Performance Analysis of Threshold Selection in Energy Detector Working Over Noise Uncertainty Channel for Cognitive Radio Networks

TARIQ KANAAN

(Doctor, information and communication Department /Syrian Private University, Syria)

Abstract:
In this paper, noise uncertainty and its effect on energy detector performance were studied in more depth. Further, we are motivated to decide the optimal threshold allow SNR (signal-to-noise ratio) in such a way where we can jointly achieve both sensing matrices (P_{fa}=0.1 and P_{d}=0.9) and provided better sensing performance in comparison to that of the traditional constant false-alarm rate CFAR and constant detection rate (CDR) threshold selection approaches. Further, we have illustrated that at low SNR, the proposed optimal threshold selection approach has provided better throughput as compared to that of the threshold selected by traditional CDR approach.

Key Word: Cognitive Radio; Spectrum Sensing; Noise uncertainty; CONSTANT DETECTION RATE (CDR); Constant False-Alarm Rate. CFAR.

Date of Submission: 08-05-2021  Date of Acceptance: 23-05-2021

I. Introduction

The key concern of next generation communication systems (NGCS) is to fulfill the demand of spectrum for various services such as high-speed internet, internet-of-things (IoT) [1], and user-centric mobile applications [2]. The radio frequency spectrum is a scarce resource, which is already allocated to different services for example, the voice-telephony, military services, satellite and radar services etc. [3]. Therefore, this spectrum scarcity restricts the introduction of new services/devices that require the spectrum. However, a report of the Federal Communication Commission (FCC) reveals the fact that most of the allocated spectrum remains underutilized/unutilized at specific time and space [4]. This finding has motivated the concept of spectrum reuse by allowing the unlicensed/cognitive users (CUs) to utilize the licensed/allocated spectrum of the primary users (PUs) when the spectrum is temporally unexploited/underutilized. In this context, the dynamic spectrum allocation (DSA) [5, 6] allows the CUs to utilize the spectrum in such a way that the licensed user/PUs communication remains impervious [7–9]. The cognitive radio (CR) is a framework, which supports the DSA mechanism by exploiting the cognitive cycle that comprises four elements [10] namely, (1) spectrum sensing, (2) spectrum analysis and decision, (3) spectrum sharing/accessing, and (4) spectrum mobility. Initially, the CU senses its radio environment to perceive the state of channel being either active or idle by employing the spectrum sensing techniques [11]. Further, the idle sensed channels are analyzed, and the suitable idle channel is selected for essential application. Moreover, the selected channel is accessed for communication via the preferred spectrum accessing technique i.e. interweave, underlay, overlay and hybrid [12, 13]. The emergence of PU during the CU communication is a prospective event and at this instant, the CU need to stop or switch the communication on another idle channel. The process of switching the communication on another idle channel is known as spectrum mobility or handoff [14]. The spectrum sensing (SS) is a prime step of cognitive cycle which exploits the following major techniques to detect the channel states, namely, (1) energy detection spectrum sensing (EDSS) [15–18], (2) matched filter (MF) detection [19, 20], (3) cyclostationary feature detection (CFD) [21], (4) covariance absolute value detection (CAV) [22], and (5) eigen-values based detection (EVD) [23, 24]. Further, these techniques are classified as blind (EDSS, CAV, EVD) and non-blind (CFD, MF) spectrum sensing.

The non-blind spectrum sensing techniques entail the information about the PUs signal (such as the modulation type, carrier frequency, frame structure, pulse shaping etc.) at the CU terminal which is in general, difficult to yield however, in the blind spectrum sensing, there is no such prerequisite. Moreover, the comparison of different sensing approaches has been presented in [25, 26] and it is observed that the EDSS has significantly less computation and implementation complexity; therefore it is widely used spectrum sensing technique. In EDSS, the energy/test statistics (T) of the received signal is compared with the predefined threshold value (\lambda) and when the energy of received signal is greater than or equal less than the threshold value,
the sensing result is in favor of channel being active/idle, respectively. The sensing decision in the EDSS relies on the threshold value, therefore, the computation and selection of threshold is a very prominent aspect.

In addition, the key sensing performance metrics are the false-alarm probability and detection probability. The false-alarm probability \( P_{fa} \) is the probability of CR user decision in favor of channel being busy while in actual it is idle however, the detection probability \( P_d \) is the probability of CR user decision in favor of channel being busy when the PU signal is actually there. The low numerical value of \( P_{fa} \) (approx: \( \leq 0.1 \)) is required for maximum utilization of channel, while the high numerical value of \( P_d \) (approx: \( \geq 0.9 \)) is required to provide protection to PU. For example, in IEEE 802.22 (WRAN) standard, for TV signal detection, it is required to achieve 90% probability of detection and 10% probability of false-alarm at SNR level as low as -20 dB with maximum sensing time of 25 ms required in order to achieve the sensing requirement [27].

Energy detection method attracts the attention of the researchers worldwide for its low computational and implementation complexity, and it does not need any information about the primary signal, [28–29]. However, the sensing performance of the energy detector will be affected seriously by the change of noise variance or what called noise uncertainty phenomena. In fact, noise uncertainty increase the number of samples demanded to achieve sensing performance and in the certain value of SNR called SNRwall the number of sample become infinity. Therefore, for any value of SNR less than SNRwall the energy detector could not achieve the required sensing performance for any value of number of samples [30].

Many researches have been proposed approaches to reduce noise uncertainty effect on the performance of ED [31–35]. However, in [32], could not achieve the targeted detection probability. Whereas, in [32–35], have been concerned with improving the performance of the detector in terms of detection probabilities and false alarm without taking in their consideration that high computation complexity even if when the SNR has high value (greater than critical SNR).

Within this paper, we proposed a more accurate model for energy detector with dynamic double threshold detector that work to detect PU signals over noise uncertainty channel. In our model, we considered difference noise expansion coefficients and found a new equation of number of sample and SNRwall. Afterwards, Distinct thresholds are computed for CFAR and CDR approaches for a chosen number of samples (N) and received primary SNR at CU. Thereafter, the condition for a single optimal threshold is analyzed to achieve the desired values of Pf and Pd simultaneously at all SNRp. However at low SNR region, we have observed that the threshold with CFAR approach is greater than the CDR approach (\( \lambda_f > \lambda_m \)), therefore the optimality condition for the selection of threshold has not been satisfied as discussed in detail in Sect. 2.4. Further, we used the optimal number of samples and dynamic double thresholds such that the same optimality condition is satisfied even at low SNR.

Finally, The closed-form expressions of different spectrum sensing performance metrics such as the probability of detection, the probability of false-alarm, and the have been computed for the proposed approach and compared with the state-of-art work. Thereafter, throughput for the proposed approach has been computed and compared with reported literature.

This paper is structured as follows. The related work is presented in Sect. II. A noise uncertainty model of the proposed framework is described in Sect. III. Section IV comprises a performance analysis of the proposed system model and The MATLAB simulation results are presented. Finally, Sect. V concludes

II. Literature reviews:

Energy Detector:

The Energy Detector is a special case of the Estimator Correlator in the case of modeling the noise signal as a Multivariate Gaussian Distribution [36]. It is also considered as the optimal detector when detecting Independent and Identified Distributed IID signals, and it is one of the most popular detectors for its simplicity as well as not requiring prior information about the signal to be detected.

The basic concept of Energy detection (ED) method in [15] is depicted in Fig. 1. It involves continuous monitoring of the received signal energy by the SUs and its comparison to a threshold to make decision on the spectrum opportunity by the fusion center.

\[
\begin{align*}
\text{Received Signal} & \xrightarrow{\text{BPF}} A/D & \sum X(n)^2 & > y & \rightarrow H_1 \\
& & & < y & \rightarrow H_2
\end{align*}
\]

Fig 1 Spectrum sensing via ED

The local decision of presence or absence of a vacant spectral space is confirmed by testing binary hypotheses (H0 and H1) by each cognitive radio user [37] as described in (1):

\[
\begin{cases}
    y(n) = \sim(n) \text{under } H_0 \\
    y(n) = \tilde{\sim}(n) + \sim(n) \text{under } H_1
\end{cases}
\]  

(1)

DOI: 10.9790/1676-1603010721 www.iosrjournals.org 8 | Page
Where: \( y(n) \) represents the received signal at the secondary receiver at the moment \( n \), \( \hat{s}(n) \) is the PU signal, \( \sigma^2 \) represents an additive white Gaussian noise signal with zero mean and variance denoted by \( \sigma^2 \).

The binary hypothesis \( H_0 \) and \( H_1 \) are considered to identify the status of channel i.e. idle and active, respectively. The test statistics \( T_N \) for EDSS is given as [13]:

\[
T_N(y) = \frac{1}{N} \sum_{n=0}^{N-1} y^2(n) \quad (2)
\]

Where \( N \) represents the number of samples taken during the sensitivity period.

The probability density function (PDF) of test statistics \( T_N \) under hypothesis \( H_0 \) and \( H_1 \) follows a Chi square distribution with \( N \) degree of freedom for real valued noise. For a sufficient high number of samples (\( N > 256 \)), the PDF of \( T_N \) under hypothesis \( H_0 \) and \( H_1 \) followed the Gaussian distribution [37]. In such a case, the probability of detection (\( P_D \)) and the probability of false alarm (\( P_{FA} \)), respectively as follows [31]:

\[
P_{FA} = P_r(T_N(y) > \lambda; H0) = Q_N\left(\frac{\lambda}{\frac{2}{N}\sigma^2_n}\right) \quad 3
\]

\[
P_D = P_r(T_N(y) > \lambda; H1) = Q_N\left(\frac{\lambda}{\frac{2}{N}(\sigma^2 + \sigma^2_s)}\right) \quad 4
\]

Where \( Q(x) = \frac{1}{\sqrt{\pi x}} \int_x^\infty e^{-\frac{u^2}{2}} du \) is the Gaussian commentary cumulative distributive function and \( \lambda \) represent the detector’s threshold.

**Degrading Effects of Noise Uncertainty:**

In the previous discussion, we assumed that the receiver knows the value of the noise power perfectly. However, in practice, this is not possible, and there is uncertainty in the noise estimation and interference is one of the most important causes of this uncertainty.

Noise uncertainty is modeled as a set of statistical distributions [39], that is, the values of noise power fall within the range \( [\sigma^2_{sL}, \sigma^2_{sH}] \). Therefore, the noise in the event of uncertainty can be modeled as Gaussian white noise of zero mean, and variance within the range \( \sigma^2_{sL} \leq \sigma^2 \leq \sigma^2_{sH} \). In this case, the test statistic is modeled as follows [40]:

\[
T_N(y) \sim N\left(\frac{\sigma^2_{sL}}{2N}\sigma^2_{sH}\right) \quad \text{under } H_0
\]

\[
T_N(y) \sim N\left((\sigma^2_{sL} + \sigma^2_{sH})\frac{2}{N}(\sigma^2_{sL} + \sigma^2_s)^2\right) \quad \text{under } H_1
\]

Similarly, we can write the relationships for \( P_D \) and \( P_{FA} \):

\[
P_{FA} = P_r(T_N(y) > \lambda; H0) = Q_N\left(\frac{\lambda - \sigma^2_{sH}}{\frac{2}{N}\sigma^2_s}\right) \quad 6
\]

\[
P_D = P_r(T_N(y) > \lambda; H1) = Q_N\left(\frac{\lambda - (\sigma^2_{sL} + \sigma^2_s)}{\frac{2}{N}(\sigma^2_{sL} + \sigma^2_s)}\right) \quad 7
\]

From the previous two equations, we can deduce the number of samples required to obtain the desired \( P_D \) and \( P_{FA} \) values:

\[
N = \frac{[Q^{-1}(P_{FA})\eta_n - Q^{-1}(P_D) - Q^{-1}(p_D)SNR]^2}{[SNR - (\eta_n - 1)]^2} \quad 8
\]

Whereas \( \eta_n = \frac{\sigma^2_{sH}}{\sigma^2_{sL}} \).

Equation (8) shows that at a certain value of the signal-to-noise ratio called the signal-to-noise ratio wall, denoted by \( \text{SNR}_{\text{wall}} \), \( \text{SNR}_{\text{wall}} = (\eta_n - 1) \), the number of samples required for a detector seeks to infinity, that is, the detector will not able to detect the Primary User signal whatever The number of samples was taken.
Threshold Selection Approaches:

The function of CU in spectrum sensing is to detect the spectrum opportunities. One of the techniques for detecting the unused licensed bands is the energy detection spectrum sensing (EDSS) for which, the selection of threshold defines sensing detector performance. In general, the fixed threshold (FT) and the dynamic threshold (DT) methods are employed for the selection of threshold in EDSS technique. In the fixed threshold, the threshold remains constant even with the change in SNR, however in order to incorporate channel variations, the dynamic threshold method has been proposed which varies its threshold with the channel conditions in order to minimize the probability of error in sensing results. Various have illustrated it researchers [33, 41, 42] that the threshold selection with DT method provides better spectrum sensing result as compare to that of the FT method.

In fixed threshold method, the threshold is mainly selected with constant false-alarm rate (CFAR) approach however, in dynamic threshold; it is selected by using either constant detection rate (CDR) approach or by minimizing error probability (MEP) approach.

CFAR approach computes the value of threshold to maximize the detection probability. In this context, Gandhi and Kassam [43] have presented that the CFAR approach is used to identify the status of target frequency band when it shows the unknown/dynamic distributions and it has been observed that its performance is highly degraded in the presence of abrupt variation in noise and interfered signal. Thereafter, Kortun et al. [44] have also illustrated that the threshold selection using CFAR approach does not perform well in the presence of noise uncertainty, hence the eigen-values based detector is employed to decide the threshold in order to enhance the sensing performance. Moreover, the throughput has been maximized by keeping fixed sensing time in the presence of noise uncertainty. Further, Lehtomaki et al. [45] have achieved a significant improvement in the sensing performance by employing forward-detection methods with CFAR when multiple PUs are presented in the chosen environment. In addition, Mahdi et al. [46] have decided the threshold using CFAR and empirical mode decomposition (EMD) techniques to maximize the detection probability and have identified multiple channels in the given spectrum band. Recently, the authors in [47] employed CFAR and improved the throughput as compared to conventional ED by employing simultaneous sensing and transmission using a single antenna at CR terminal.

Moreover, in [48], the authors have employed CDR approach to yield the detection threshold and computed the value of throughput. Further, Koley et al. in [49] have presented that CDR approach is suitable to provide sufficient protection to PU from CU however with reduced throughput in comparison to the CFAR approach. In this context, in order to provide sufficient protection to PU and high throughput to CU, Gaurav and Sahu [50] have employed the combination of CFAR and CDR approaches to decide the threshold.

In order to minimize the overall sensing error, MEP approach has been employed in various literatures. In [51, 52], the dynamic value of threshold has been achieved by minimizing the error probability with respect to the threshold for Gaussian channel. Further, Choi et al. [53] have decided the transmit power of CU and then accordingly changed its sensing threshold dynamically so that the PU and CU can communicate on the same channel without interfering to each other. However, Joshi et al. [54, 55] have used the gradient descent algorithm to minimize the error function without employing the transmitted power of CU and have found the dynamic value of threshold. Moreover, in [56], the authors have discussed the maximum allowable power that can be transmitted by the CU and have decided the threshold value according to the relative position of CU towards the base station. In addition to this, Yu et al. [57] have observed that there is spectral leakage in vacant frequency band when the PU has used high transmit power and to resolve this problem, the authors considered variable sensing duration, dynamic selection of threshold and utilized multitap-windowed FFT processing technique for the targeted value of false-alarm and detection probability. Further, it is presented that dynamic threshold is a more suitable method instead of increasing the sensing duration to yield the desired value of detection. Moreover, Ling et al. [58] have selected the dynamic threshold according to a linear function of signal to-interference plus noise ratio (SINR) and maximized the CU throughput. Further, in [59], the sensing performance has been improved with the use of prior available PU spectrum utilization information. Moreover, the authors have assumed that the previous status of the spectrum is known and selected the threshold according to the previous state information. However, Ding et al. [60] have discussed the spectrum prediction techniques based on the models of spectrum usage, sources of spectrum data and the predictability of spectrum evolution for better utilization of spectrum and to make CR more intelligent. However, Kerdabadi et al. [61] have maximized the throughput by jointly optimizing the threshold value, sensing time and user selection for sensing and data transmission. Moreover, in [62], adaptive threshold is selected using covariance based channel selection with intelligent way to minimize the probability of error with required protection to PU. Further, in order to improve the detection probability and sensing time, Benedetto and Giunta [63] have employed constant energy (CE) technique considering the signals whose energy per data-block remains fixed and the variance of the received signal energy over M block is used to decide the status of channel. Moreover, the weighted covariance- based detection (WCD) [64] is used to enhance the performance of CAV. However, the demerit
with WCD befalls when a large number of low correlated receiving antennas are there and hence results in less primary user detection. Furthermore, to overcome the aforementioned problem of WCD, Chen et al. [65] have used Ljung-Box (LB) test to detect the presence of PU signal in the above-mentioned scenario and provided significantly better performance under noise uncertainty. furthermore, Xiong et al. [66] have presented adaptive spectrum sensing strategy (ASSS) which utilized the PU traffic pattern to find the channel to be sensed having more possibility of being idle. Further to improve the energy efficiency in full-duplex cognitive radio (FDCR), Bayat and Ai’ssa [67] used the concept of contiguous sensing by inserting sleep period between sensing without any significant degradation in throughput parameter. Recently, Kumar et al [68], reveal an excellent algorithm for optimal threshold selection. However, their model did not take noise uncertainty phenomena in their consideration. whereas, Mahendru et al [69], proposed a mathematical model of ED working in noise uncertainty environment based the correlation between number of samples and signal to noise ratio but their model was only based increasing the number of samples and lacked of determining threshold that needed to successful performance.

Optimal Threshold Condition for Threshold Selection:

From Fig. 2, it is perceived that the false-alarm probability \( (P_{FA}) \) and miss-detection probability \( (P_m) \) shows a direct and an inverse relation with the threshold \( (\lambda) \). In the CFAR and CDR approach, we have fixed the maximum permissible value of false-alarm and miss-detection probability and computed the corresponding value of the threshold \( \lambda_f \) and \( \lambda_m \) respectively. In Fig. 2, it is clear that to minimize the false-alarm, the threshold \( \lambda_f \) (threshold for CFAR approach) needs to be as high as possible while to minimize the miss-detection, the threshold \( \lambda_m \) (threshold for CDR approach) needs to be as low as possible. Therefore, it has been observed from the above discussion that the above two conditions will be satisfied only when \( \lambda_m \geq \lambda_f \) (optimal threshold condition) which satisfy both the false alarm and miss-detection probabilities, simultaneously [70].

![Fig.2 Threshold selection in hypothesis model](image)

III. Our new model for noise uncertainty

Our New Model for Noise Uncertainty:

In previous researches, the noise uncertainty was studied by setting limitations on the noise expansion coefficients by assuming that the lower noise expansion coefficients are equal to the reciprocal of the noise upper expansion coefficient [71]. However, under noise uncertainty conditions, the fluctuation of noise power leads to the deterioration of the detector’s performance differently, and we can distinguish between two cases. The first case occurs when the noise power decreases, and this lead to a decrease in the received signal power below the threshold level, thus the decrease of the value of the probability of detection \( p_D \) will be below the nominal value. whereas, the second cases occurs when the noise power increased higher than detector’s threshold, and this increase the power of received signal and may cause to detect noise signals as primary user signals and which lead to an increase in the probability of false alarm \( p_{FA} \) values.

Therefore, within this paper, the noise uncertainty problem was analysed in more detail with assuming that noise uncertainty model had different noise expansion coefficients \( \rho_L \) and \( \rho_H \), where \( \rho_L \) denotes to a the lower noise expansion coefficient, while the \( \rho_H \) denoted to the higher noise expansion coefficient. Whereas, in detector side, we assumed a new dynamic double threshold detector with two different threshold coefficients, the lower coefficient denote by \( \rho_{L}' \), that used to enhance the probability of \( p_D \) through decreasing the threshold value when the noise power decreased, whereas the upper coefficient \( \rho_H \) that used to enhance \( p_{FA} \) through increasing the threshold value when the noise power increased. So that the limits of our proposed detector can be written as:
\[
\begin{align*}
\lambda \in \left[\frac{\lambda}{\rho_L}, \rho_H \lambda\right] \\
\lambda^* \in \left[\frac{\lambda}{\rho_L}, \rho_H \lambda\right] \\
\end{align*}
\]
Probability of detection can be defined as considering both threshold values:
\[
p_D = \min_{\lambda^* \in \left[\frac{\lambda}{\rho_L}, \rho_H \lambda\right]} \min_{\lambda \in \left[\frac{\lambda}{\rho_L}, \rho_H \lambda\right]} Q \left(\frac{\lambda - (p + \sigma_w^2)}{\frac{2}{N} (p + \sigma_w^2)}\right) = Q \left(\frac{\rho_H \lambda - \rho_H \lambda^*}{\frac{2}{N} \rho_H \sigma_w^2}\right) \\
\]
\[
p_{FA} = \max_{\lambda^* \in \left[\frac{\lambda}{\rho_L}, \rho_H \lambda\right]} \max_{\lambda \in \left[\frac{\lambda}{\rho_L}, \rho_H \lambda\right]} Q \left(\frac{\lambda^* - \sigma_w^2}{\frac{2}{N} \sigma_w^2}\right) = Q \left(\rho_H \lambda - \rho_H \lambda^*\right)
\]
Now by using equation 11 to find the value of \(\lambda\):
\[
\lambda = \frac{\rho_H \sigma_L^2}{\rho_H} \left(\sqrt{\frac{2}{N} Q^{-1}(p_{FA}) + 1}\right)
\]
For getting the equation that represent Receiver Operation Characteristic ROC, we substitute the value of \(\lambda\) in equation:
\[
p_D = Q \left(\frac{\rho_H}{\rho_L \rho_H} \left(\sqrt{\frac{2}{N} Q^{-1}(p_{FA}) + 1} - \frac{\text{SNR} + \frac{1}{\rho_L}}{\sqrt{\text{SNR} + \frac{1}{\rho_L}}}\right)\right)
\]
From previous equation, we find the number of samples as follow:
\[
Q^{-1}(p_D) = \sqrt{\frac{2}{N} \left(\text{SNR} + \frac{1}{\rho_L}\right)} = \frac{\rho_H}{\rho_L \rho_H} \left(\sqrt{\frac{2}{N} Q^{-1}(p_{FA}) + 1} - \left(\text{SNR} + \frac{1}{\rho_L}\right)\right)
\]
\[
Q^{-1}(p_D) = \sqrt{\frac{2}{N} \left(\text{SNR} + \frac{1}{\rho_L}\right)} = \frac{\rho_H}{\rho_L \rho_H} Q^{-1}(p_{FA}) + \frac{\rho_H}{\rho_L \rho_H} - \left(\text{SNR} + \frac{1}{\rho_L}\right)
\]
\[
Q^{-1}(p_D) = \sqrt{\frac{2}{N} \left(\text{SNR} + \frac{1}{\rho_L}\right)} = \frac{\rho_H}{\rho_L \rho_H} Q^{-1}(p_{FA}) - \left(\text{SNR} + \frac{1}{\rho_L}\right)
\]
\[
N^* = 2 \left[\frac{\rho_H}{\rho_H} Q^{-1}(p_{FA}) - Q^{-1}(p_D) \left(\rho_L \text{SNR} + \frac{\rho_L}{\rho_L}\right)\right]^2 \left[\left(\rho_H \text{SNR} + \frac{\rho_H}{\rho_H}\right) - \frac{\rho_H}{\rho_H}\right]^2
\]
The previous equation leads us to drive a new equation for the signal-to-noise ratio that makes the number of samples infinite or what is called the signal to ratio wall \(\text{SNR}_{\text{wall}}\):
\[
\text{SNR}_{\text{wall}} = \left(\frac{\rho_H}{\rho_H} - \rho_H\right) \left(\frac{1}{\rho_L}\right) \left(\frac{1}{\rho_H}\right)
\]
In CDR approach, the miss-detection probability is fixed ($P_m$ fixed) and the corresponding value of threshold with the help of (10) has been computed as follows:

$$\lambda_f = \rho_H \lambda_f = \rho_H \sigma_w^2 \left( \sqrt{\frac{2}{N}} Q^{-1}(p_{f_H}) + 1 \right)$$

In (16), $\lambda_f$ is the optimality threshold condition.

The condition for critical SNR ($SNR_c$) under noise uncertainty:

Critical SNR is defined as the SNR below which $\lambda_f \geq \lambda_m$ and optimality threshold condition will not be satisfied. For the fixed value of $N$, we have computed the minimum $SNR_p$ at which the optimality condition is satisfying and is computed by equating Eqs. (16) and (17) as follows:

$$SNR_c = \frac{\rho_H \sigma_w^2 \left( \sqrt{\frac{2}{N}} Q^{-1}(p_{f_H}) + 1 \right)}{\left( \sqrt{\frac{2}{N}} Q^{-1}(p_D) + 1 \right)} - \frac{1}{\rho_L}$$

Optimal number of samples under noise uncertainty:

As we mentioned, determining the optimal number of sample is very important because it is used to determine threshold when $SNR < SNR_c$.

The figure 3 depicted the number of samples curves as a function of received SNR for various values of noise uncertainty coefficient. These curves also show the contribution of both coefficient to the noise uncertainty, and that any change in any of these two factors cause a deterioration in the overall detector performance due to an increase in the signal-to-noise ratio wall, which makes it necessary for the energy detector to use a dynamic threshold and estimate each of the two coefficient separately.

From the previous curves, we can see that we can find finite value of number of sample that achieve detector performance in case the SNR greater than $SNR_{wall}$. Therefore, we proposed algorithm to find both $N^*$ and $\lambda_{opt}$ by selected $\lambda_{opt} > \lambda_f$ when $SNR > SNR_c$ and applying dynamic double threshold detector on other $\lambda_{opt} < \lambda_f$ case as follow:

The following pseudocode depict the algorithm that we have adopted to determine $N^*$:

Algorithm 1: Our proposed energy detector with adaptive double thresholds

1. Input $P_0, P_{f_H}, \rho_H, \rho_L, SNR, \rho_D, \rho_H$
2. Output $N^*, \lambda_{opt}$
3. Compute $\lambda_f, \lambda_m$ from equations (17) and (18) respectively
If $\lambda_f \leq \lambda_m$
5. $\lambda_{opt} \leftarrow \lambda_f$
6. Else $\lambda_m < \lambda_f$
7. $\lambda_{opt}$ is not possible
8. Compute Number of samples $N^*$ from equation 14
9. $N^* \leftarrow N$
10. Compute $\lambda_f, \lambda_m$ from equations 16 and 17 respectively for $N^*$
11. Select $\lambda$ from range $[\frac{\lambda^2}{\mu_L}, \mu_H\lambda]$ correspond to the deviate value of $\sigma^2_w$ in the range $[\frac{\lambda^2}{\mu_L}, \mu_H\sigma^2_w]$
12. $\lambda_{opt} \rightarrow \lambda$
13. END

System model and Throughput computation:

The integration of CUs with lesser priority which should transmit their messages in a way that PU of the licensed channel would not be adversely affected is the key concern of CRN. The spectrum sensing is a critical aspect of CR systems that intend to identify the working state of PU before allowing the CU temporarily accesses the channel without causing harmful interference to the PU. In the proposed system model, we are considering single band spectrum sensing with a pair of PU and CU transceiver and assumed that the PU receiver is in the range of CU transmitter as shown in Fig. 4(a). In an anticipated band of interest, the probability of Fig. 4 (b) The proposed a system model and b frame structure of cognitive user [73] Wireless Networks

The throughput has been categorized into two cases as follows. In case-1, the PU is absent in the channel and no false-alarm is generated while in case-2, the PU is present in the channel and it is not detected by the CU. The throughput of first and second cases are denoted by $R_0(T_s)$ and $R_1(T_s)$, correspondingly. In a chosen frequency band, we have considered that $P(H_1)$ and $P(H_0)$ are the probability of channel being active and idle, respectively and average throughput, $R(T_s)$ for CU has been computed as follows [73]:

$$R_0(T_s) = \left(\frac{T - T_s}{T}\right) (1 - p_{FA}) \log_2 (1 + SNR_s)$$

$$R_1(T_s) = \left(\frac{T - T_s}{T}\right) (1 - p_D) \log_2 \left(1 + \frac{SNR_p}{1 + SNR_p}\right)$$

Where the $SNR_s$ is the SNR for the secondary link. The total average throughput for CU is as:

$$R(T_s) = P(H_0)R_0(T_s) + P(H_1)R_1(T_s)$$

IV. Results and Discussion

In this section, we have presented the numerically simulated results for sensing performance parameters i.e. the probability of detection, the probability of false alarm and throughputs. Further, the numerically simulated results for the threshold values and number of samples.

The simulation environment is yielded using the MATLAB 2010 [74]. Moreover, the values of simulation parameters are selected based on IEEE 802.22 wireless regional area network (WRAN) standard where the minimum number of samples assumed is more than 256. Indeed, we considered four cases of noise variation; case1: when there is no noise uncertainty and this represented by ($\mu = 1, \mu_L = 1$), case2 when the...
noise power is equal or higher than and this represent by (\( \rho_H = 1.4, \rho_L = 1.4 \)), case3 when the noise power is equal or less than and that represent as (\( \rho_H = 1, \rho_L = 1.4 \)), case4 when the noise power fluctuated between \( \rho_H \) and \( 1/\rho_L \) that represent as (\( \rho_H = 1.4, \rho_L = 1.4 \)).

The variations in threshold value for the CFAR when \( \rho_H = 1 \) and \( \rho_H = 1.4 \), CDR when \( \rho_L = 1 \) and \( \rho_L = 1.4 \). with received primary SNR are presented in Fig. 5. The threshold is constant with SNR in CFAR approach. However, the value of threshold \( \lambda_f \) in CFAR increased when \( \rho_H = 1.4 \) whereas the value of threshold \( \lambda_f \) in CDR decreased when \( \rho_L = 1.4 \).

According to noise uncertainty cases, we can distinguish between four different values of critical SNR (SNRe) unlike when there is no noise uncertainty where there is only one value of SNRe [50]. For the case1 SNRe is denoted by SNRe1 and it has the lowest value between other values of SNRe, whereas, in the second cases of noise uncertainty the SNRe is denoted by SNRe2 and it has the value less than SNRe3 in the case3 and the highest value of SNRe is SNRe4 and it occurs in case4.

![Graph showing variations in threshold value with SNRp for CFAR and CDR approaches at N=15000](image)

We have defined the critical SNR (SNRe) as that SNRp value below which \( \lambda_f \geq \lambda_m \). Further, it is depicted from the Fig. 5 that at higher value of SNR(SNRp > SNRe), the optimal threshold condition (\( \lambda_f \leq \lambda_m \)) is verified. However, at low SNR (SNRp <SNRe), the optimal threshold condition is not satisfied as is illustrated in Fig. 5. Further, the variations in sensing performance parameters (Pf, Pd) with SNRp for CFAR and CDR approaches are presented in Fig. 6.

In CFAR approach, the Pf value is constant (0.1) at every value of SNRp, while the value of Pd in case1 (no noise uncertainty) is less (0.9) and approximately (0) for other cases when SNRp > SNRe. However, pd achieve detector’s performance when SNRp >= SNRe for all cases.
In CDR approach, the $P_d$ value is constant (0.9) for all values of $\text{SNR}_p$, while the value of $P_f$ is higher than (0.1) in case1 and equal to (1) for all other cases when $\text{SNR}_p < \text{SNR}_c$ and it decreases and achieve detector's performances when $\text{SNR}_p > \text{SNR}_c$ for all noise uncertainty cases.

It is clear that when $\text{SNR} < \text{SNR}_c$, CFAR and CDR does not meet detector performance for all any cases. Moreover, any changing in the value of noise power than nominal value could deteriorate the detector performance drastically. Further, the variation in the achievable throughputs of CU with $\text{SNR}_p$, for CFAR and CDR approaches with fixed number of samples ($N=15,000$) are presented in Fig. 7. It is clear from Fig. 7 that in CFAR approach, the throughput value decreases with $\text{SNR}_p$ from 7.539 to 4.674 bps/Hz and afterwards become constant in case1 whereas in case2, case3 and case4 the throughput value decreases with $\text{SNR}_p$ from 7.916 to 4.674 bps/Hz and then become constant.

While in CDR, as illustrated in fig, the throughput increases from 0.9256 bps/Hz to 5.319 bps/Hz and thereafter remains constant in case1 whereas the throughputs curves have lesser value when $\text{SNR} < \text{SNR}_c$ in other cases where they start from 0.3252 bps/Hz to 5.319 bps/Hz and thereafter remains constant.
We have proposed an approach in Sect. 3 to outcome the optimal number of samples ($N^*$) to get desired value of $P_{FA}$ and $P_{D}$, simultaneously. In this context, the performance of sensing parameters ($P_{FA}$, $PD$) when $\rho_H = 1.4, \rho_L = 1.4$ with SNR$p$ for the proposed approach is compared with [50] and presented in Fig. 8. In the proposed approach, we have achieved both targeted values of $P_{FA} = 0.1$ and $P_{D} = 0.9$, simultaneously when SNR$p <$SNR$c$. While in [50], authors have fixed one of the sensing parameter (either $P_{FA}$ or $P_{D}$) and have tried to improve the other ($P_{D}$ or $P_{FA}$), as is illustrated in Fig. 10. However, the required sensing performance improvement was not achieved in [50] when SNR$p < $SNR$c$ besides authors did not take noise uncertainty in their consideration.
Fig. 10. Sensing parameters variation with SNRp for the proposed approach

Moreover, the comparison of throughput variation with SNRp are presented in Fig. 11. It is clear that CFAR approach has the highest throughputs when SNR < SNRc but the pd value is far less than 0.9 as shown in Fig. 10. However, our proposed method achieve both of sensing performance and has better throughput than CDR approach when SNR < SNRc and when SNR > SNRc proposed approach has throughput performance equal to throughput in CDR approach that is the highest throughput when SNR > SNRc.

Fig. 11. Throughput variation with SNRp for the proposed approach

V. Conclusion and future scope:

In this paper, we have exploited the threshold computation using CFAR and CDR approaches in noise uncertainty environment. We have supposed the noise uncertainty model has two separate expansion coefficients and found the new equation to calculate the number of sample with reducing the effect of SNRwall. Thereafter, we have analyzed the optimality condition for threshold and selected the appropriate threshold that has been achieved with the anticipated values of Pp and Pd simultaneously. Further, the computation of SNRp as a critical SNR (SNRc) below which the optimality condition is not satisfied and increased based upon expansion coefficients of noise uncertainty besides the highest value of SNRc occurs when both coefficient have highest values. Moreover, we have proposed an approach in order to satisfy the optimality condition even though at low SNR (SNRp<SNRc) and have computed the throughput for the proposed approach. It has been perceived that at low SNR, the throughput for proposed approach is higher than CDR approach. However, less
than that of CFAR approach. Moreover, the throughputs achieved using CDR and CFAR approaches are not satisfying the desired $P_F$ and $P_D$ values when compared with the proposed approach.

Hence, in this proposed approach, we have achieved the maximum throughput while achieving the desired $P_F$ and $P_D$ simultaneously at all SNR. However, in this paper, our proposed approach does not fully blind, because the detector needs to know the values of noise expansion coefficients and the value of noise derivation form nominal values. However, we can overcome this issue by exploiting the probed detector in the terms of cooperative spectrum sensing. Moreover, the multiband spectrum sensing in the proposed approach can also be considered when PU changes its state during the sensing period.

References


DOI: 10.9790/1676-1603010721 www.iosrjournals.org 21 | Page