

## **Second order Integral Sliding Mode Control: an approach to speed control of DC Motor**

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**Abstract:** *In this paper, a second order sliding mode controller is applied for single-input single-output (SISO) uncertain system. The presented controller successively overcomes the variations caused by the uncertainties and external load disturbances although an approximate model of the system is used in the design procedure. An integral type sliding surface is used and the stability and robustness properties of the controller are proved by means of Lyapunov stability theorem. The chattering phenomenon is significantly reduced adopting the switching gain with the known parameters of the system. Thus, the presented controller is suitable for long-term application to the systems those are having fast response closed-loop behaviours. The performance of the developed control scheme is validated by simulation in Mathworks MATLAB and the results are compared with the similar controllers presented in the literature. In order to verify the performance comparison of second order integral sliding mode approach, first, a sliding mode control system with a PID sliding surface is adopted to control the speed of an electromechanical plant. In this, a sliding mode controller is derived so that the actual trajectory tracks the desired trajectory despite uncertainty, nonlinear dynamics, and external disturbances. The sliding mode controller is chosen to ensure the stability of overall dynamics during the reaching phase and sliding phase. The stability of the system is guaranteed in the sense of the Lyapunov stability theorem. Second, the sliding mode approach for stable systems which is available in the literature is designed with tuning parameters as per their guidelines. And finally conventional controller is designed based on the Zeiglar-Nicholas approach. All controllers applied to the electro-mechanical system to verify the performance in terms of time domain and performance error indices. The chattering problem is almost removed in the presented approach in comparison with other controllers. This problem is overcome using a hyperbolic function in the switching control considering the small constant term in case of the zero error to avoid the problem of zero division.*

**Keywords:** *DC Motor, integrating sliding surface, PID Controller, stability of the systems, non-singular terminal sliding surface, second order sliding mode control, parametric uncertainty.*

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### **I. Introduction**

Many of the industrial processes have complex dynamics with time delay and non-linear behaviour in addition to known or unknown disturbances. The performance of low order control system design, such as with proportional-integral-derivative (PID) controller is less effective for complex order processes or processes with parametric uncertainty. In literature, the design methods of PI/PID controllers for complex behaviour processes are adopted from time to time, resulted from the research algorithms, e.g. [1]- [3]. Most of the PI/PID tuning relations for complex behaviour systems, reported in literature are based on the reduced first order plus delay time (FOPDT) or second order plus delay time (SOPDT) plant models [4,5].

The process model order reduction introduces plant-model mismatch and hence the designed controller may affect the stability and robustness of the system, especially in the presence of disturbances. To control processes with complex behaviour dynamics, sliding mode control (SMC) plays a significant role since it ensures stability, disturbance rejection and insensitivity to parameter variations [6]. The design procedure of SMC starts with defining the sliding surface passing through the origin of phase plane to reduce the error equal to zero. SMC comprise of two phases, namely reaching phase and sliding phase. In reaching phase, the system variables start from initial condition and reach the sliding surface within finite time while in sliding phase, they travel along the sliding surface and reach the zero error condition with the motion governed by the sliding surface [7]. The two terms in control law corresponding to two phases, a continuous control term (equivalent control) brings system variables on sliding surface and a discontinuous control term (switching control) maintains them on the sliding surface, need to be derived separately. However the implementation of SMC introduces undesired high frequency oscillations called chattering phenomena, due to discontinuous switching function (signum) which causes control signal to oscillate around the sliding surface [8]. The chattering results in excessive wear and tear of the actuators and even may excite the unmodeled high frequency dynamics of the system. The problem can be overcome by replacing signum function with a smooth function such as saturation or hyperbolic tangent (tanh) function considering an ultimate boundedness of the error within some

predetermined boundary layer [8]- [13]. Many researchers have developed SMC using FOPDT or SOPDT process models e.g. [9], [10], [14]- [16].

In this paper, the performance comparisons between the conventional sliding mode and PID controllers with integral second order sliding mode control (IS-SMC) have been analyzed. A IS-SMC is applied to a electromechanical system, that is DC motor. The comparisons of the performance responses for all control schemes are analysed in terms of which technique results an excellent robustness in responses to system parameter uncertainties, load disturbances and in case of noisy measurement. The simulation results show that the IS-SMC performs better compared to Conventional SMC and the classical PID controller.

This paper is organized as follows: In section II, modelling of DC motor followed by the design of the Sliding mode controllers, conventional controller and PID controller is included. Section III includes, implementation of SOISM in Matlab2009 as well as implementation of conventional and PID controller. In this section, the detail analysis of performance of various controller is included to show the usefulness of SOISM approach. Finally, conclusion and direction to the further work is included in section IV.

## II. Modeling Of Electro-Mechanical System

The system consists of a dc motor and some loads on the shaft of the motor. The DC motors are widely used in the industrial applications since the useful property of easy adjustability of the position and speed control under load disturbance [3]. In general, the feedback is provided with a tachogenerator directly connected to the shaft. The tachogenerator produces output voltage proportional to the shaft speed. The system can be modelled using first-order plus dead-time model(FOPDT) since its effective way for the real systems where dead-time is relatively small. The procedure is based on the process reaction curve method which is simple to understand so that any operator can easily apply. The FOPDT model of the system is obtained as follows:

$$\frac{Y(s)}{U(s)} = \frac{k.e^{-t_d s}}{(\tau s + 1)} \tag{1}$$

If the time delay is so small compared with time constant, the system model can be approximated as follows:

$$\frac{Y(s)}{U(s)} = \frac{K}{(\tau s + 1)(t_d s + 1)} = \frac{C_n}{s^2 + A_n s + B_n} \tag{2}$$

In Equation 2, Taylor series approximation is used for delay time. The procedure to obtain the model of the experimental system is given as follows:

1. Use data acquisition card (DAQ) card to interface the real system to computer.
2. Connect tachogenerator's output to analog input channel of the DAQ. This is for the measurement of the speed.
3. Apply step input from DAQ to DC motor, say 5 V from the PC via DAQ to the DC motor, Record the output speed using step 2 above.
4. Plot the graph of the output, in this case, speed of the motor.
5. For example, 5.5 V DC is given to the motor and the speed (Output in terms of tachogenerator voltage) is plotted and the model parameters are obtained using process reaction curve.

In this paper, the model parameters as per Furat and Eker [32] are used extensively to design the SOISM. The model of the system is as

$$G(s) = \frac{Y(s)}{U(s)} = \frac{0.86e^{-0.0035s}}{(0.145s + 1)} \tag{3}$$

or after delay approximation

$$\frac{Y(s)}{U(s)} = \frac{0.86}{(0.145s + 1)(0.0035s + 1)} = \frac{1694.6}{s^2 + 292.6s + 1970} \tag{4}$$

The transfer function of the system is a second-order over-damped transfer function with two poles located on the left half part of the complex plane. Rearranging the system model as

$$\ddot{y}(t) + 292.6\dot{y}(t) + 1970y(t) = 1694.6u(t) \tag{5}$$

If the uncertainty and external disturbances to the system are introduced into Eq. 5,

$$\ddot{y}(t) = -292.6\dot{y}(t) - 1970y(t) + 1694.6u(t) + D(t, u(t)) \tag{6}$$

$D(t, u(t))$  is less than or equal to  $D_{max}$ , where  $D_{max}$  is the upper bound on the uncertainty with  $D_{max} > 0$ .

In this paper, the proposed sliding surface is a type of I-SMC, as follows:

$$\sigma(t) = [kp + \frac{ki}{s} + kds]^{(n-1)} \psi(E(s)) \tag{7}$$

Where, the last term indicates the integral sliding surface while  $K_j$  is the PID tuning parameters which help to define sliding surface given in Eq. 7 and selected by the designer, and determines the performance of the system on the sliding surface.

Second Derivative of the error is

$$\ddot{e}(t) = \ddot{r}(t) - \ddot{y}(t)$$

With Eq. 6, and considering set point is constant term, the equivalent controller is given by

$$u_{eq}(t) = \frac{1}{k_d C_n} [k_p \dot{e}(t) + k_i e(t) + k_d \ddot{r}(t) + k_d A_n \dot{y}(t) + k_d B_n y(t)] \quad (8)$$

The control input in the conventional SMC has the following form:

$$u(t) = u_{eq}(t) + u_{sw}(t) \quad (9)$$

The switching control taken in this work is as follows

$$u_{sw}(t) = k_{sw} r^2(t) e(t) \operatorname{sgn}\left(\frac{k_{sf}}{e(t)} \dot{\sigma}(t)\right) + \frac{1}{k_d C_n} \operatorname{sgn}(\sigma(t)) \quad (10)$$

Where  $k_{sw}$  is a positive gain used to limit chattering keeping the tracking performance.

### III. SMC Parameters And Results

In this section, parameters of a second-order integral sliding mode controller, conventional SMC and SMC with PID sliding surface are obtained for electromechanical system. The model of the electromechanically system is given in previous section and the obtained parameters for the sliding modes are used in the simulation work to obtain the performance. It is observe that the second-order integral sliding mode controller has better performance as compared to other SMC approaches and PID controller. The results of integral second order sliding mode approach are compared with the conventional SMC, SMC for stable processes and Zeigler-Nicholas PID controller in terms of transient response and performance error indices.

Table 1: Parameters of second-order integral sliding mode control

Terms	parameters value	Remark
Sliding surface	$k_p = 1.2, k_i = 10^{-4}, k_d = 0.0024$	-
Equivalent control	$r = \text{set point}, y(t) = \text{Output}$	$A_n, B_n, C_n$ plant parameters
Switching Control	$k_{sw} = 40, k_{sf} = 0.025$	$\epsilon_1 = 10^{-20}$

Table 2: Parameters of conventional sliding mode control

Terms	parameters value	Remark
Sliding surface	$\lambda_1 = 292.6, \lambda_0 = 2.1405 \times 10^4$	-
Equivalent control	$r = \text{set point}, y(t) = \text{Output}$	$\tau, t_d, K$ plant parameters[Ref. Eq. 3.3]
Switching Control	$K_D = 10.05, \delta = 304.2$	-

Table 3: Parameters of PID sliding surface sliding mode control

Terms	parameters value	Remark
Sliding surface	$k_p = 260, k_i = 0.01, k_d = 24$	-
Equivalent control	$r = \text{set point}, y(t) = \text{Output}$	$A_n, B_n, C_n$ plant parameters
Switching Control	$\beta = 25, k_{sw} = 100$	-

Table 4: Parameters of the PID controller

Type of Controller	$K_p$	$T_i$	$T_d$
PID	58.38	0.0055	0.0014

#### IV. Performance Analysis

In this section, suppose the parameters of the system given in Eq. 4 are considered. The sliding mode approaches with parameters given in previous section. The sliding mode controllers are implemented using the above parameters and performance of the system is analyzed. The reference speed is 1200 RPM (revolution per minute). The speed of the motor with various approaches is shown in Fig.1, the input to system is shown in Fig. 2. The sliding surfaces with all SMC approaches are shown in Fig. 3 while the error signal in RPM are shown in Fig. 4.

From the Fig. 1, it is clear that the performance of the proposed controller is far superior than the approach of Eker [9], Camacho [10] and conventional PID controller. The rise time is less and there is no overshoot by the proposed controller. The conventional Zeigler-Nicholas tuned PID controller gives large overshoot and it is concluded that such type of response is not suitable for applications where DC motor is used as a main controlled system. The response given by the Eker is sluggish with larger settling time while the response of the Camacho approach is fast compared to Eker but slow as compared to proposed approach. From Fig. 2, it can be seen that the control effort by the proposed controller is smooth while that of the Eker and Conventional PID controller is continuously oscillatory. The control signal by the Camacho is smooth and similar to the proposed approach. Even though the control signal by Camacho approach is smooth, but the sliding surface cannot go to zero as time tends to infinity as shown in Fig. 3. The sliding surface of the proposed controller goes toward zero as time tends to infinity.

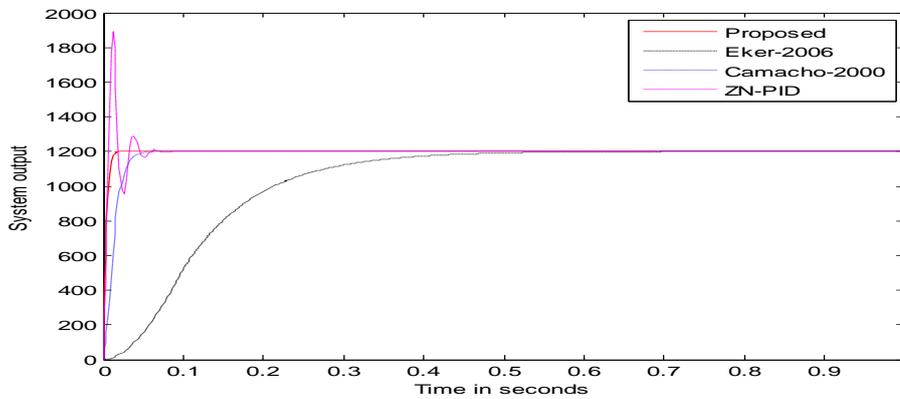


Fig. 1 The speed of the motor

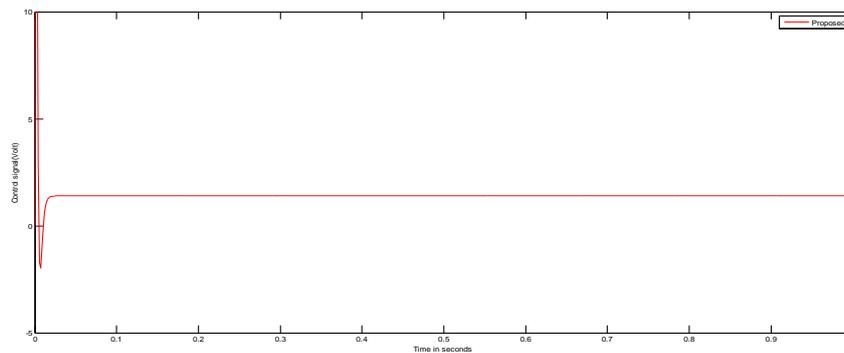


Fig. 2 a) The input to the motor by proposed controller

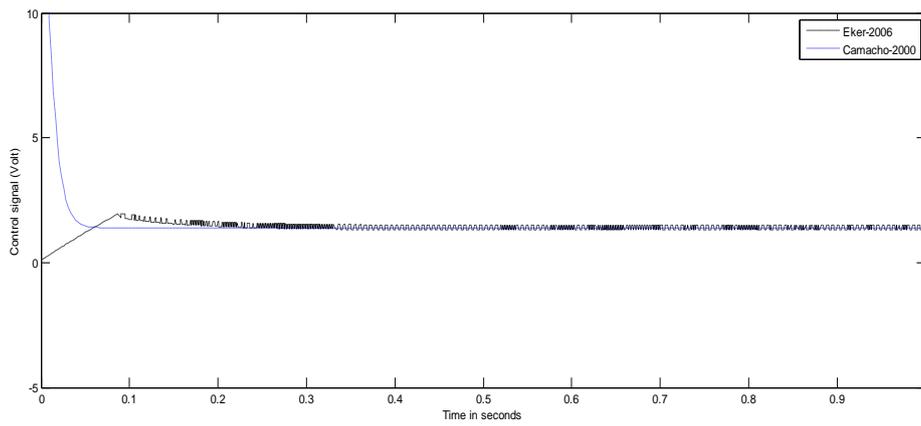


Fig. 2 b) The input to the motor by Eker and Camacho controller

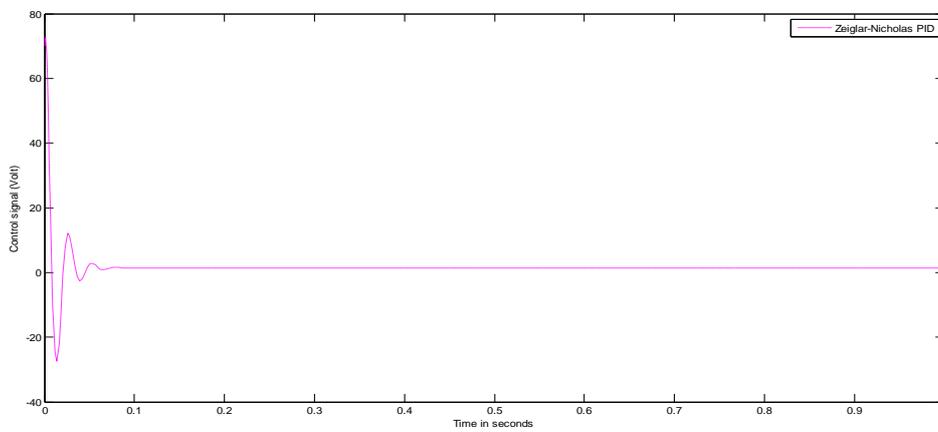


Fig. 2 c) The input to the motor by Zeiglar-Nicholas PID controller

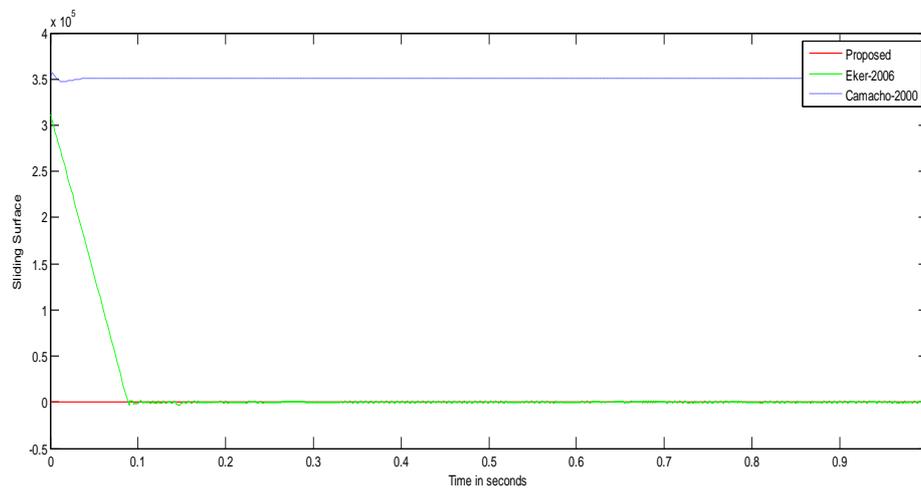


Fig. 3 The sliding surface

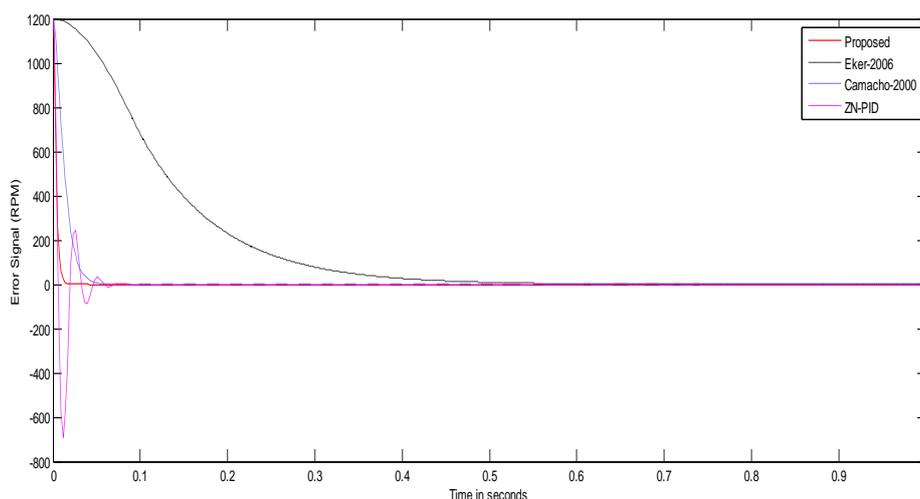


Fig. 4 The error signal

## V. Conclusion

The method of second order integral sliding mode, conventional SMC, SMC with PID sliding surface and PID controller for electromechanical system is explained. The results obtained for various approaches gives the conclusion that the performance given by the second order integral sliding mode approach is far superior than that of the other approaches. The second order integral sliding mode approach algorithm for effective speed control of electromechanical system is clearly discussed. The second order integral reaching law approach has been successfully used to design the sliding control laws for the mathematical model of electromechanical system. A comparisons with conventional SMC, SMC with PID sliding surface and Zeigler-Nicholas based PID controller are used for the speed loop have also been discussed. Simulation results demonstrate that the developed second order integral sliding control scheme has a much better performance related to reduction in steady state error, faster settling time, smaller overshoot in the speed response and much better disturbance rejection capabilities. Chattering however persists and further work can be done to eliminate it by selecting suitable parameters of the switching law.

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