

## **A Context for Training - Based Approximation in Randomly Correlated Rician Mimo Channels with Rician Disruption**

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**Abstract:** An antenna array on the transmit side provides the system with an extra spatial dimension that can be utilized for coding both in the spatial as well as the temporal domain. The recent development of such space time codes shows that there are ways of exploiting multiple transmit antennas while completely avoiding traditional beam-forming techniques need of accurate channel state information. In this project, we generate a framework for training based channel estimation under different channel and interference statistics. The minimum mean square error (MMSE) estimator for channel matrix estimation in Rician fading multi-antenna systems is analyzed, and exclusively the proposal of mean square error (MSE) minimizing training structures. By considering Kronecker-structured systems with a grouping of noise and interference and random training sequence length, we gather and simplify numerous earlier results in the framework. We simplify the circumstances for attaining the optimal training sequence structure and show when the spatial training power distribution can be explained unambiguously. We also prove that spatial correlation improves the estimation performance and establish how it determines the optimal training sequence length. The analytic results for Kronecker-structured systems are used to derive a heuristic training sequence under general unstructured statistics. The MMSE estimator of the squared Frobenius norm of the channel matrix is also derived and shown to provide far better gain estimates than other approaches. It is shown under which conditions training sequences that minimize the non-convex MSE can be derived explicitly or with low complexity. Numerical examples are used to evaluate the performance of the two estimators for different training classifications and system statistics. We also elucidate how the finest length of the training sequence often can be shorter than the number of transmit antennas.

**Keyword:** mse, mmse estimator, kronecker-structured systems.

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### **I. Introduction**

In this paper, we consider training-based estimation of instantaneous CSI in multiple-input multiple-output (MIMO) systems. Thus, the estimation is conditioned on the received signal from a known training sequence, which potentially can be adapted to the long-term statistics. By nature, the channel is stochastic, which motivates Bayesian estimation—that is, modeling of the current channel state as a realization from a known multi-variate probability density function (PDF). There is also a large amount of literature on estimation of deterministic MIMO channels which are analytically tractable but in general provide less accurate channel estimates, as shown in [7], [8]. Herein, we concentrate on minimum mean square error (MMSE) estimation of the channel matrix and its squared Frobenius norm, given the first and second order system statistics.

Training-based MMSE estimation of MIMO channel matrices has previously been considered for Kronecker-structured Rayleigh fading systems that are either noise-limited [9]–[11] or interference-limited [12]. In these papers, optimization of the training sequence was considered under various limitations on the long-term statistics, and analogous structures of the optimal training sequence were derived. These results reduce the training optimization to a convex power allocation problem that can be solved explicitly in some special cases. When mentioning previous work, it is worth noting that simplified channel matrix estimators have been developed in [8] and [13] and claimed to be MMSE estimators, but we show herein that these estimators are in general restrictive.

Although estimation of the channel matrix is important for receive and transmit processing, knowledge of the squared Frobenius norm of the channel matrix provides instantaneous gain information and can be exploited for rate adaptation and scheduling [17], [18]. The squared norm can be determined indirectly from an estimated channel matrix, but as shown in [16] this approach gives poor estimation performance at most signal-to-interference-and-noise ratios (SINRs). The MMSE estimator of the squared channel norm was introduced in [16] for Kronecker-structured Rayleigh fading channels, assuming the same training structure as for channel matrix estimation. Herein, the estimator is proved and generalized to Rician fading channels, along with the

design of MSE minimizing training sequences. Although the MSE is non-convex, we show that the optimal training sequence can be determined with limited complexity.

## II. System Model

We consider flat and block-fading MIMO systems with a transmitter equipped with an array of transmit antennas and a receiver with an array of receive antennas. The symbol-sampled complex baseband equivalent of the flat fading channel when transmitting at channel use is modelled as

$$y(t) = Hx(t) + n(t) \quad (1)$$

A measure of the spatial channel correlation is the eigenvalue distribution of the channel covariance matrix; weak correlation is represented by almost identical eigenvalues, while strong correlation means that a few eigenvalues dominate. Thus, in a highly correlated system, the channel is approximately confined to a small eigensubspace, while all eigenvectors are equally important in an uncorrelated system. In urban cellular systems, base stations are typically elevated and exposed to little near-field scattering. Thus, their antennas are strongly spatially correlated, while the non-line-of-sight mobile users are exposed to rich scattering and have weak antenna correlation if the antenna spacing is sufficiently large [19].

There are many reasons for estimating the channel matrix at the receiver. Instantaneous CSI can, for example, be used for receive processing (improved interference suppression and simplified detection) and feedback (to employ beamforming and rate adaptation). In this section, we consider MMSE estimation of the channel matrix from the observation during training transmission. In general, the MMSE estimator of a vector from an observation is

$$\hat{h}_{MMSE} = E\{h/y\} = \int hf(h/y)dh \quad (2)$$

We stress that the general MMSE estimator in (6) is in fact linear (affine), but nonetheless it has repeatedly been referred to as the linear MMSE (LMMSE) estimator [10]–[12] which is correct but could lead to the incorrect conclusion that there may exist better non-linear estimators. The MMSE estimator in (6) is also the maximum a posteriori (MAP) estimator

Observe that the MSE depends on the training matrix and on the covariance matrices of the channel and disturbance statistics, while it is unaffected by the mean values. Thus, the training matrix can potentially be designed to optimize the performance by adaptation to the second order statistics [9]–[12]. The intuition behind this training optimization is that more power should be allocated to estimate the channel in strong eigendirections (i.e., large eigenvalues). Observe that training optimization is useful in systems with dedicated training for each receiver, while multiuser systems with common training may require fixed or codebook-based training matrices (if users do not have the same channel statistics).

For general channel and disturbance statistics, the MSE minimizing training matrix will not have any special form that can be exploited when solving (9). However, if the covariance matrices are structured, the optimal may inherit this structure. Previous work in training optimization has showed that in Kronecker-structured systems with either noise-limited [9]–[11] or interference-limited [12] disturbance, the optimal training matrix has a certain structure based on the transmit-side channel covariance and temporal disturbance covariance. Herein, this result is generalized by showing that the same optimal structure appears in systems with both noise and interference. Then, we will show how the training matrix behaves asymptotically and under which conditions there exist explicit solutions to (9). Finally, we analyze how the statistics and total training power determines the smallest length of the training sequence necessary to achieve the minimal MSE. To summarize the results of this section, we have showed the structure of the MSE minimizing training matrix in Kronecker-structured systems and analyzed the allocation of power between the eigendirections. Based on these results, we propose a heuristic training matrix that can be applied under general system conditions. Observe that even when Kronecker-structured approximations are used in the training sequence design, the general MMSE estimator in (6) should always be applied without these approximations.

## III. Mmse Estimation Of Squared Channel Norms

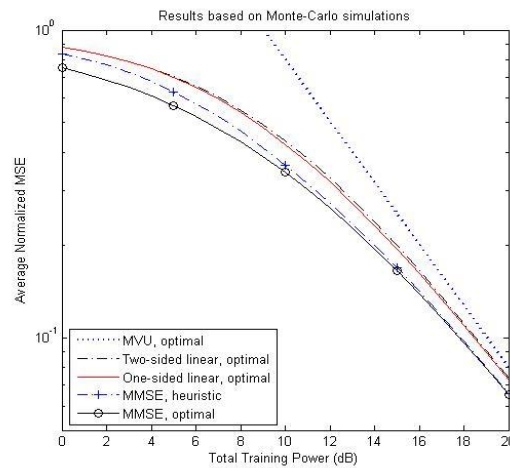
In many applications, it is of great interest to estimate the squared Frobenius norm of the channel matrix. This norm corresponds directly to the SINR in space-time block coded (STBC) systems and has a large impact on the SINR in many other types of systems [7], [8]. The channel norm can be estimated indirectly from an estimated channel matrix, for example using the estimator in (6). This will however lead to suboptimal performance and gives poor estimates at low training power. Thus, we consider training-based MMSE estimation of in this section

Analysis of the squared channel norm is considerably more involved than for the channel matrix. The next theorem gives a general expression for the MMSE estimator and its MSE, and special expressions for Kronecker-structured systems. In order to derive these expressions, we limit the analysis to training

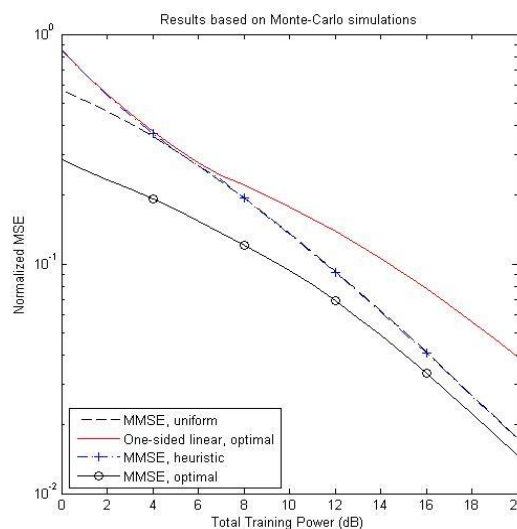
matrices with the structure . It is our conjecture that the MSE minimizing training matrix has this form,3aswas proved in Theorem 1 for channel matrix estimation. This training matrix structure is also of most practical importance,since the same training signalling will be used to estimate..low training power can be derived explicitly. Observe that theMSE depends on the mean value of the channel, while theMSE for channel matrix estimation is independent of the mean.The limiting solutions are however similar in the sense that allpower is allocated in a single Eigen direction at low power and are spread in all spatial direction at high power. The definition of the strongest direction at low training power and theproportional power distribution at largepower are however different, which means that the MSE minimizing training matricesusually are different for matrix and squared norm estimation.

In this section, the performance of the MMSE estimators andthe training sequence design will be illustrated numerically. TheMSE performance of the channel matrix estimator was thoroughly evaluated in [12] for interference-limited Kroneckerstructured systems. Thus, we consider the opposite setting of a noise-limited non-Kronecker-structured system, and we willcompare the MMSE estimation performance with other recently proposed estimators. This section will also illustrate the advantage of direct MMSE estimation of the squaredchannel norm over indirect calculation from an estimated channel matrix. Finally, we will illustrate how the smallest necessary length of thetraining sequence depends on the spatial correlation and available training power.

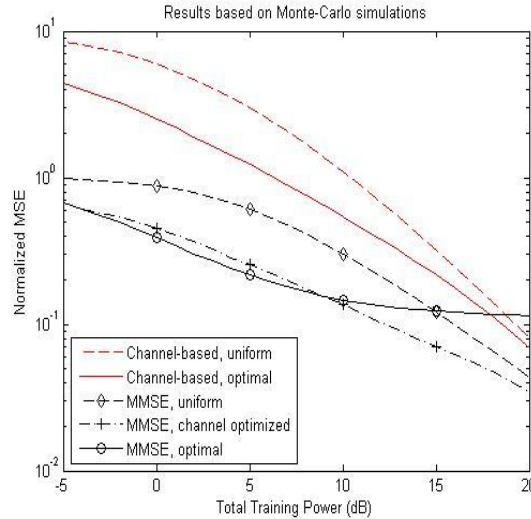
### OUTPUTS



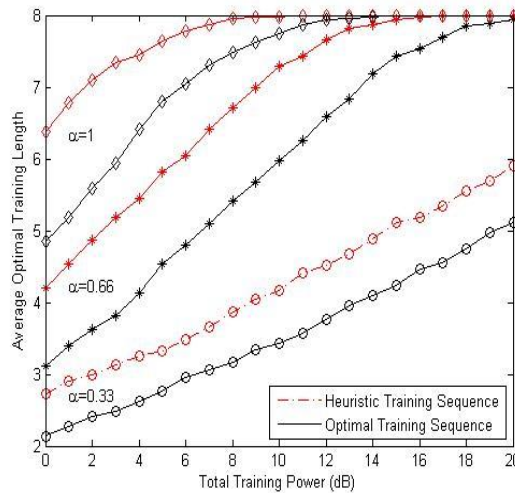
**Fig 1**The normalized MSEs of channel matrix estimation as a function of the total training power in a system with the Weichselberger model and the coupling matrix proposed



**Fig 2**The normalized MSEs of channel squared norm estimation as a function of the total training power in a system with uncorrelated receive antennas and a transmit antenna correlation of 2.4.



**Fig 3** The MMSE estimator with three different training matrices is compared with the one-sided linear estimator



**Fig 4** The average optimal training sequence length (smallest length that minimizes the MSE) as a function of the total training power

#### IV. Conclusion

The performance analysis given throughout this thesis is based on a large collection of new random matrix theory results, which were presented. The key utility of these results is that, in contrast to many existing results in random matrix theory, they involve simple finite expressions, and can be easily and efficiently evaluated. This is quite remarkable for many of the results, such as those involving noncentral matrix-variate quadratic forms, given the huge complexity of the underlying matrix-variate distributions. The capacity were derived based on the new random determinant properties of complex noncentral Wishart matrices and matrix-variate quadratic forms. The properties for the Wishart case were derived using matrix-variate integrals and various determinant operations, as well as a new determinant representation for the classical hypergeometric function  ${}_1F_1(\cdot)$  of a single complex matrix argument. The properties for the quadratic form case were derived by first using determinant operations to re-formulate the problems into ones involving only Wishart matrices, and then solving these with the help of the Wishart properties. The ergodic capacity bounds were initially obtained using two classical inequalities from information theory due to Jensen and Minkowski. In order to evaluate the bounds in closed form however, using the results of, it was first necessary to re-formulate the required expectations using an m.g.f. approach. A similar method was also employed to obtain the capacity variance results. The capacity results were derived based on the new expression for the joint unordered distribution of two (jointly) correlated complex Wishart matrices, given. This was obtained by marginalizing a joint eigenvalue distribution, using some determinant expansions. The performance results in Chapter 5 were derived based on the new expressions for the maximum eigenvalue distribution of complex central matrix-variate quadratic forms, given. The derivations in this case were particularly involved.

Specifically, these results were derived by directly marginalizing the joint eigenvalue distribution given in, which involves a  ${}_0F_0(\cdot)$  hypergeometric function of three matrix arguments; for which there is no equivalent determinant representation. Thus the marginalization was required to be carried out using the very complicated zonal polynomial infinite series representation given. The key tools used for handling this expression were advanced Vandermonde determinant operations, and the Cauchy-Binet Theorem (this was employed for re-summing the infinite series at the end).

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