Design and Implementation of Sliding Mode Controller using Coefficient Diagram Method for a nonlinear process

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Abstract: The method proposed in this paper uses a second order Sliding Mode Control Algorithm(SMC), is designed and implemented for a non linear process which makes use of the Coefficient Diagram Method(CDM) technique to find the controller parameters. The proposed controller is inherently 2 degree of freedom structure. This work considers the liquid level control of a spherical tank system. Simulation of the process model has been carried out by Mat lab Simulink software. Along with simulation results, real time responses of spherical tank liquid level control process also presented for evaluation and validation of proposed method.

Keywords: Adaptive control, Second order SMC, 2 Degree of freedom, SMC-CDM, Spherical tank level process.

INTRODUCTION I.

The Sliding Mode Control approach is recognized as one of the efficient tools to design robust controllers for complex high-order nonlinear dynamic plant operating under uncertainty conditions. The major advantage of sliding mode is its low sensitivity to plant parameter variations and disturbances which eliminates the necessity of exact modeling. Sliding mode control enables the decoupling of the overall system motion into independent partial components of lower dimension and, as a result, reduces the complexity of feedback design. The basic properties are the order reduction, invariance principle and its effectiveness towards chattering in real time applications make it more popular. The SMC approach consists of two steps:

- The first step is the choice of a manifold in the state space such that, once the state trajectory is 1) constrained on it, the controlled plant exhibits the desired performance.
- The second step is represented by the design of a discontinuous state-feedback capable of forcing the 2) system state to reach, in finite time, such a manifold (accordingly called "SLIDING MANIFOLD").

In this work, the design and implementation of SMC using Coefficient diagram method is presented to improve standard designs in adaptive control schemes. Section 2 describes the design of SMC algorithm for SISO plants. Section 3 is devoted to the development of 2 degree of freedom SMC using CDM for SISO plants. Section 4 presents the results and analysis of performance comparison of controllers. The conclusion is presented in section 5.

II. SECOND ORDER SLIDING MODE CONTROL ALGORITHM

Consider an uncertain SISO nonlinear system described by the vector differential equation

 $\dot{x} = \emptyset(t, x(t)) + \gamma(t, x(t)) u(t)$

(1)

Which is affine in the control u and where $x \in X \subset \mathbb{R}^n$ is a state vector, $u \in U \subset \mathcal{R}$ is a bounded input and t is the independent time variable. Select a sliding surface as follows, (2)

s = S(t, x)

Such that by zeroing it, the control objective is achieved. Further assume that the sliding surface s, has relative degree two with respect to the control input i.e. $\frac{\partial}{\partial x}S(t; x)(t; x) = 0$. Thus the system dynamics can be written in the following form

$$\ddot{s} = f(t, s, \dot{s}, x) + g(t, s, \dot{s}, x)u$$

The dynamics in equation (2) are assumed to satisfy the following bounding conditions $0 < G_{min} \leq g(t, s, \dot{s})$ $\leq G_{max}$ and $|f(t, s, \dot{s})| \leq F$; $|s| \leq s_0$. Where G_{min} , G_{max} , s_0 and F are some positive constants. Given the physical limits of most practical engineering systems, the assumed bounding conditions are not unduly restrictive.

A Second order sliding control algorithm is applied to stabilize the dynamics without the knowledge of the derivative of the sliding variable (\dot{s}) . The algorithm is given as follows

 $u(t) = -\lambda sign(s) + u_2(t)$

(4)

(3)

$$\dot{u}_2 = \begin{cases} -ku, & |u| > u_0 \\ -Wsign(s), & |u| \le u_0 \end{cases}$$
(5)

The corresponding sufficient conditions for finite time convergence are

$$u_0 = \frac{r}{G_{min}}; \lambda > u_0; \qquad k, W > 0$$

III. 2 DEGREE OF FREEDOM ADAPTIVE SMC USING CDM TECHNIQUE

The proposed method which is used to design a sliding mode controller satisfying the required time and frequency domain specifications consists of two steps. In the first step, a method is proposed to compute the global and the local stability regions using the stability boundary locus approach. In the second step, the CDM method is used to design SMC controllers for which the step responses have a required overshoot and an acceptable settling time. As a result of combining these two steps, the Frequency and Time Domain Plot(FTDP) map is obtained. The FTDP map, which is a graphical tool, shows the relation between the stabilizing parameters of the SMC controller and the chosen frequency and time domain performance criteria on the same λ , u_2 plane. Thus, one can choose a SMC controller providing all of the desired GM, PM, MO, and t_s specification values together.

A general schema of the 2DOF control system is shown in Fig.1.



Fig. 1. A 2DOF control system structure.

Where

$$G_a(s) = G(s)e^{-\theta s} = \frac{N(s)}{D(s)}e^{-\theta s}$$
(6)

The closed-loop characteristic polynomial P(s) of the system, i.e., the numerator of $1 + C(s) G_a(s)$, can be written as

$$P(s) = sD(s) + (\lambda sign(s)s + u_2) N(s)e^{-\theta s}$$
(7)

where all or some of the coefficients a_i , i = 0, 1, 2, ..., n are the function of λ , u_2 , and $e^{-\theta s}$ depending on the order of N(s), and D(s) polynomials. The characteristic polynomial is given in the following form

 $P(s) = a_n s^n + \dots + a_1 s + a_0 = \sum_{i=0}^n a_i s^i$ The stability index γ_i , the equivalent time constant τ , and stability limit γ_i^* are defined as follows.

$$\gamma_i = \frac{a_i^2}{a_{i+1}a_{i-1}}, \qquad i = 1 \sim n - 1 \tag{9}$$

$$\tau = \frac{a_1}{a_0} \tag{10}$$

$$\gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}}, \quad \gamma_n = \gamma_0 = \infty$$
 (11)

From these equations the following relations are derived.

$$\frac{a_{i+1}}{a_i} = \frac{\frac{1}{a_{j-1}}}{\gamma_i \gamma_{i-1} \dots \gamma_{j+1} \gamma_j} \qquad i \ge j$$

$$a_i = \frac{a_0 \tau^i}{\gamma_{i-1} \gamma_{i-2}^2 \dots \gamma_2^{i-2} \gamma_1^{i-1}} \qquad (12)$$

Then characteristic polynomial will be expressed by a_0 , τ and γ_i as follows.

$$P(s) = a_0 \left[\left\{ \sum_{i=2}^n (\prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^j}) (\tau s)^i \right\} + \tau s + 1 \right]$$
(13)

When the performance specifications are given, usually the rise time, the settling time, the overshoot, and the peak time are used for the time response specification. However from the CDM design point of view, only the settling time t_s is meaningful, because it gives upper bound of τ , where $t_s = 2.5 \sim 3 \tau$. The frequency response specifications are used for the high frequency attenuation characteristics and the low frequency disturbance rejection characteristics.

(8)

IV. Results And Analysis

The designed SMC is implemented in real time also compared with a PI controller. The set point is given in terms of percentage of level. 20% of level is given as nominal operating value, after 14000(s) the set point has been changed to 30%. The load is applied at the valve in outlet, for 10 liter/min change in outflow. For the sampling time, 1 sec is selected.

Table 1. Plant parameters					
Process variables	Nominal operating conditions				
Level(h)	1 m				
Flow rate, (F _{in})	0.2215 m ² /sec				
Radius of the tank (r)	1 m				
Constant of the outlet valve(c _s)	0.05 m ²				
Outlet valve stem position(x _s)	1				
Gravitational acceleration (g)	9.8 ms ⁻¹				
Maximum level	2 m				

Table 1 provides the description of Spherical tank parameters where table 2 shows the PI controller parameters for various linearized plant models as different operating levels 10%, 50% and 66% of tank level. Table 3 provides the controller settings for SMC-CDM.

Linearized models	Transfer function models	PI Controller parameters					
		K _c	Ki				
Model 1	$4e^{-120s}$	0.825	0.002				
	440s + 1						
Model 2	$6e^{-130s}$	1.385	0.003				
	1200s + 1						
Model 3	$2.75e^{-150s}$	2.29	0.005				
	1050s + 1						

Table 2. PI controller parameters(ZN method) concerning plant models

Table 3. SMC-CDM controller parameters for simulation

SET POINT (LEVEL %)	γ	τ	λ	u ₂
10	20	1.5	0.9	6.5868e-04



Figs. 2 and 3 shows that the controller output of sliding mode controller uses CDM technique and PI controller. And the following figs. 4 shows that the comparison of plant responses with corresponding controller. It can be seen that the time domain and error criteria are well satisfied in SMC-CDM controller. And table 4 provides the comparison of controller performances.



Fig 4. Comparison of servo and regulatory responses with SMC-CDM and PI controller

Table 4. Performance comparison	of SMC and PI controller
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CONTROLLER	LEVEL (%)		IAE	ISE	OVERSHOOT(%)	SETTLING T	TIME (Sec)
	Set point	Load				Set point	Load
SMC	10	3	3700	2.258e+04	10.83	2135	4209
PI	10	3	5912	2.975e+04	16.62	2590	5000

Real Time Implementation And Responses

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Table 5. Controller settings of SMC-CDM and PI controller(ZN method)						
LEVEL (CM)	SMC-CDM	PI controller				

LEVEL (CM)	SMC-CDM				PI controller	
SET POINT	γ	γ τ Λ u_2				k_i
20	20	0.3	0.9	0.0016	2.5	0.0024



Fig 5. Real time setup of Spherical tank level process

To study the servo and regulatory characteristics of the SMC controller set point change and load is applied to the process. Set point is changed from 20% of the level to 30% of the level, and Load is applied at the 2.624e+5th Sampling Instant. The controller output is shown in the fig (6) and the corresponding plant response is shown in fig (7).



Fig 6. SMC-CDM Controller output in real time



Fig 7. Servo and regulatory responses of plant with SMC-CDM controller in real time

To study the servo and regulatory characteristics of the PI controller set point change and load is applied to the process. Set point is changed from 20% of the level to 30% of the level, and Load is applied at the 3.261e+5th Sampling Instant. The PI controller output is shown in the fig (8) and the corresponding plant output is shown in fig (9). Fig 10 shows that the comparison of SMC-CDM and PI controller performance in closed loop.



Fig 9. Servo and regulatory response of plant with PI controller in real time



Fig 10. Comparison of plant responses with SMC-CDM and PI controllers

The following table 6 provides the performance comparison of SMC-CDM and PI controller in real time of Spherical tank level process.

CONTROLLER	LEVEL (%)		IAE	ISE	OVERSHOOT(%)	SETTLING	TIME (Sec)
	Initial Set point	Changed set point				Initial Set point	Changed set point
SMC	20	30	8902	8.913e+04	24.48	1.374e+5	2.546e+05
PI	20	30	9749	10.53e+04	27.62	1.397e+5	3.208e+05

Table 6. Performance comparison of SMC-CDM and PI controllers in real time

From the results (Simulation and Real Time) obtained (as shown in this chapter) it is very much clear that the SMC controller performs better than the conventional PI controller in all cases.

V. Conclusion

Sliding mode controller for a non-linear process (Spherical Tank) is designed as an algorithm that does not require the derivative of the sliding surfaces is proposed, CDM technique for optimizing controller parameters is implemented and pperformance of designed controller was compared (Both Simulation Real Time). As per the Control algorithm of SMC, it takes more time to find out the optimum values of controller parameters by using Trial and error method, in order to reduce complexity, CDM technique is used to find the parameters. CDM technique is a simple and effective method to find the unknown parameters. It proves that the proposed controller is better one.

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