

# Out-of-focus Point Spread Functions of Optical Systems in the Presence of Primary Spherical aberration

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## Abstract

The out-of-focus point spread function of optical systems in the presence of primary spherical aberration has been investigated. Considerable reduction of the central maximum intensity and the presence of non-zero first minimum put a severe restriction for Rayleigh criterion to be applied for two-point resolution studies for these systems.

**Key Words:** PSF, Primary Spherical Aberration, Defocusing.

## I. Introduction

It is now well known that the diffraction image of a point never a point. It is a patch of light distributed over a finite region in the image plane. This is known as the Point Spread Function (PSF) of the optical system and is commonly known as Airy pattern when the optical system is purely diffraction-limited. The PSF of an optical system is an important parameter for studying its imaging characteristics. In other words, the performance of an optical system significantly depends on its PSF [1].

The study of imaging properties of optical systems suffering from aberrations from the knowledge of the PSF has become an important method in the design and testing of such systems [2]. In our present study, we have investigated the PSF of an optical system suffering from primary spherical aberration and in the presence of defect-of-focus.

## II. Theory

The far-field effects due to a circular aperture in an optical system can be derived from its amplitude response or the amplitude PSF. The diffracted light amplitude associated with a rotationally symmetric pupil is given by [3],

$$A(Z) = 2 \int_0^1 f(r) \exp \left[ i \left( -Y \frac{r^2}{2} - Y_1 \frac{r^4}{4} \right) \right] J_0(Zr) r dr \quad (1)$$

where  $f(x)$  is the pupil function of the optical system;  $Y$  and  $Y_1$  are respectively defocusing and primary spherical aberration parameters in dimensionless co-ordinates and  $J_0(Zr)$  is the zero order Bessel function of the first kind.  $R$  is a reduced co-ordinate on the exit-pupil of the system. For our present analysis, we have considered the shaded apertures whose pupil function can be represented by

$$f(r) = (1 - \beta^2 r^2) \quad (2)$$

where  $\beta$  is the apodising parameter controlled the non-uniform transmission of the pupil.  $\beta=0$  corresponds to a diffraction-limited system and in this paper; we have restricted ourselves to this case only. The intensity PSF  $B(Z)$  which is the real measurable quantity can be obtained by taking the squared modulus of  $A(Z)$ . Thus,

$$B(Z) = |A(Z)|^2 \quad (3)$$

### III. Results and Discussions

Expressions (1), (2) and (3) have been used to compute the intensity PSF  $B(Z)$  by employing the standard twelve-point gauss quadrature method. In Figure 1, we have plotted the computed values of  $B(Z)$  versus  $Z$  for various values of the primary spherical aberration parameter  $Y_1=0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4$  and for a fixed value of the defocusing parameter  $Y=2\pi$ . In the Tables 1 to 5, we have shown the locations and the intensity values of the central maximum, first and secondary maxima and also the first, second and the third minima for various values of  $Y$  and  $Y_1$ . In all these cases,  $\beta$  has been kept equal to zero. This means that we have considered a diffraction-limited optical system suffering from both defocusing and primary spherical aberration. From the figure and the tables, we find that there is a significant effect of the combined influence of defocusing and spherical aberration on the PSF. For example, the central maximum reduces to 40% of its original peak value when  $Y=2\pi$  and even  $Y_1=0$ . The situation is still worse as the PSF drops from a value of 0.80016 when  $Y=0$  and  $Y_1=2\pi$  to a significantly low value of 0.09382 when  $Y=2\pi$  and  $Y_1=2\pi$ . In general, for a fixed value of  $Y$ , the PSF central maximum reduces considerably as the spherical aberration increases.

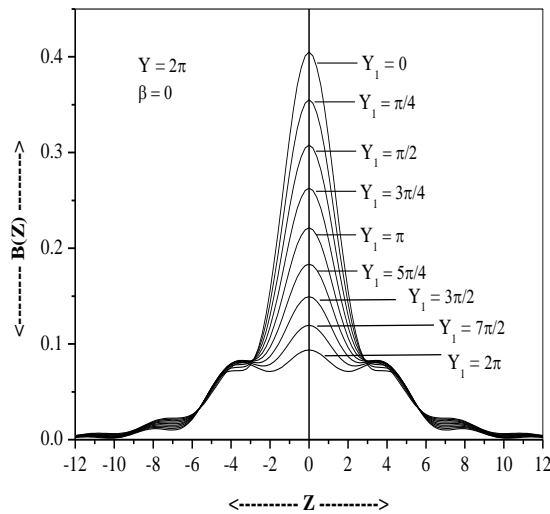


Fig.1 At defocused plane  $Y = 2\pi$ , intensity distribution in the image of a point object in the presence of various amount of primary spherical aberration.

Table: 1

| Y = 0, $\beta = 0$ |             |        |           |        |           |        |            |        |            |        |           |        |
|--------------------|-------------|--------|-----------|--------|-----------|--------|------------|--------|------------|--------|-----------|--------|
| Y1                 | Central Max |        | First Min |        | First Max |        | Second Min |        | Second Max |        | Third Min |        |
|                    | Pos         | Val    | Pos       | Val    | Pos       | Val    | Pos        | Val    | Pos        | Val    | Pos       | Val    |
| 0                  | 0.0000      | 1.0000 | 3.8317    | 0.0000 | 5.1339    | 0.0175 | 7.0155     | 0.0000 | 8.4066     | 0.0042 | 10.1732   | 0.0000 |
| $\pi/4$            | 0.0000      | 0.9966 | 3.8336    | 0.0004 | 5.1307    | 0.0176 | 7.0138     | 0.0001 | 8.4050     | 0.0042 | 10.1721   | 0.0000 |
| $\pi/2$            | 0.0000      | 0.9864 | 3.8393    | 0.0015 | 5.1164    | 0.0178 | 7.0086     | 0.0003 | 8.3899     | 0.0043 | 10.1670   | 0.0000 |
| $3\pi/4$           | 0.0000      | 0.9695 | 3.8493    | 0.0034 | 5.0924    | 0.0181 | 6.9998     | 0.0006 | 8.3629     | 0.0045 | 10.1593   | 0.0001 |
| $\pi$              | 0.0000      | 0.9464 | 3.8645    | 0.0059 | 5.0577    | 0.0187 | 6.9892     | 0.0010 | 8.3313     | 0.0047 | 10.1504   | 0.0002 |
| $5\pi/4$           | 0.0000      | 0.9174 | 3.8863    | 0.0090 | 5.0094    | 0.0194 | 6.9736     | 0.0016 | 8.2947     | 0.0050 | 10.1358   | 0.0003 |
| $3\pi/2$           | 0.0000      | 0.8829 | 3.9166    | 0.0127 | 4.9448    | 0.0203 | 6.9552     | 0.0022 | 8.2373     | 0.0054 | 10.1186   | 0.0005 |
| $7\pi/4$           | 0.0000      | 0.8436 | 3.9606    | 0.0166 | 4.8569    | 0.0215 | 6.9301     | 0.0029 | 8.1812     | 0.0059 | 10.1103   | 0.0006 |
| $2\pi$             | 0.0000      | 0.8002 | 4.0302    | 0.0208 | 4.7343    | 0.0230 | 6.9036     | 0.0038 | 8.1147     | 0.0065 | 10.0869   | 0.0008 |

Table: 2

$Y = \pi/2, \beta = 0$

| Y1       | Central Max |        | First Min |        | First Max |        | Second Min |        | Second Max |        | Third Min |        |
|----------|-------------|--------|-----------|--------|-----------|--------|------------|--------|------------|--------|-----------|--------|
|          | Pos         | Val    | Pos       | Val    | Pos       | Val    | Pos        | Val    | Pos        | Val    | Pos       | Val    |
| 0        | 0.0000      | 0.9496 | 3.8316    | 0.0072 | 5.0268    | 0.0209 | 7.0149     | .0004  | 8.3880     | 0.0045 | 10.1722   | 0.0001 |
| $\pi/4$  | 0.0000      | 0.9220 | 3.8426    | 0.0107 | 4.9737    | 0.0221 | 7.0048     | 0.0008 | 8.3479     | 0.0047 | 10.1652   | 0.0001 |
| $\pi/2$  | 0.0000      | 0.8887 | 3.8601    | 0.0147 | 4.9062    | 0.0236 | 6.9913     | 0.0013 | 8.3134     | 0.0051 | 10.1546   | 0.0002 |
| $3\pi/4$ | 0.0000      | 0.8505 | 3.8862    | 0.0191 | 4.8244    | 0.0253 | 6.9765     | 0.0019 | 8.2676     | 0.0055 | 10.1437   | 0.0004 |
| $\pi$    | 0.0000      | 0.8079 | 3.9273    | 0.0238 | 4.7166    | 0.0274 | 6.9568     | 0.0027 | 8.2183     | 0.0060 | 10.1304   | 0.0005 |
| $5\pi/4$ | 0.0000      | 0.7615 | 3.9973    | 0.0286 | 4.5716    | 0.0299 | 6.9357     | 0.0035 | 8.1569     | 0.0065 | 10.1181   | 0.0007 |
| $3\pi/2$ | 0.0000      | 0.7123 | -----     | -----  | -----     | -----  | 6.9132     | 0.0045 | 8.0853     | 0.0072 | 10.1074   | 0.0009 |
| $7\pi/4$ | 0.0000      | 0.6608 | -----     | -----  | -----     | -----  | 6.8862     | 0.0055 | 8.0183     | 0.0079 | 10.0894   | 0.0012 |
| $2\pi$   | 0.0000      | 0.6081 | -----     | -----  | -----     | -----  | 6.8553     | 0.0065 | 7.9475     | 0.0087 | 10.0697   | 0.0015 |

Table: 3

$Y = \pi, \beta = 0$

| Y1       | Central Max |        | First Min |        | First Max |        | Second Min |        | Second Max |        | Third Min |        |
|----------|-------------|--------|-----------|--------|-----------|--------|------------|--------|------------|--------|-----------|--------|
|          | Pos         | Val    | Pos       | Val    | Pos       | Val    | Pos        | Val    | Pos        | Val    | Pos       | Val    |
| 0        | 0.0000      | 0.8104 | 3.8312    | 0.0263 | 4.7143    | 0.0320 | 7.0144     | 0.0018 | 8.2975     | 0.0055 | 10.1686   | 0.0003 |
| $\pi/4$  | 0.0000      | 0.7651 | 3.8612    | 0.0316 | 4.6003    | 0.0349 | 6.9995     | 0.0026 | 8.2506     | 0.0060 | 10.1528   | 0.0004 |
| $\pi/2$  | 0.0000      | 0.7166 | 3.9142    | 0.0369 | 4.4534    | 0.0381 | 6.9825     | 0.0035 | 8.1856     | 0.0066 | 10.1477   | 0.0006 |
| $3\pi/4$ | 0.0000      | 0.6656 | -----     | -----  | 4.0762    | 0.0420 | 6.9633     | 0.0045 | 8.1164     | 0.0073 | 10.1285   | 0.0008 |
| $\pi$    | 0.0000      | 0.6131 | -----     | -----  | -----     | -----  | 6.9422     | 0.0057 | 8.0433     | 0.0081 | 10.1164   | 0.0011 |
| $5\pi/4$ | 0.0000      | 0.5597 | -----     | -----  | -----     | -----  | 6.9213     | 0.0069 | 7.9734     | 0.0090 | 10.1045   | 0.0014 |
| $3\pi/2$ | 0.0000      | 0.5063 | -----     | -----  | -----     | -----  | 6.8922     | 0.0081 | 7.8863     | 0.0099 | 10.0876   | 0.0017 |
| $7\pi/4$ | 0.0000      | 0.4536 | -----     | -----  | -----     | -----  | 6.8712     | 0.0094 | 7.8033     | 0.0110 | 10.0644   | 0.0021 |
| $2\pi$   | 0.0000      | 0.4023 | -----     | -----  | -----     | -----  | 6.8412     | 0.0107 | 7.7153     | 0.0120 | 10.0466   | 0.0026 |

Table: 4

$Y = 3\pi/2, \beta = 0$

| Y1       | Central Max |        | First Min |        | First Max |        | Second Min |        | Second Max |        | Third Min |        |
|----------|-------------|--------|-----------|--------|-----------|--------|------------|--------|------------|--------|-----------|--------|
|          | Pos         | Val    | Pos       | Val    | Pos       | Val    | Pos        | Val    | Pos        | Val    | Pos       | Val    |
| 0        | 0.0000      | 0.6147 | 3.8302    | 0.0507 | 4.2263    | 0.0512 | 7.0144     | 0.0049 | 8.1355     | 0.0076 | 10.1686   | 0.0007 |
| $\pi/4$  | 0.0000      | 0.5618 | -----     | -----  | -----     | -----  | 6.9972     | 0.0061 | 8.0603     | 0.0085 | 10.1524   | 0.0010 |
| $\pi/2$  | 0.0000      | 0.5086 | -----     | -----  | -----     | -----  | 6.9792     | 0.0075 | 7.9813     | 0.0095 | 10.1427   | 0.0013 |
| $3\pi/4$ | 0.0000      | 0.4559 | -----     | -----  | -----     | -----  | 6.9613     | 0.0089 | 7.8956     | 0.0105 | 10.1288   | 0.0017 |
| $\pi$    | 0.0000      | 0.4043 | -----     | -----  | -----     | -----  | 6.9412     | 0.0104 | 7.8033     | 0.0117 | 10.1055   | 0.0021 |
| $5\pi/4$ | 0.0000      | 0.3546 | -----     | -----  | -----     | -----  | 6.9162     | 0.0119 | 7.7073     | 0.0129 | 10.0884   | 0.0025 |
| $3\pi/2$ | 0.0000      | 0.3073 | -----     | -----  | -----     | -----  | 6.8922     | 0.0134 | 7.6053     | 0.0141 | 10.0665   | 0.0030 |
| $7\pi/4$ | 0.0000      | 0.2630 | -----     | -----  | -----     | -----  | 6.8732     | 0.0149 | 7.5043     | 0.0154 | 10.0554   | 0.0035 |
| $2\pi$   | 0.0000      | 0.2222 | -----     | -----  | -----     | -----  | 6.8513     | 0.0164 | 7.4034     | 0.0167 | 10.0245   | 0.0041 |

Table: 5

$Y = 2\pi, \beta = 0$

| Y1       | Central Max |        | First Min |        | First Max |        | Second Min |        | Second Max |        | Third Min |        |
|----------|-------------|--------|-----------|--------|-----------|--------|------------|--------|------------|--------|-----------|--------|
|          | Pos         | Val    | Pos       | Val    | Pos       | Val    | Pos        | Val    | Pos        | Val    | Pos       | Val    |
| 0        | 0.0000      | 0.4050 | 3.5942    | 0.0720 | 3.8293    | 0.0721 | 7.0166     | 0.0100 | 7.8779     | 0.0114 | 10.1691   | 0.0016 |
| $\pi/4$  | 0.0000      | 0.3552 | -----     | -----  | -----     | -----  | 6.9952     | 0.0117 | 7.7763     | 0.0127 | 10.1475   | 0.0020 |
| $\pi/2$  | 0.0000      | 0.3077 | -----     | -----  | -----     | -----  | 6.9822     | 0.0134 | 7.6733     | 0.0141 | 10.1255   | 0.0025 |
| $3\pi/4$ | 0.0000      | 0.2629 | -----     | -----  | -----     | -----  | 6.9622     | 0.0150 | 7.5743     | 0.0155 | 10.1055   | 0.0030 |
| $\pi$    | 0.0000      | 0.2214 | 3.1652    | 0.0820 | 3.4743    | 0.0821 | 6.9504     | 0.0167 | 7.4616     | 0.0170 | 10.0928   | 0.0036 |
| $5\pi/4$ | 0.0000      | 0.1834 | 2.9182    | 0.0819 | 3.4723    | 0.0828 | 6.9334     | 0.0183 | 7.3345     | 0.0184 | 10.0756   | 0.0042 |
| $3\pi/2$ | 0.0000      | 0.1494 | 2.6532    | 0.0803 | 3.4603    | 0.0828 | 6.9314     | 0.0198 | 7.2035     | 0.0199 | 10.0646   | 0.0048 |
| $7\pi/4$ | 0.0000      | 0.1195 | 2.3552    | 0.0769 | 3.4444    | 0.0820 | 6.9356     | 0.0213 | -----      | -----  | -----     | -----  |
| $2\pi$   | 0.0000      | 0.0938 | 2.0132    | 0.0712 | 3.4263    | 0.0805 | -----      | -----  | -----      | -----  | 10.0074   | 0.0062 |

As a consequence, the intensity values in the first and the second diffraction maximum increases with the increase in the values of  $Y_1$ . It is also observed that, not only the diffraction maxima, but also the various minima including the first minimum, show an increase in intensity values. Thus, as the value of  $Y_1$  is increased, a perfectly dark minimum becomes a non-zero minimum even when  $Y=0$ . This imposes a severe restriction to the Rayleigh criterion to be applied for two-point resolution studies of these systems.

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