

Backstepping Control with Integral Action of PMSM Integrated According to the MRAS Observer

A. Larbaoui¹, B. Belabbes¹, A. Meroufel¹, A. Tahour¹, and D. Bouguenna^{2,*}.

1(Intelligent Control Electronic Power System Laboratory (I.C.E.P.S), University of Djilali Liabes, BP 98, Sidi Bel Abbes, Algeria).

2(Department of LMD Matter Sciences (SM), Faculty of Sciences and Technology, University of Mascara, Road of Mamounia, 29000 Mascara, Algeria).

Abstract: The work developed in this article, has aims to control design law is based on Backstepping method to ensure the servo-control of the permanent magnet synchronous motors (PMSM). The slow convergence of adaptation can result, in some cases; irreversible instabilization loop, especially in the presence of nonlinearity and/or couplings. Hence, the Backstepping method coupled with introduction of integral actions is an alternative choice. Moreover, the Backstepping theory is a recursive design methodology that makes use of the Lyapunov stability theory. Strong properties of overall and asymptotic stability can be achieved. The second part of this work is a direct application of the control proposed to associate an observer, is the Model Reference Adaptive System (MRAS) to observe the position and the rotor speed of the MRAS. Furthermore, the simulation under the Matlab/Simulink software allows highlighting the performance of the control strategy adopted.

Keywords: Backstepping control, Lyapunov theorem, Integral action, Permanent Magnet Synchronous Motor (PMSM), Model Reference Adaptive System (MRAS), observer.

I. Introduction

During several years, the industry has been gradually replacing DC motors in a wide range of drive applications such as machine tools and industrial robots offer the main advantage to be easily controllable through decoupling flux and torque control. However, the presence of the collector and brushes have been always a great disadvantage of motor among other limiting increasingly use.

Address this limitation; PMSM is more attractive because they meet all the requirements. Furthermore, the coupling existing between the flux and torque makes the PMSM hardly controllable. With nonlinear control it is possible to obtain variable speed AC as well as drives speed of DC motors [1]. When the part controlled is submitted for perturbations and for changing the system parameters, a self adaptive solution should be considered by adjustment of controller parameters of regulators, which allow keeping the performances fixed advance in presence of perturbations and of change in the system parameters. This solution present has a disadvantage of requiring an implementation often complex. Thus, it is possible to consider a more simple solution, using a special nonlinear control system class called, Backstepping control [1, 3, 4].

The technical proposed in our work is to control design law for the PMSM to achieve high statics and dynamics performance. This technique is based on the Backstepping technique which establishes relations successive causal iteratively to build a systematic and robust control law asymptotically stable according to the Lyapunov theory of stability [5]. The influence of the change some parameters and of the perturbation of charge can be greatly reduced by the introduction of an integral action in each step, in order to ensure high accuracy speed control [6].

The paper is organized as follows: in Section II an overview of the description of PMSM mathematical model in ($d-q$) reference coordinate system. In Section III the control design law is based on Backstepping control theory in integral action. Then, the Section IV will be devoted to the design of the control without speed sensor associated with MRAS observer for PMSM with salient poles. Section V presents some simulation results and we conclude the paper in the last Section of this paper.

II. PMSM mathematical model

Electric and dynamic behavior of electrical machines can't be studied if it is defined by a mathematical model. To obtain a simpler formulation and reduce the complexity of the machine model, the establishment of its mathematical model will be developed based on simplifying assumptions.

Knowing that:

- The machine is symmetrical,
- Operate in the unsaturated regime,
- The various losses are negligible,

- The excitation is done by permanent magnets, such as the excitation flux is considered constant, [1,3]. Furthermore, the magnet is considered a winding without resistance or inductance or mutual, but as a flux source.

Using transformations Park, the PMSM mathematical model in rotating reference frame (d - q), is defined by the following system of equations:

$$\begin{cases} \frac{d i_d}{dt} = -\frac{R_s}{L_d} i_d + \frac{L_q}{L_d} p \omega_r i_q + \frac{1}{L_d} v_d \\ \frac{d i_q}{dt} = -\frac{R_s}{L_q} i_q - \frac{L_d}{L_q} p \omega_r i_d - \frac{\psi_f}{L_q} p \omega_r + \frac{1}{L_q} v_q \\ \frac{d \omega_r}{dt} = \frac{3p}{2j} (\psi_f i_q + (L_d - L_q) i_d i_q) - \frac{1}{j} T_L - \frac{B_m}{j} \omega_r \\ \frac{d \theta}{dt} = p \omega_r \end{cases}, \quad (1)$$

with:

- R_s stator resistance,
- L_d, L_q d and q-axis inductances,
- ψ_f permanent-magnet flux linkage,
- i_d, i_q stator currents,
- v_d, v_q stator voltages,
- ω_r rotor speed in angular frequency,
- θ rotor angular position,
- j moment of inertia of the motor and load,
- B_m friction coefficient of the motor,
- p number of pole pairs,
- T_L load torque.

III. Backstepping control with integral action for PMSM

This part of the article studies the strategy of speed control by the Backstepping technique. This technique of non-linear control has been recently developed; it can be effectively used for linearizing a non-linear system in the presence of uncertainties [6-9].

The control technique is a nonlinear Backstepping control having properties of strength. The pursuit of speed takes place with a high yield by the control voltage v_q as long as the current i_d is kept equal to zero.

The basic idea of the type Backstepping control is to make curly equivalent subsystems of order 1 systems in cascade stable within the meaning of Lyapunov, which gives them the qualities of robustness and a global asymptotic stability [1], [5]. In other words, this is a multi-step method. At each step of the process, a virtual control is also generated to ensure the convergence of system to its equilibrium state. This can be reached from Lyapunov functions to ensure step by step the stabilization of each synthesis step. The idea is to compute a control law to ensure the Lyapunov function that is positive definite and its derivative is always negative.

The calculation of Lyapunov function is performed in a recursive way. It is based on the previous system state. A new Control Lyapunov Function (CLF) is constructed by the increase of CLF of the previous step [8]. This procedure calculates allow us to ensure overall system stability. The improvement of the robustness of this technique by incorporation of introduction in terms integrations of the control design of PMSM thereafter.

III.1. Speed loop

To solve speed tracking problem, define the following speed tracking error as:

$$\varepsilon_\Omega = \omega_r \text{ref} - \omega_r + k'_\Omega \int_0^t (\omega_r \text{ref} - \omega_r) dt, \quad (2)$$

where $k'_\Omega \int_0^t (\omega_r \text{ref} - \omega_r) dt$: is an integral action term added to the control in Backstepping to ensure the convergence of the tracking error to zero in spite of the uncertainties of such piecewise constant at each step of the algorithm.

The virtual control input $i_q \text{ref}$ used to ensure the stability of the speed loop. The error dynamics of speed from Eq. (2) is given by:

$$\dot{\varepsilon}_\Omega = \dot{\omega}_{r\text{ref}} - \frac{p}{j} (L_d - L_q) i_d i_{q\text{ref}} + \frac{B_m}{j} \omega_r - \frac{p}{j} \psi_f i_{q\text{ref}} + \frac{1}{j} T_L + k'_\Omega (\omega_{r\text{ref}} - \omega_r). \quad (3)$$

Consider the following Lyapunov candidate function: $V_\Omega = \frac{1}{2} \varepsilon_\Omega^2$ its derivative is given by the following equation:

$$\dot{V}_\Omega = \varepsilon_\Omega \left\{ \dot{\omega}_r - \frac{p}{j} (L_d - L_q) i_d i_{q\text{ref}} + \frac{B_m}{j} \omega_r - \frac{p}{j} \psi_f i_{q\text{ref}} + \frac{1}{j} T_L k'_\Omega (\omega_{r\text{ref}} - \omega_r) \right\}. \quad (4)$$

Following the Backstepping methodology, and in order to ensure the stability of speed tracking, the virtual control $i_{q\text{ref}}$ is given by the following equation:

$$i_{q\text{ref}} = \frac{j}{p \psi_f + p (L_d - L_q) i_d} \left[k_\Omega \varepsilon_\Omega + \dot{\omega}_{r\text{ref}} + \frac{B_m}{j} \omega_r + k'_\Omega (\omega_{r\text{ref}} - \omega_r) + \frac{1}{j} T_L \right]. \quad (5)$$

Then $\dot{V}_\Omega = -k_\Omega \varepsilon_\Omega^2$, with $k_\Omega > 0$.

III.2. Current loop i_q

Since the virtual input $i_{q\text{ref}}$ is designed to stabilize the dynamics Eq. (3), now to design the control input v_q we define the tracking error in the current as follows:

$$\varepsilon_q = i_{q\text{ref}} - i_q + \varepsilon'_q, \quad (6)$$

where $\varepsilon'_q = k'_q \int_0^t (i_{q\text{ref}} - i_q) dt$ is an integral action. Consider the following Lyapunov candidate function:

$$V_q = V_\Omega + \frac{1}{2} \varepsilon_q^2 + \frac{1}{2} \varepsilon_q'^2, \quad (7)$$

By taking the time derivative of V_q is given by:

$$\dot{V}_q = -k_\Omega \varepsilon_\Omega^2 + \varepsilon_q \left\{ \frac{di_{q\text{ref}}}{dt} - \frac{di_q}{dt} + k'_q (i_{q\text{ref}} - i_q) \right\} + \varepsilon'_q k'_q (i_{q\text{ref}} - i_q), \quad (8)$$

Replacing by: $\frac{di_q}{dt} = -\frac{R_s}{L_q} i_q - \frac{L_d}{L_q} p \omega_r i_d - \frac{\psi_f}{L_q} p \omega_r + \frac{1}{L_q} v_q$, then Eq. (8) becomes:

$$\dot{V}_q = -k_\Omega \varepsilon_\Omega^2 + \varepsilon_q \left\{ \frac{di_{q\text{ref}}}{dt} + \frac{R_s}{L_q} i_q - \frac{1}{L_q} v_q + p \frac{L_d}{L_q} \omega_r i_d + p \frac{1}{L_q} \psi_f \omega_r + k'_q (i_{q\text{ref}} - i_q) \right\} + \varepsilon'_q k'_q (i_{q\text{ref}} - i_q). \quad (9)$$

By choosing the control input v_q as:

$$v_q = L_q \left[k_q \varepsilon_q + 2 \frac{p}{j} \psi_f \varepsilon_\Omega + p \frac{\psi_f}{L_q} \omega_r + \frac{R_s}{L_q} i_q + \frac{di_{q\text{ref}}}{dt} \right]. \quad (10)$$

It is obtain that:

$$\dot{V}_q = -k_\Omega \varepsilon_\Omega^2 - k_q \varepsilon_q^2 + \{\varepsilon_q + \varepsilon'_q\} k'_q (i_{q\text{ref}} - i_q). \quad (11)$$

Replacing by: $i_{q\text{ref}} - i_q = \varepsilon_q + \varepsilon'_q$, then Eq. (11) becomes:

$$\dot{V}_q = -k_\Omega \varepsilon_\Omega^2 - k_q \varepsilon_q^2 + \{\varepsilon_q + \varepsilon'_q\} k'_q (i_{q\text{ref}} - i_q), \quad (12)$$

with $\dot{V}_q \leq -\bar{K}_q V_q$,

where $\bar{K}_q = \min\{k_\Omega; \{k_q + k'_q\}; k'_q\}$. Then, under the control action v_q the current i_q tracks input $i_{q\text{ref}}$, and by choosing the virtual control input $i_{q\text{ref}}$ is calculated which the speed tracking its reference: $(\omega_r \rightarrow \omega_{r\text{ref}})$.

III.3. Current loop i_d

To eliminate the reluctance torque, the current reference is fixed to zero, $i_{d\text{ref}} = 0$, it meets another objective of our control that is elimination of the reluctance effect. We obtain a similar operation to that of a DC machine separately excited; let us define the following tracking error: $\varepsilon_d = i_{d\text{ref}} - i_d + \varepsilon'_d$,

where: $\varepsilon'_d = k'_d \int_0^t (i_{d\text{ref}} - i_d) dt$ is an integral action. Defining the following candidate Lyapunov function:

$$V_d = \frac{1}{2} \varepsilon_d^2 + \frac{1}{2} \varepsilon_d'^2, \tag{13}$$

the time derivative is computed as:

$$\dot{V}_d = \varepsilon_d \left\{ -\frac{di_d}{dt} - k'_d i_d \right\} + \varepsilon_d' \{ -k'_d i_d \}. \tag{14}$$

Since $i_{d\text{ref}} = 0$, then replacing $i_d = \varepsilon_d - \varepsilon_d'$ in the above equation, we obtain

$$\dot{V}_d = \varepsilon_d \left\{ \frac{R_s}{L_d} i_d - p \frac{L_q}{L_d} \omega_r i_q - \frac{1}{L_d} v_d \right\} + k'_d \{ \varepsilon_d + \varepsilon_d' \} \{ \varepsilon_d - \varepsilon_d' \}. \tag{15}$$

Then, by choosing the control law v_d as follows:

$$v_d = L_d \left[k_d (i_{d\text{ref}} - i_d + k'_d \int_0^t (i_{d\text{ref}} - i_d) dt) + \frac{R_s}{L_d} i_q - p \frac{L_q}{L_d} i_q \omega_r \right], \tag{16}$$

and by substituting in Eq. (15), we obtain:

$$\dot{V}_d = -\{k_d - k'_d\} \varepsilon_d^2 - k'_d k_d'^2. \tag{17}$$

By choosing $\bar{K}_d = \min\{k_d + k'_d; k'_d\}$ it follows that:

$$\dot{V}_d \leq -\bar{K}_d V_d.$$

Then, this implies that under the control law v_d the current i_d tracks the desired reference:

$$i_d \rightarrow i_{d\text{ref}} = 0.$$

Finally, combining the action of the control inputs v_d with $i_{d\text{ref}}$, the control objectives are achieved.

Proposition 1. We consider the system of the PMSM Eq (1), with references, $\omega_{r\text{ref}}$ and $i_{q\text{ref}}$, assume to be differentiable and bounded. Then, the trajectories of the closed loop system with orders tracking Eq. (10) and Eq. (16) following the exponentially references $\omega_{r\text{ref}}$ and $i_{d\text{ref}}$.

IV. Control without speed sensor of PMSM integrated according to the MRAS

MRAS (Model Reference Adaptive System) method is a technique belonging to the category of indirect speed estimation by exploiting tensions (v_d, v_q) and the stator currents (i_d, i_q) [10], [11]. It implements this technique using two independent models. The first is the reference model; it is used to determinate the two components of the stator current along the direct and quadrature axes (in the Park reference). Furthermore, the adaptive method for estimating the speed and the stator resistance are expressed by the second is the adjustable model. It is used to estimate the two components of the stator currents from the direct measurement of the stator voltages and currents. Canceling the discard between the stator currents of reference model and the adjustable model, we can estimate the rotor speed in dynamic mode. This discard is used by the adaptive mechanism to generate the estimated value and the converging towards the reference value.

IV.1. MRAS Synthesis

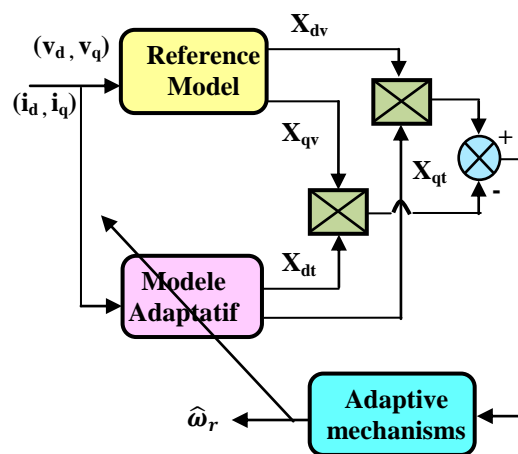


Fig. 1 Structure of MRAS for speed estimation.

The first is the reference model; it is used to determinate the two components of the stator current along the direct and quadrature axes (in the Park reference). Furthermore, the adaptive method for estimating the speed and the stator resistance are expressed by the second is the adjustable model. It is used to estimate the two components of the stator currents from the direct measurement of the stator voltages and currents. Canceling the discard between the stator currents of reference model and the adjustable model, we can estimate the rotor speed in dynamic mode. This discard is used by the adaptive mechanism to generate the estimated value and the converging towards the reference value.

The adjustable model of this MRAS observer to salient pole ($l_d \neq l_q$) is given by:

$$\begin{cases} \frac{d\hat{i}_d}{dt} = \frac{v_d}{L_d} - \frac{\hat{R}_s}{L_d} \hat{i}_d + p \hat{\omega}_r \hat{i}_q \\ \frac{d\hat{i}_q}{dt} = \frac{v_q}{L_q} - \frac{\hat{R}_s}{L_q} \hat{i}_q - p \hat{\omega}_r \hat{i}_q - p \frac{\psi_f}{L_q} \hat{\omega}_r + k_e \frac{\hat{R}_s}{L_q} \hat{i}_q \end{cases}, \quad (18)$$

where $\hat{R}_s = -\frac{1}{p} \int_0^t (e_d \hat{i}_d + e_q \hat{i}_q) dt$.

with :

$$\begin{cases} e_d = i_d - \hat{i}_d \\ e_q = \dot{i}_q - \dot{\hat{i}}_q \end{cases}, \quad (19)$$

Moreover, the adaptive method for estimating the speed and the stator resistance is expressed by:

$$\hat{\omega}_r = \frac{1}{p} \left(k_p e + k_i \int_0^t e dt \right), \quad (20)$$

with:

$$e = i_d \hat{i}_d - i_q \hat{i}_q - \frac{k_e}{L_d} e_q. \quad (20a)$$

where k_i et k_p are the gains of the corrector PI, e_d and e_q are errors in the stator currents.

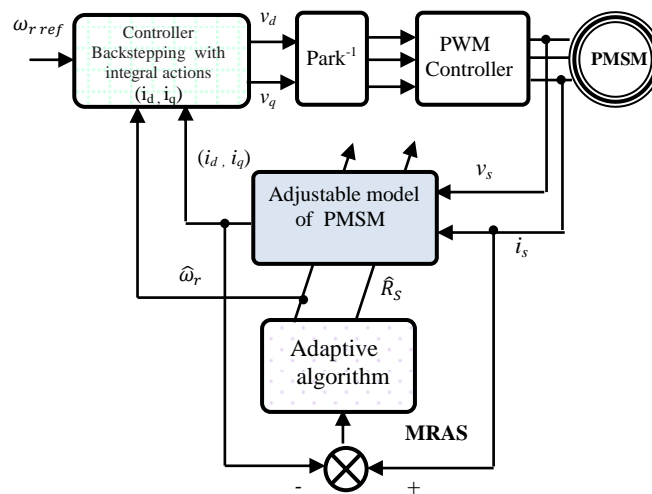


Fig. 2 Overall block diagram of the control.

The MRAS may be introduced into a control loop by Backstepping with integral action of PMSM, to observe its state variables. The diagram above shows a general structure of a MRAS related with a block control of PMSM by Backstepping with integral action.

V. Simulation results

To illustrate the mathematical analysis and, hence to investigate the performance of the proposed PMSM control of Backstepping with integral action according to the MRAS observer, simulations are carried out following the overall block diagram of the control shown in Fig. 2. Table 1 gives the nominal parameters of the PMSM which is used in the simulation tests. The simulation has been carried out using Matlab/ Simulink software. The machine started on the vacuum, then a load torque is applied of 5 (N.m) at time 0.1 s, and then eliminated at the moment has 0.2 s. At time t = 0.3 s, the direction of rotation is reversed at 150 (rad/s) to -50 (rad/s).

Table 1. The parameters of the PMSM.

ω_r	R_s	L_d	L_q	ψ_f	p	j	f
157 rad/s	0.6 Ω	1.4 mH	1.8 mH	0.12 Wb	4	11x10 ⁻⁴ kg.m ²	14x10 ⁻⁴ N.ms/rad

Gains control used in this paper are: $k_\Omega = 0.01$, $k'_\Omega = 139$, $k_q = 2900$, $k'_q = 150$, $k_d = 100$, and $k'_d = 900$. In these results the actual motor speed was calculated from the end position by the encoder.

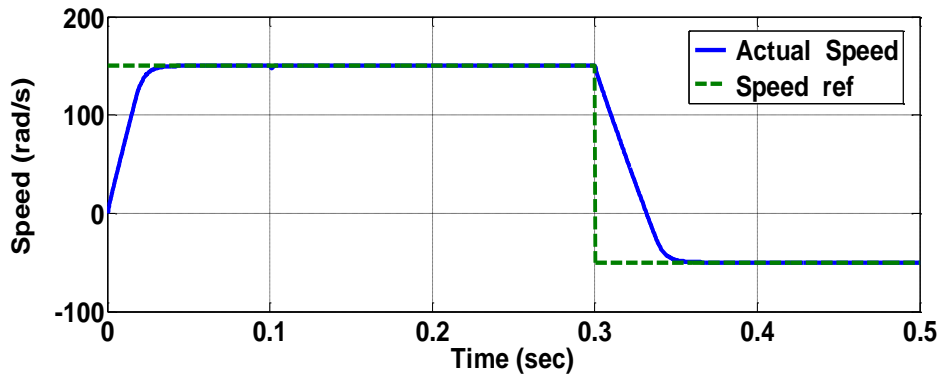


Fig. 3 Speed rotation [rad/sec].

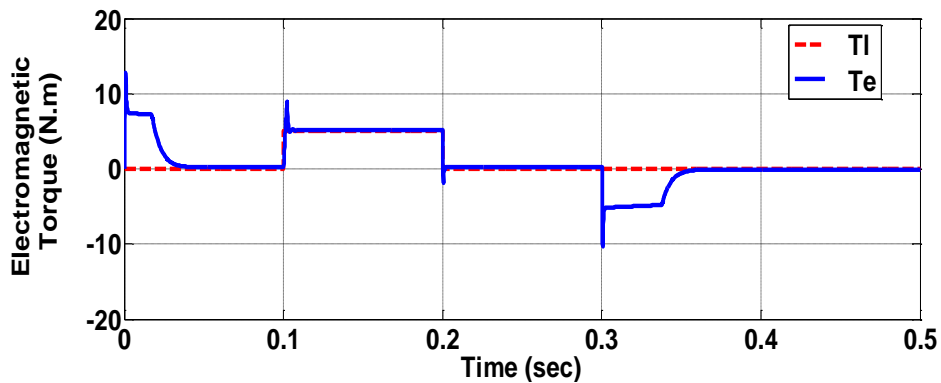


Fig. 4 Electromagnetic torque [N.m].

Figures 3 and 4 are noted that according to shape of the speed perfectly follows its reference which is achieved very rapidly, with a very fast response. The effect of the perturbation is rapidly and the electromagnetic torque stabilizes the load torque value.

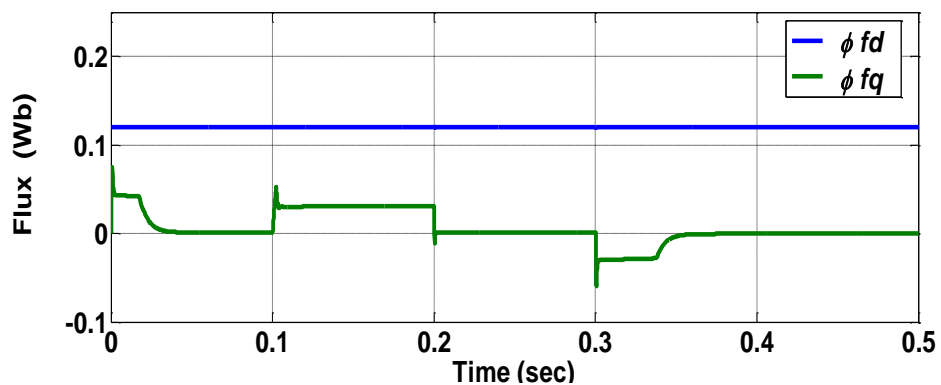


Fig. 5 Flux [Wb].

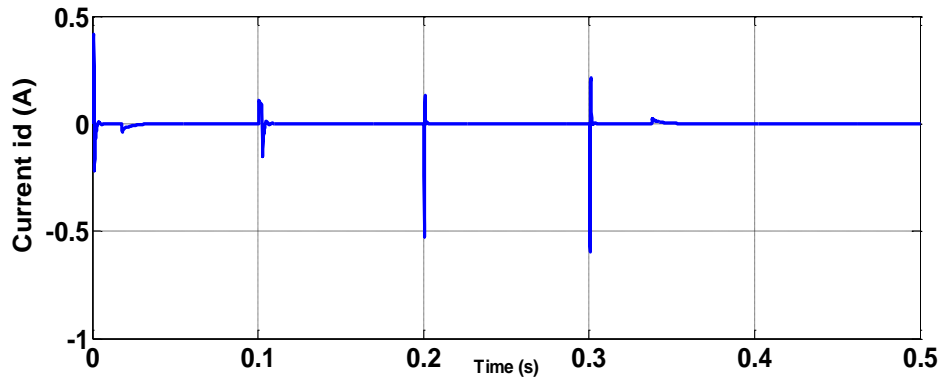


Fig. 6 Current i_d [A].

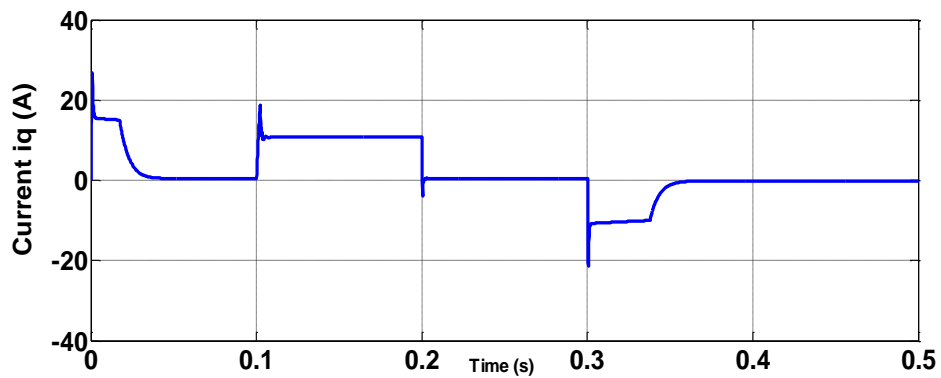


Fig. 7 Current i_q [A].

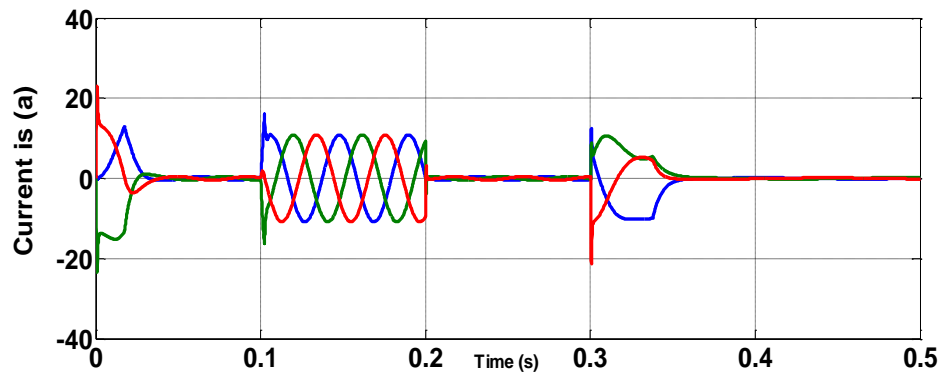


Fig. 8 Stator current [A].

Figures 5 to 7 illustrate the response of the two current components i_d and i_q show good decoupling introduced by PMSM control (the current $i_d = 0$). The current i_q is the couple image, and the phase sinusoidal shape of the current with a peak at start-up.

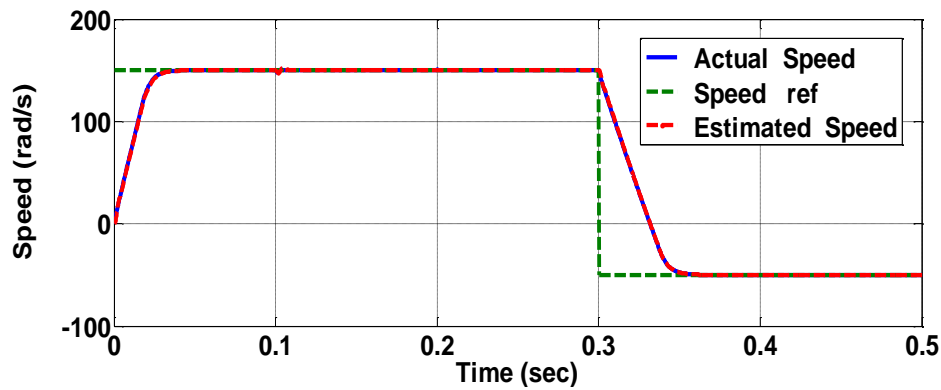


Fig. 9 Speed of rotation [rad/sec].

Figures 9 to 11 illustrate the dynamics behaviors of PMSM according MRAS observer at a servo-control of speed at the vacuum and a load torque is applied at 5 (N.m) at time 0.1 s and eliminated at 0,2 s. At time $t = 0.3$ the direction of rotation is reversed at 150 (rad/s) to -50 (rad/s), this time the values of the speed used in the feedback loop are estimated values.

These results are obtained with the same conditions as those obtained without MRAS observer (Fig. 3).

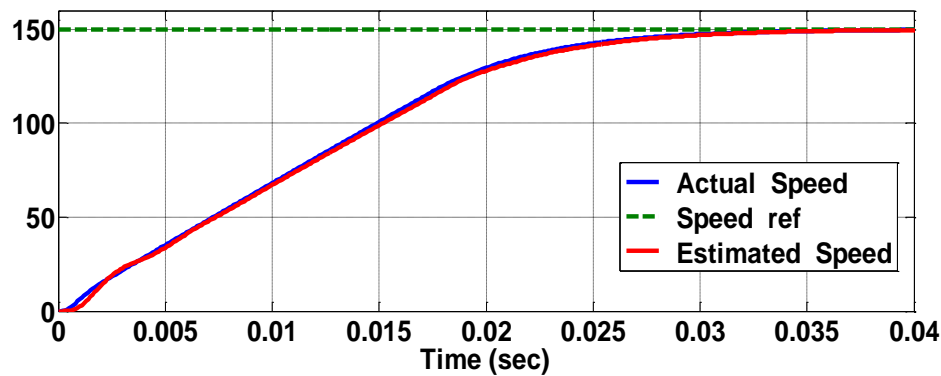


Fig.10 Zoom speed of rotation [rad/sec].

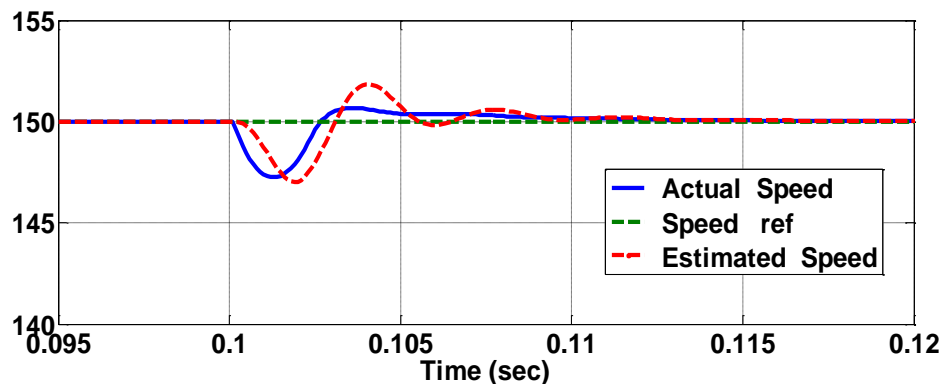


Fig. 11 Zoom speed of rotation [rad/sec].

Moreover, figures 10 and 11 show a zoom speed evolution in which the notes set is perfectly superimposed on the estimated speed.

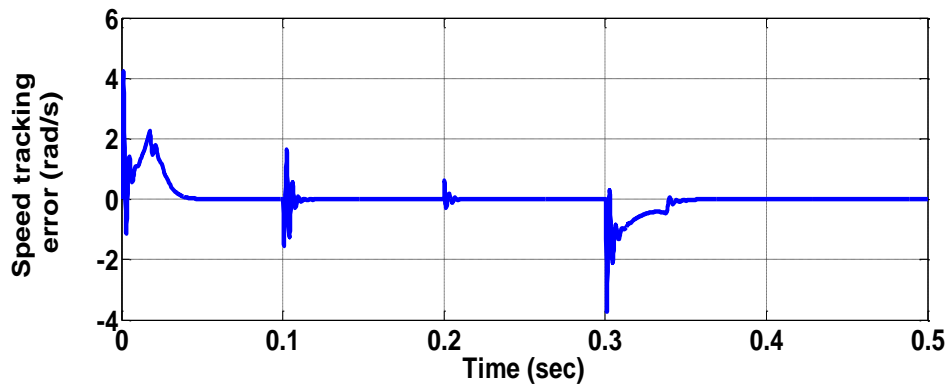


Fig. 12 Speed tracking error [rad/sec].

Figure 12 shows the speed tracking error. The speed estimation error, it is also shown that this estimated speed follows the actual speed with precision on permanent regime. Against by the transitional regime (starting and reversing the direction of rotation), this error is acceptable and does not affect the static and dynamic behavior of the control and also the overall drive system.

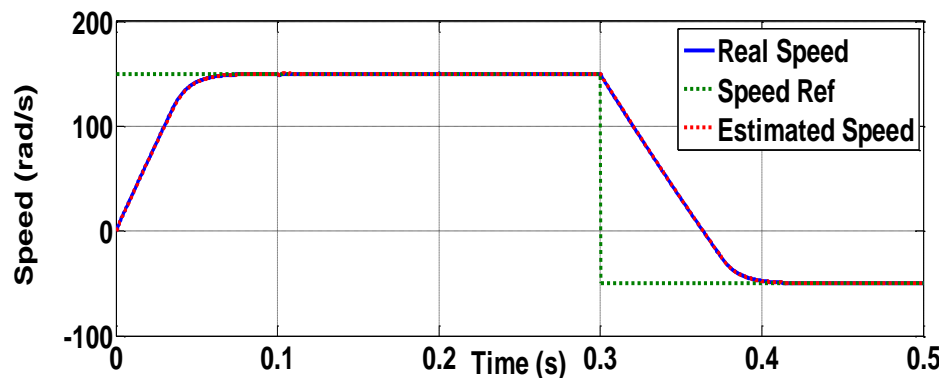


Fig. 13 Simulation results during the variation of inertia j .

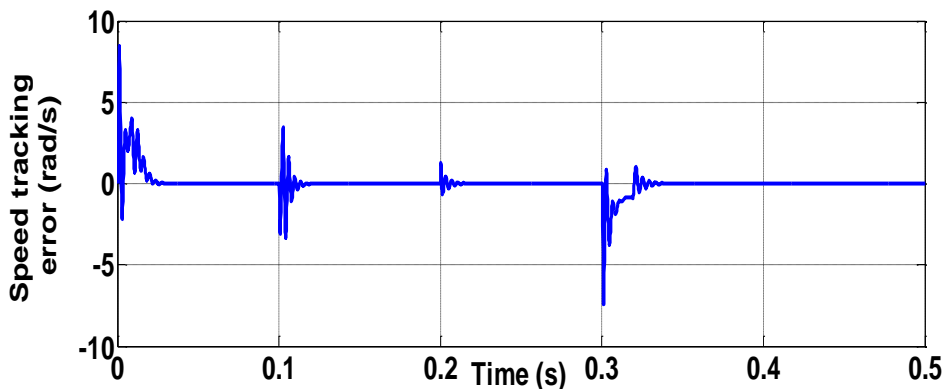


Fig. 14 Speed tracking error [rad/sec].

Figures 13 and 14 show error of +100 % on the nominal value the moment of inertia does not affect the static and dynamic performance of the Backstepping control and the MRAS observer.

Finally, we present the results obtained from actual simulation runs that, the MRAS observer is successfully established for a nonlinear robust sensorless towards the load change.

VI. Conclusion

This paper based on Backstepping control with an integral action for PM synchronous motor integrated according to the MRAS observer, which is addressed in a hand, as a tool for a new nonlinear control speed PMSM, and in the other hand as a tool for studying dynamic stability.

However, the simulation results exhibited a significant improvement in performance. This improvement manifests itself at the speed of signal quality, and the level of almost total rejection of the perturbation (charge couple). Moreover, to overcome the problem of control without mechanical sensors, we used a method based on MRAS technique, for estimating speed from the measured voltages and currents. The speed elimination sensor reduces the constraints and gives more flexibility to control the machine. In the light of the simulation results, we can conclude that the objective of this study is achieved. Thus, the Backstepping technique to the integral action combined with vector control offers high control performance. The intrinsic robustness Backstepping is strengthened through full terms added to it.

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