

Synchronization Analysis of a New Autonomous Chaotic System with Its Application In Signal Masking

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Abstract: In this paper synchronization of a new chaotic system is analyzed and simulated using Matlab-Simulink and Orcad-Pspice programs. Firstly a new chaotic system is designed and analyzed, then Pecora-Carroll identical synchronization method is applied to design a drive and response subsystems. After synchronization, it is demonstrated that the new system can be applied to signal masking communication.

Keywords : chaotic systems, chaotic attractors, identical synchronization, chaotic masking.

I. Introduction:

Chaos is a complex nonlinear phenomenon which is intensively studied over the past two decades within science, mathematics and engineering communities. Recently chaos is found to be useful in many fields like power system protection, biometric system analysis, flow dynamics and liquid mixing and encryption. Chaotic signals are broadband, uncorrelated and difficult to predict thus they have potential for use in signal masking and as modulating waveform in spread spectrum system. Chaotic systems are deterministic systems which are characterized by high sensitivity to initial conditions and defined by simple ordinary differential equations. Many chaotic systems are defined and studied in Ref [1 - 11].

In 1990 Pecora and Carroll discovered that chaotic systems can be made to synchronize by linking them with a common signal, this is also known as identical synchronization. The idea of synchronization is to use the output of the master system to control the slave system so that the output of response system follows the output of the drive system asymptotically [12-14]. Generalized synchronization is experimentally demonstrated using Chua's circuit by Kocarev and U. Parlitz in 1992 and phase synchronization of two coupled systems are given by Michael G. Rosenblum [15, 17]. Synchronization of Lorenz system and its circuit implementation is given by Cuomo and Oppenheim in 1993. Using the concepts of self synchronization, two major applications are demonstrated with Lorenz circuit first is a chaotic signal masking and second is the chaotic binary communication applications [18-20]. Synchronization between chaotic signals has received considerable attention and led to various communication applications. Many researchers demonstrated with simulation that chaos can be synchronized and applied to secure communication schemes [21-29].

This paper deals with the signal masking application of chaotic signals. Here a chaotic signal is generated with a new autonomous three dimensional chaotic system and this signal is used as a masking signal. Information signal is added to the chaotic signal at transmitter and at receiver the masking signal is regenerated and subtracted from the receiver signal. For synchronization of transmitter and receiver, Pecora – Carroll method of identical synchronization technique is used. In this method, receiver consists of two subsystems and with one state received from transmitter, receiver is able to generate the same masking signal as the signal generated at transmitter. In section 2, a new chaotic system is defined by three ordinary differential equations and the complete analysis of the system is given. Using Simulink and Pspice simulation chaotic attractors of new system is shown. In section 3 Pecora – Carroll method is applied to new chaotic system and simulation results are shown. In section 4 masking application of chaotic system is given using Matlab-Simulink and Orcad Pspice programs. In section 5 conclusions are drawn.

II. Analysis and design of New Chaotic System:

The new system is described by following three ordinary differential equations

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -ax - by - cz - x^2\end{aligned}\tag{1}$$

System equation can also be written as jerk equation

$$\ddot{x} + c\dot{x} + bx + ax + x^2 = 0\tag{2}$$

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where x, y and z are the state variables and a, b, c are positive real constants. The system displays a typical chaotic attractor when $a = 1, b = 1.1$ and $c = 0.42$. The new system has two equilibrium points $(0, 0, 0)$ and $(-1, 0, 0)$.

The Jacobian of the system is

$$J = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2x - a & -b & -c \end{bmatrix}$$

(3)

The eigenvalues are $\lambda_1 = -0.745, \lambda_2 = 0.162 + 1.147 i, \lambda_3 = 0.162 - 1.147 i$ at the equilibrium point $(0, 0, 0)$ and $\lambda_1 = 0.589, \lambda_2 = -0.504 + 1.20 i, \lambda_3 = -0.504 - 1.20 i$ at the equilibrium point $(-1, 0, 0)$. As eigen values has positive real parts which implies chaos [30]. The system model is simulated in Matlab-Simulink with initial condition $x = 0.1, y = 0$ and $z = 0.1$. The system is realized with three integrators, one multiplier and one subtractor block. The system model and chaotic attractors of the new system are also shown in the Fig. 1.

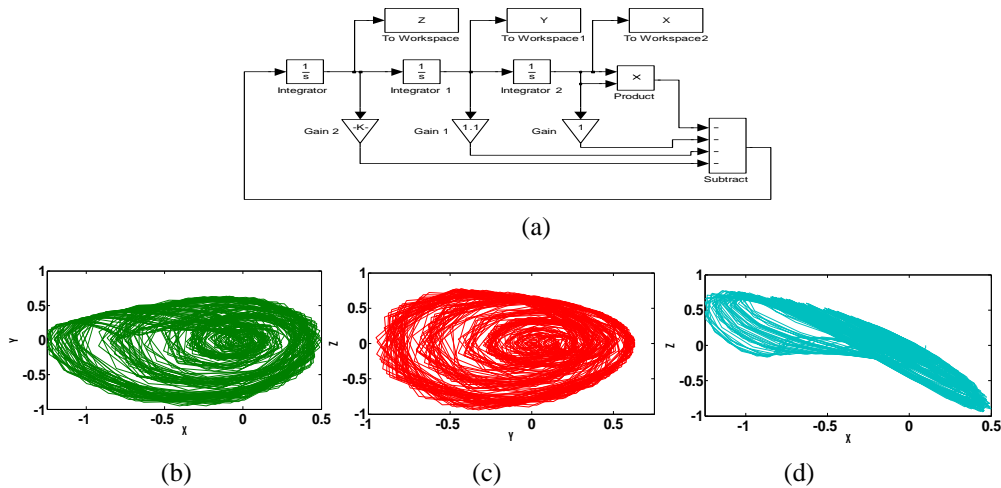


Figure 1. (a) Matlab – simulink model of new system, (b) $x - y$, (c) $y - z$, (d) $x - z$ phase portraits of chaotic attractors.

Circuit realization of new system is done using orcad pspice. The circuit is designed with five opamps, three capacitors, one multiplier AD633/AD and ten resistors. D.C. Power supply of $\pm 15V$ is used for Op-amp and multiplier AD633. Circuit Diagram and plots of chaotic attractors are shown in fig 2. Simulink and pspice simulation results are similar (Fig.1 and Fig.2).

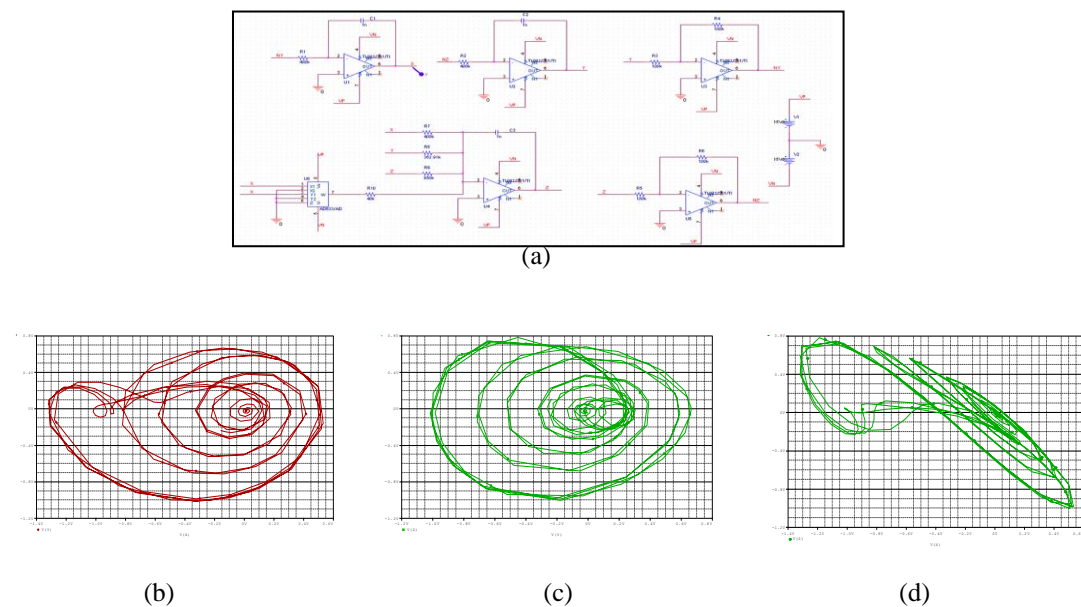


Figure 2. Pspice Circuit Schematic of new system and (a) $x - y$, (b) $y - z$, (c) $x - z$ phase portraits of chaotic attractors.

III. Synchronization of new system

To use a chaotic signal in communications, it is immediately led to the requirement that somehow the receiver must have a duplicate of the transmitter's chaotic signal, or better yet, synchronize with the transmitter. In fact, synchronization is a requirement of many types of communication systems, not only chaotic ones. The new system is synchronized using Pecora -Carroll method of identical synchronization. The drive response system is shown in fig.3 (a). Here the response system breaks into two subsystems to reproduce all states at receiver under the influence of a single chaotic signal from the drive system. Here x state is fed to the response subsystem1 and then y state of subsystem1 is applied to subsystem2. The output state of subsystem2 i.e. x_2 is same as the output state of drive system when system is synchronized.

The response subsystems of the drive system defined by equation (1) are defined by following equations

$$\begin{aligned} \dot{y}_1 &= z_1 \\ \dot{z}_1 &= -ax - by_1 - cz_1 - x^2 \quad \text{and} \\ \dot{x}_2 &= y_1 \\ \dot{z}_2 &= -ax_2 - by_1 - cz_2 - x_2^2 \end{aligned} \tag{4}$$

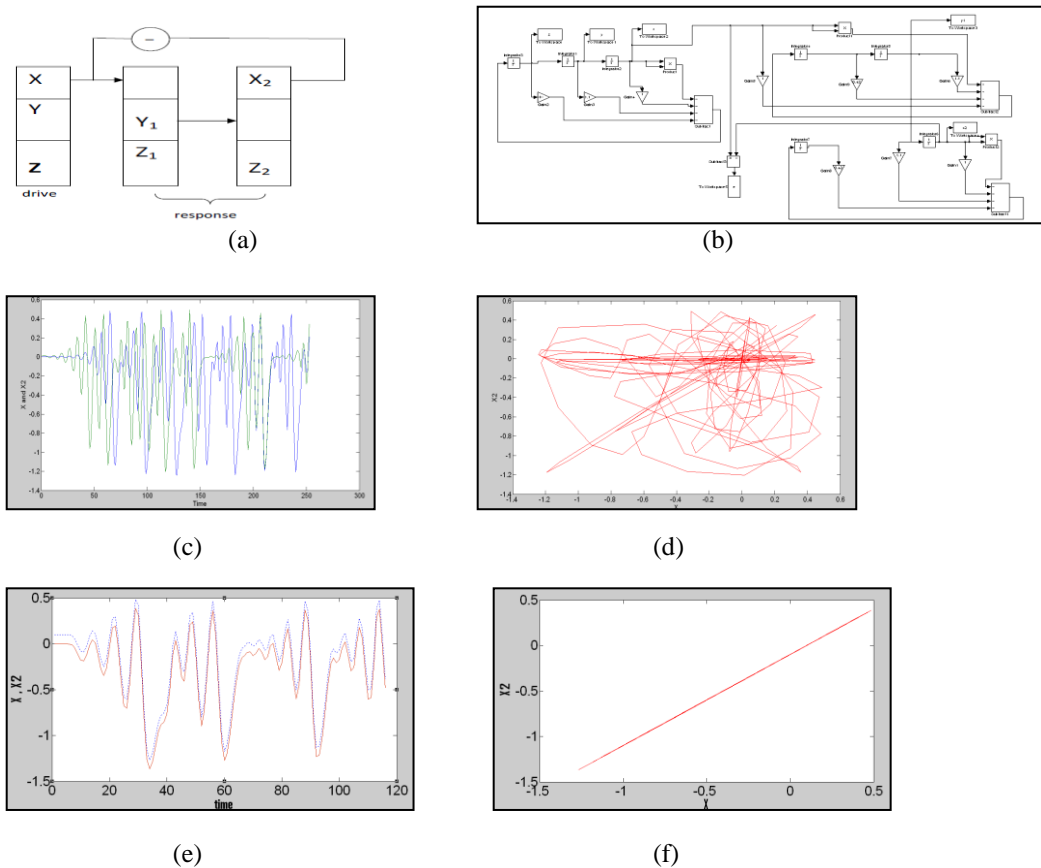


Fig.3 (a) Block diagram of drive and response subsystem, (b) Simulink model of new system with Pecora – Carroll synchronization , (c) Drive and response chaotic signals before synchronization , (d) Unsynchronized graph between x and x_2 , (e) Drive and response chaotic signals after synchronization , (f) Synchronization graph between x and x_2 .

When it is required to deal with coupled identical systems, synchronization appears as the equality of the state variables while evolving in time. If a stable response subsystem has been chosen, then the response's dynamic variables will converge to their counterparts in the drive and will remain synchronized with them as long as the drive continues to supply the one state variable to the response subsystem i.e. the key idea for identical synchronization is to choose a stable subsystem. The regime of identical chaotic oscillations is stable, when the synchronized trajectories are stable for the perturbations in the transverse direction to the synchronization manifold. This analysis is called transversal stability analysis [14]. To find out the stability of our subsystem let the difference system for $\Delta y = y - y_1$ and $\Delta z = z - z_1$ in matrix form is given in (5)

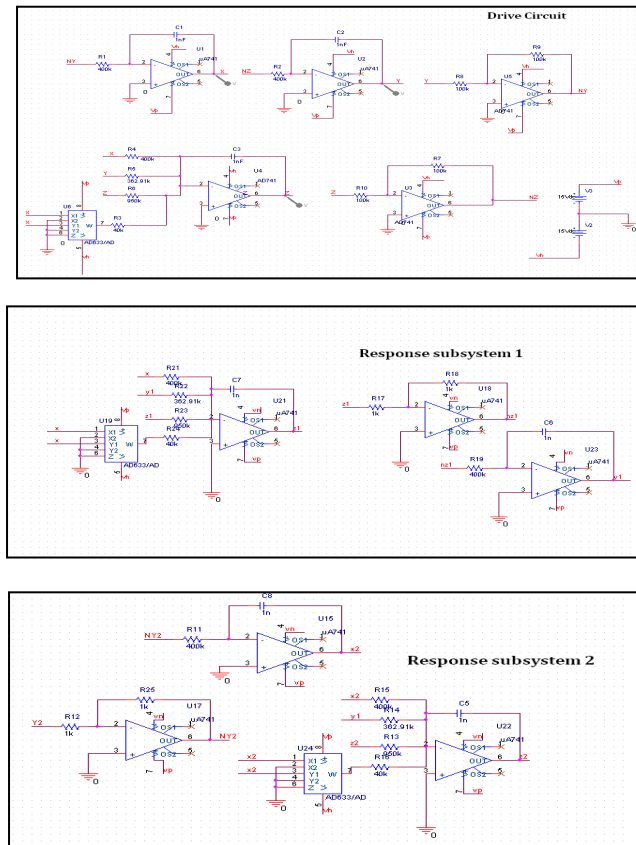


Fig. 4 Pspice circuit of synchronization of new system

$$\begin{bmatrix} \Delta \dot{y} \\ \Delta \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b & -c \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta z \end{bmatrix}$$

(5)

Equation (5) is the basic equation for describing the stability of the perturbation transverse under x drive signal from the drive dynamical system. Here the Jacobian matrix defined on difference system is a constant matrix, thus the eigen values for this matrix are $-0.21 \pm 1.027i$. The eigen values are negative for the onset of synchronization. The solution of (5) is given in (6).

$$\Delta y = \Delta z = e^{-0.21t}(k_1 \cos 1.027t + k_2 \sin 1.027t)$$

(6)

Here k_1 and k_2 are constants of integration. It is clear that as $t \rightarrow \infty$, $\Delta y = \Delta z = 0$, Hence drive and response systems are synchronized if x state is selected as reference signal for receiver system. Simulink circuit blocks for new system for implementing synchronization are shown in fig.3 (b). The drive and Response system are identical. The initial value of drive system is (0.1, 0.1, 0.1) and the initial value of the response system1 is (0.1, 0.1) and subsystem 2 is (0.1, 0.1). The simulation output results of synchronize case and unsynchronized case for the new system is given in Fig 3(c) – (f). Figure 4 shows the circuit diagram for implementing synchronization of new system using Pspice. Figure 5 shows Pspice simulation results for synchronize and unsynchronized cases. Simulink and Pspice simulation give the same conclusion.

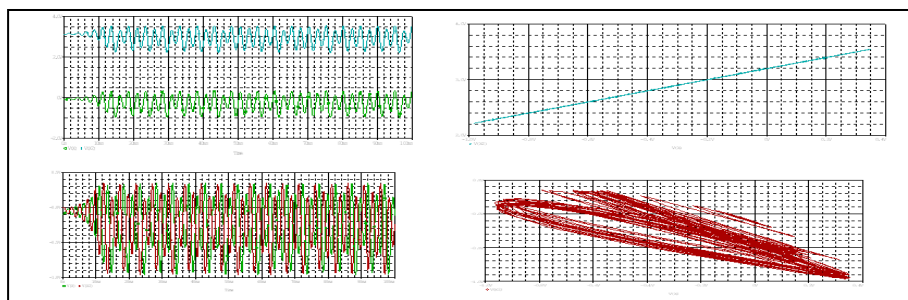


Fig 5. Pspice simulation output of synchronized and unsynchronized chaotic signals and their phase portraits

IV. Chaotic Masking Circuit using Orcad Pspice and Matlab Simulink:

By using Pecora and Carroll method the response system is able to generate the same chaotic signal as signal generated by drive circuit. Thus the system can be used for chaotic masking circuit. Fig. 6(a) shows the block diagram of transmitter and receiver system and Fig. 6(b) shows the simulink model of the system with signal masking. The parameters of transmitter and receiver are identical for implementing chaotic masking application. The information signal $I(t)$ is a sinusoidal signal of amplitude 0.02V. The information signal is added to the $x(t)$ chaotic signal and a sum signal $= x(t) + i(t)$ is feed to the receiver. In the receiver subsystem2 chaotic signal $x_2(t)$ is generated and it is subtracted from sum signal to get the required received information signal $i_r(t)$. Due to synchronization of drive and response circuit $x_2(t) = x(t)$ and $i_r(t) = i(t)$. Figure 6 (c) shows the x and x_2 signal. Fig. 6 (d) shows the sum signal and information signal $i(t)$ and $i_r(t)$ is shown in Fig.6 (e).

Pspice simulation of masking circuit is shown in figure 7-8. Transmitter and receiver circuits are identical except their initial conditions. The initial condition for transmitter is (0.1, 0, 0) and receiver subsystems is (0, 0) and (0, 0). Fig. 9 shows the Pspice simulation results of the chaotic masking circuit. Information signal is completely recovered at receiver. Pspice results and Simulink results are same.

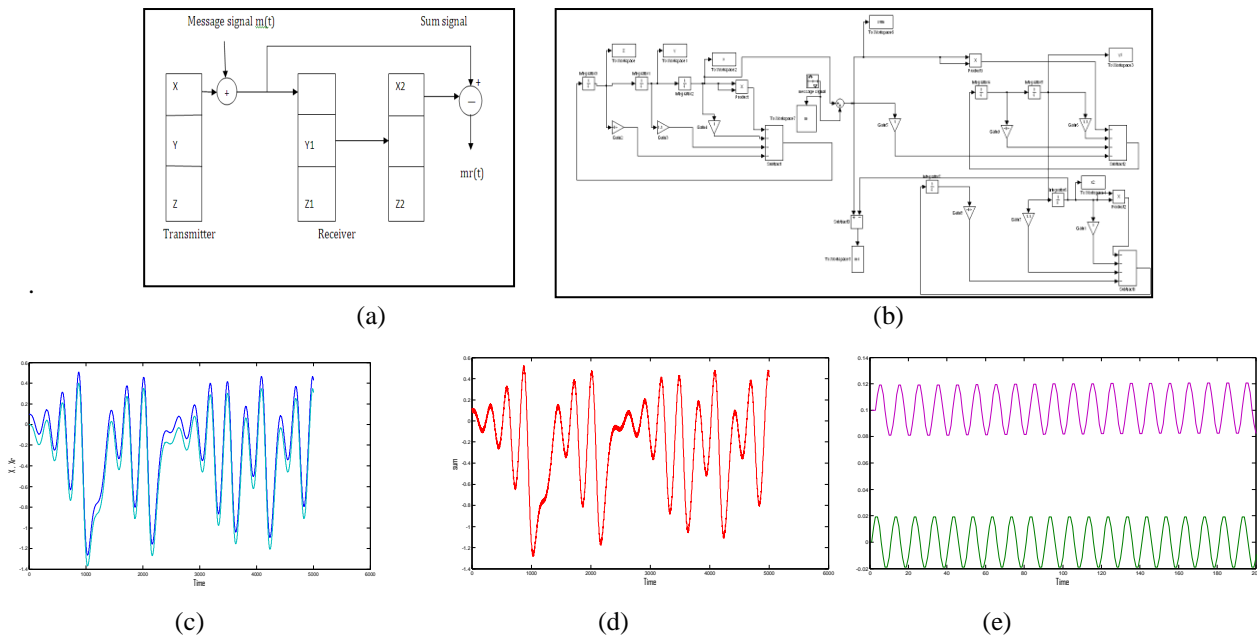


Fig.6(a)Block diagram of chaotic signal masking application , (b) Simulink model for chaotic masking circuit, (c) x and x_2 signal, (d) Transmitted Sum signal $= x(t) + i(t)$, (e) Message signal $i(t)$ and retrieved message signal $i_r(t)$.

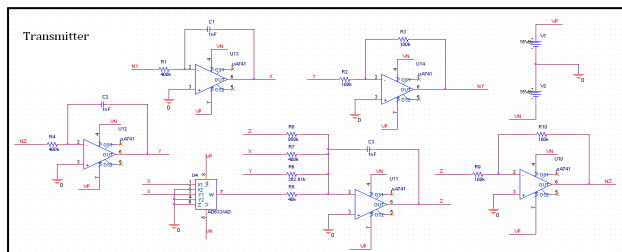
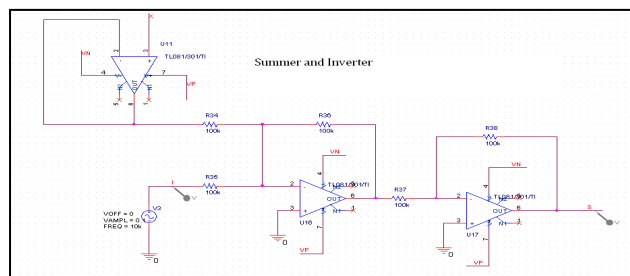


Fig.7 Transmitter circuit



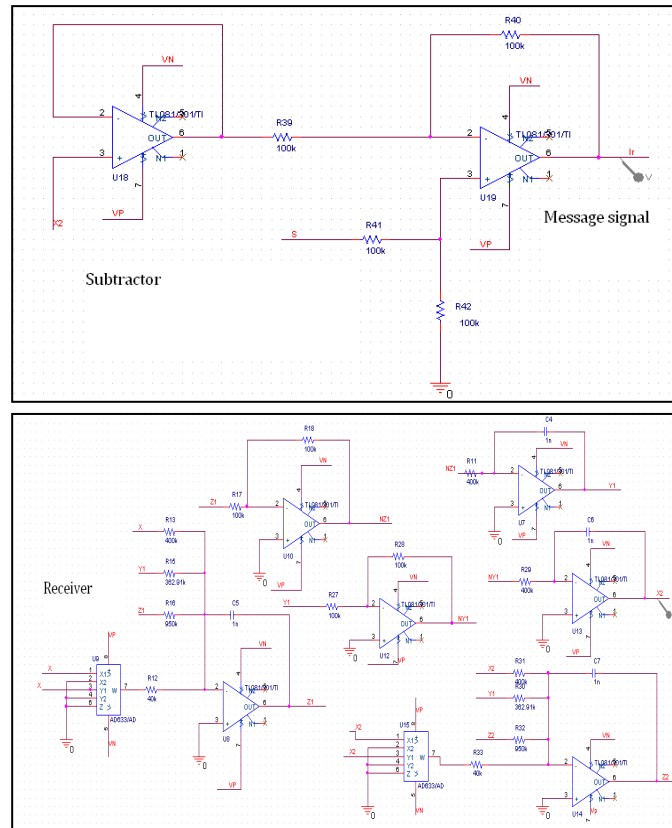


Fig 8 summer and Inverter, Subtractor circuit and receiver circuit

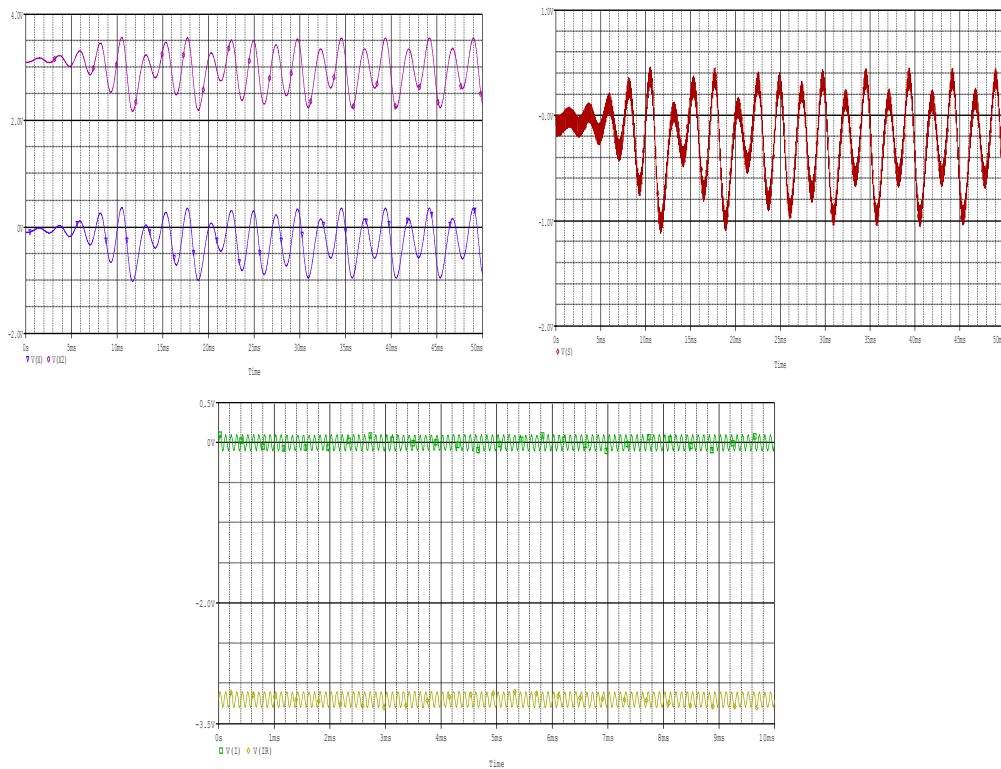


Fig.9.Pspice outputs of Receiver circuit x and x_2 signal, Sum signal, $i(t)$ and received signal $i_r(t)$.

V. Conclusion:

In this paper identical synchronization in new chaotic system has been investigated by implementing Pecora-Carroll technique. The stability of identical synchronization has also been investigated using extensive transversal stability analysis. It is also demonstrated with simulation that the synchronized drive and response system can be used in signal masking application. The simulation results show that the new system is made for use in secure communication.

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