seen from the Fig. 3 that the received signal at time  $i$  can be shown as

$$r_i = c_i \cdot s_i + n_i$$

where,  $n_i$  indicate zero mean Gaussian noise having variance  $N_0/2$  and  $c_i$  is the complex channel gain, which can be expressed as follows using phasor notation.

$$c_i = a_i \cdot e^{j\phi_i}$$

$a_i$  and  $\phi_i$  indicate the amplitude and the phase respectively.

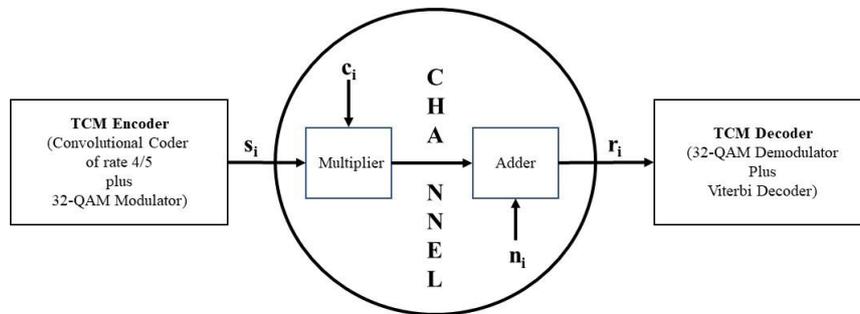


Fig. 3 Simplified model of Fig. 2

Suppose the receiver performs coherent detection, then the channel phase shift gets compensated by the receiver then  $r_i$  can be rewritten as

$$r_i = a_i s_i + n_i$$

where  $a_i$  is the noise amplitude.

When there is only diffused multipath component in a channel then the fading amplitude is modeled as Rayleigh distributed having the PDF

$$p_A(a) = 2ae^{-a^2}, \quad a \geq 0.$$

And when in addition to multipath fading if, there is a single dominant, nonfading component offered by the channel then, the amplitude  $a_i$  is Rician distributed, having PDF as

$$p_A(a) = 2a(1 + K)e^{-(K+a^2(1+K))} I_0(2a\sqrt{K(1+K)}) \quad , \quad a \geq 0$$

where  $K$  is the indicative of Rician parameter, which is the ratio of the received signal energy of multipath components in the direct and diffused paths and  $I_0(\cdot)$  is the zero-order modified Bessel function of the first kind.

### III. Proposed Design

The design of TCM coding scheme having rate 4/5, 32 states in respect of 32 QAM will be enumerated in this section. The design has been optimized for fading channel. The rules those are being framed have been based on the rules of earlier TCM Coding scheme having rate 2/3, 8 states developed by Ungerboeck for fading channel [9], in respect of 8 PSK modulation and optimized for fading channel. This scheme provided an effective length  $L$  of 2 and  $d_{free}^2$  of 4.586 times  $E_s$ . As performance of QAM is comparatively better in comparison with PSK, many researchers in the past have tried to develop the TCM codes for QAM. In [10] Periyalwar S has explained one such design of TCM for MQAM. By taking this design into consideration, following guidelines are proposed for designing the TCM scheme optimized for the fading channel, having rate 4/5, 32-states, utilizing the modulation scheme of 32-QAM.

First, a method of designing the signal set partitioning is being presented. The 32 QAM signal set will be partitioned into two subsets as shown in the Fig 4. These sets are designated as  $A_0$  and  $A_1$ , where  $A_0$  consists of  $S_0, S_2, S_4, S_6, S_8, S_{10}, S_{12}, S_{14}, S_{16}, S_{18}, S_{20}, S_{22}, S_{24}, S_{26}, S_{28}, S_{30}$  and  $A_1$  consists of  $S_1, S_3, S_5, S_7, S_9, S_{11}, S_{13}, S_{15}, S_{17}, S_{19}, S_{21}, S_{23}, S_{25}, S_{27}, S_{29}$  and  $S_{31}$ . The value of  $S_0$  to  $S_{31}$  is depicted in table 1.

Symbol	Value	Symbol	Value	Symbol	Value	Symbol	Value
$S_0$	00000	$S_8$	01000	$S_{16}$	10000	$S_{24}$	11000
$S_1$	00001	$S_9$	01001	$S_{17}$	10001	$S_{25}$	11001
$S_2$	00010	$S_{10}$	01010	$S_{18}$	10010	$S_{26}$	11010
$S_3$	00011	$S_{11}$	01011	$S_{19}$	10011	$S_{27}$	11011
$S_4$	00100	$S_{12}$	01100	$S_{20}$	10100	$S_{28}$	11100
$S_5$	00101	$S_{13}$	01101	$S_{21}$	10101	$S_{29}$	11101
$S_6$	00110	$S_{14}$	01110	$S_{22}$	10110	$S_{30}$	11110
$S_7$	00111	$S_{15}$	01111	$S_{23}$	10111	$S_{31}$	11111

Table 1. The binary values of symbols used in set partitioning

Second aspect of TCM is the generation of these signals whenever an input is provided. It can also be seen that whenever an input is given the state changes, therefore, all outputs are best represented by the matrix of states. In the present case as there are 32 states, 32x32 matrix will depict all transitions between the states. In order to represent the output of the TCM coder while state changes from  $i$  to  $j$  is indicated by the  $ij^{th}$  element of the matrix.

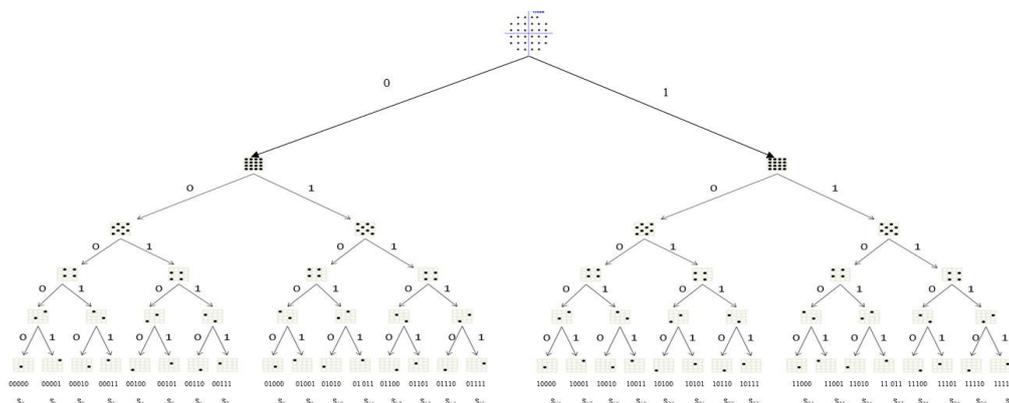


Fig. 4 Partitioning of 32 QAM Signal set

It can be seen that there are 32 States and the input are only 16, which means that all transitions cannot take place, therefore it is proposed to follow the undermentioned rules for associating the Signals with the change in the states.

- a. In any row or column Signal should occur only once.
- b. Path will be valid only if LSB of starting state i.e., 0 or 1 is equal to the MSB of the next state.

c. For the given initial state to all possible transition states, the signals will be from either of the two sets  $A_0$  and  $A_1$  defined previously. Output matrix of this design has been shown in the Fig. 5.

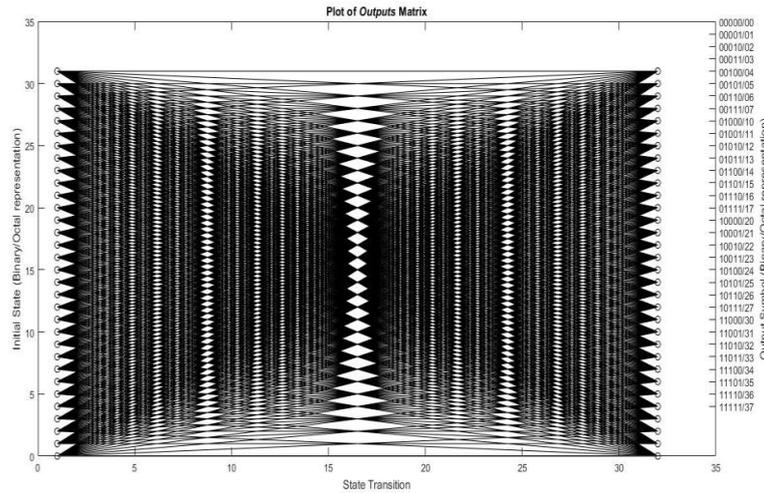


Fig. 5 Output matrix

d. The State Transition matrix for the proposed design is shown in Table 2.

States	00000	00001	00010	00011	00100	00101	00110	00111	01000	01001	01010	01011	01100	01101	10000	10001	10010	10011	10100	10101	10110	10111	11000	11001	11010	11011	11100	11101	11110	11111		
00000	S <sub>00</sub>	S <sub>01</sub>	S <sub>02</sub>	S <sub>03</sub>	S <sub>04</sub>	S <sub>05</sub>	S <sub>06</sub>	S <sub>07</sub>	S <sub>08</sub>	S <sub>09</sub>	S <sub>0A</sub>	S <sub>0B</sub>	S <sub>0C</sub>	S <sub>0D</sub>	S <sub>0E</sub>	S <sub>0F</sub>	S <sub>00</sub>	S <sub>01</sub>	S <sub>02</sub>	S <sub>03</sub>	S <sub>04</sub>	S <sub>05</sub>	S <sub>06</sub>	S <sub>07</sub>	S <sub>08</sub>	S <sub>09</sub>	S <sub>0A</sub>	S <sub>0B</sub>	S <sub>0C</sub>	S <sub>0D</sub>	S <sub>0E</sub>	S <sub>0F</sub>
11111	S <sub>1000</sub>	S <sub>1001</sub>	S <sub>1002</sub>	S <sub>1003</sub>	S <sub>1004</sub>	S <sub>1005</sub>	S <sub>1006</sub>	S <sub>1007</sub>	S <sub>1008</sub>	S <sub>1009</sub>	S <sub>100A</sub>	S <sub>100B</sub>	S <sub>100C</sub>	S <sub>100D</sub>	S <sub>100E</sub>	S <sub>100F</sub>	S <sub>1010</sub>	S <sub>1011</sub>	S <sub>1012</sub>	S <sub>1013</sub>	S <sub>1014</sub>	S <sub>1015</sub>	S <sub>1016</sub>	S <sub>1017</sub>	S <sub>1018</sub>	S <sub>1019</sub>	S <sub>101A</sub>	S <sub>101B</sub>	S <sub>101C</sub>	S <sub>101D</sub>	S <sub>101E</sub>	S <sub>101F</sub>

Table 2. State transitions Matrix

#### IV. Construction of Code

The construction of code for rate 4/5, 32-state, 32-QAM will now be discussed. The design will be based on the rules enumerated in the preceding section

The second rule states that path will be valid only if LSB of starting state is equal to the MSB of the next state, therefore in such cases, wherever transitions are not permitted should be removed and the subset  $A_0$  to be associated with the  $LSB = 0$  and subset  $A_1$  to be associated with the  $LSB = 1$ .

Which implies that for the signals indicated in the rows having even number subset  $A_0$  will be utilized and in case of the rows having odd numbers subset  $A_1$  will be utilized. For  $ij^{th}$  element of the transition matrix, we can choose any signal point from the subset  $A_0$  as the first element of the first row and similarly from subset  $A_1$  as the first element of second row. For the sake of simplicity let us take them to be as  $s_0$  and  $s_1$  respectively.

After this, in accordance with the rule mentioned above, the signals from the subset  $A_0$  and  $A_1$  are assigned to the first row to the sixteenth row.

Subsequently, any number from subset  $A_0$  i.e.  $s_8$ , or  $s_9$  can be put as first element of seventeenth row and eighteenth row respectively. Now by taking the second and third rule into consideration remaining signals are assigned in the rows from seventeenth to thirty second rows. The trellis diagram for this design of code is shown in Fig 6.

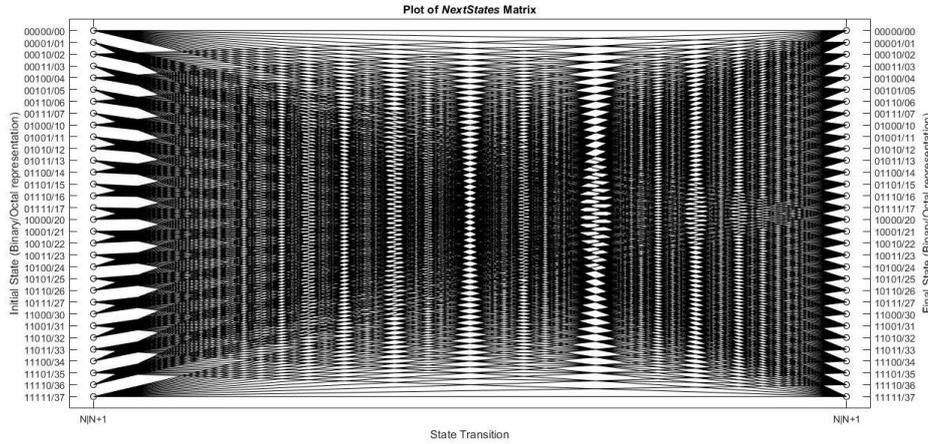


Fig. 6 Trellis diagram of the code having Rate 4/5 and 32-state

### V. Performance Analysis

For undertaking the performance analysis in simplistic manner, let us assume that detection is coherent, CSI is perfect and fading is independent in respect of each symbol. Now, if Rician channel model is considered then the upper bound on the pairwise error probability [6] for above system is given as

$$P_2(s_l, \hat{s}_l) \leq \prod_{i=1}^l \frac{1+K}{1+K + \frac{1}{4N_0} |s_i - \hat{s}_i|^2} \exp \left[ -\frac{K \frac{1}{4N_0} |s_i - \hat{s}_i|^2}{1+K + \frac{1}{4N_0} |s_i - \hat{s}_i|^2} \right]$$

where  $\hat{s}_l$  is the symbol decoded in respect of symbol  $s_l$  that was transmitted and  $P_2(s_l, \hat{s}_l)$  indicates the probability of this decoding. At high SNR this simplifies to

$$P_2(s_l, \hat{s}_l) \leq \prod_{i \in \eta} \frac{(1+K)e^{-K}}{\frac{1}{4N_0} |s_i - \hat{s}_i|^2}$$

where  $\eta$  indicates all those values of  $i$  where  $S_i$  is not equal to  $\hat{S}_i$ .

If all these values where  $S_i$  is not equal to  $\hat{S}_i$  denoted  $l_\eta$ , then the above equation can be further simplified as

$$P_2(s_l, \hat{s}_l) \leq \frac{((1+K)e^{-K})^{l_\eta}}{\left(\frac{1}{4N_0}\right)^{l_\eta} d_P^2(l_\eta)}$$

where,

$$d_P^2(l_\eta) = \prod_{i \in \eta} |s_i - \hat{s}_i|^2$$

is referred to as the squared product distance of the signals where  $S_i$  is not equal to  $\hat{S}_i$ , in the error event  $(s_l, \hat{s}_l)$  path. In the Fig. 7. the effective length of the error event  $(s_l, \hat{s}_l)$  has been shown.

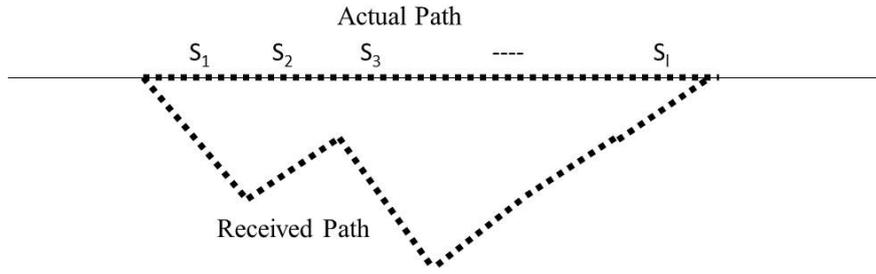


Fig. 7 Effective length of error event

To calculate the upper bound in respect of error event probability, the use of union bound can be made and the error event probability is summed up for all values of  $l$  by taking all transmitted sequence into consideration then this can be calculated as

$$P_e \leq \sum_{l=1}^{\infty} \sum_{s_i} \sum_{\hat{s}_i \neq s_i} P(s_i) P_2(s_i, \hat{s}_i)$$

in which  $P(s_i)$  denotes the *a priori* probability of transmitting symbol  $s_i$ . Substituting  $P_2(s_i, \hat{s}_i)$ , as mentioned above for high SNR's then in respect of Rician fading channel, the upper bound can be rewritten as

$$P_e \leq \sum_{l_{\eta}} \sum_{d_p^2(l_{\eta})} \alpha(l_{\eta}, d_p^2(l_{\eta})) \frac{((1+K)e^{-K})^{l_{\eta}}}{\left(\frac{1}{4N_0}\right)^{l_{\eta}} d_p^2(l_{\eta})}$$

where  $\alpha(l_{\eta}, d_p^2(l_{\eta}))$  depicts the average number of code sequences which have the effective length  $l_{\eta}$  & the squared product distance as  $d_p^2(l_{\eta})$ . For further simplification  $\min(l_{\eta})$  is depicted by  $L$  and the corresponding squared product distance by  $d_p^2(L)$ , then the error event probability can be approximated by

$$P_e \approx \alpha(L, d_p^2(L)) \frac{((1+K)e^{-K})^L}{\left(\frac{1}{4N_0}\right)^L d_p^2(L)}$$

If the channel is considered to be Rayleigh, then  $K$  is equal to 0, and the above equation can be written as

$$P_e \approx \frac{\alpha(L, d_p^2(L))}{\left(\frac{1}{4N_0}\right)^L d_p^2(L)}$$

If the channel is considered to be AWGN then  $K$  is equal to  $\infty$  and the equation of  $P_e$  can be written as follows [6]

$$P_e \approx \frac{1}{2} N(d_{free}) \operatorname{erfc} \left( \sqrt{\frac{d_{free}^2}{4N_0}} \right)$$

wherein  $d_{free}$  is the free Euclidean distance of the code.

## VI. Result and Conclusion

The analysis was undertaken in MATLAB and the result has been shown in Fig 8. It can be seen from the result that approximately two to three dB gain has been achieved by using the proposed TCM coding scheme having rate 4/5 and having 32 QAM modulation, over the uncoded 16-QAM modulation scheme. The analysis was also undertaken by utilizing an interleaver in order to take care of burst errors, which is a prevalent

phenomenon in multipath channel and it is seen that a further gain of two to three dB has been obtained over and above the gain achieved through TCM coding. It can be concluded that the proposed coding scheme will facilitate the communication at higher data rate over the fading channel.

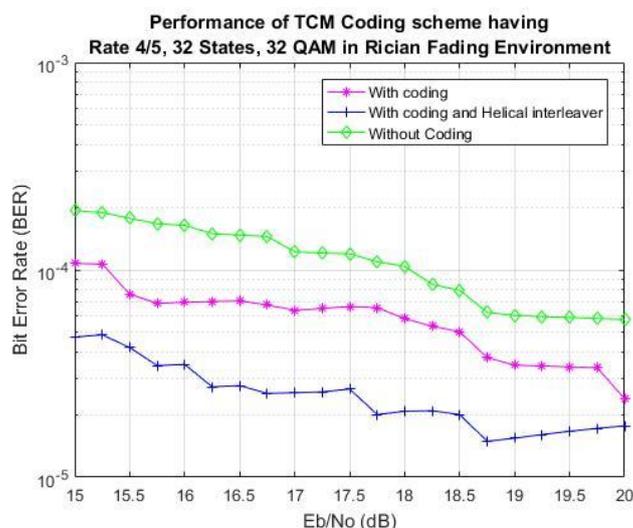


Fig. 8 Bit Error Rate Vs  $E_b/N_0$

### References

- [1]. Ezio Biglieri, Dariush Divsalar, Peter J. McLane and Marvin K. Simon, Introduction to Trellis-Coded Modulation with Applications, Maxwell Macmillan International Editions, 1991.
- [2]. Christian B. Schlegel and Lance C. Perez, Trellis and Turbo Coding, John Wiley & Sons Inc., Publication, 2004
- [3]. Baharuddin et al Performance Analysis of Trellis Coded Modulation and Diversity Combining on Wireless Channel, 2021 IOP Conf. Ser.: Mater. Sci. Eng. 1041 012023
- [4]. Zhuqing Yue et al Trellis coded modulation with three-dimension 9QAM mapping based on a novel probabilistic shaping method, Optical Communication Volume 449, 15 October 2019, Pages 45-52
- [5]. L. Hanzo, S. X. Ng, T. Keller, and W. Webb, Quadrature amplitude modulation: From basics to adaptive trellis-coded, turbo-equalised and space-time coded OFDM, CDMA and MC-CDMA systems, pp. 746-748. Wiley-IEEE Press, 2nd ed., December 15, 2004.
- [6]. Jamali Hamidreza S. and Tho Le Ngoc, Coded Modulation Techniques for Fading Channels, Boston: Kluwer Academic Publishers, 1994.
- [7]. Öztürk, E., Aygözü, Ü. A Combined Interleaving Technique for Trellis Coded MPSK Systems in Rayleigh Fading Channels. Wireless Personal Communications 16, 245-257 (2001).
- [8]. John G. Proakis and Dimitris G. Manolakis, Digital Signal Processing: Principles, Algorithms and Applications, Second Edition, New Delhi: Prentice Hall of India, 1995.
- [9]. Rajkumar Goswami, Sasi Bhusana Rao, Rajan Babu, Ravindra Babu, "8 State Rate 2/3 TCM Code Design for Fading Channel" IEEE conference On Control, Communications and Automation, Vol-II, pp. 323 -326, Dec 2008.
- [10]. Periyalwar, S. and Fleisher, S.M., "A modified design of trellis-coded MQAM for the fading channel", IEEE Trans. Commun., Vol.41, No.6, June 1993 pp.874-882

Raj Kumar Goswami. "Design of 32 States, Rate 4/5, 32 QAM, TCM Code for Fading Channel." *IOSR Journal of Electronics and Communication Engineering (IOSR-JECE)*, 16(5), (2021): pp 01-08.