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General Model of Interest Account

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Abstract:In the paper we present a general model of interest account for which the standard simple and compound interest accounts as well as the continuous one are only special cases. We show how the interest rates in these standard models are derived from the general model.
Key Words:*Interest, interest account, interest rate.*

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I. Introduction and motivation

Interest account and interest calculation is a basic task in everyday financial affairs. Earnings of banks and financial institutions are based on interest. Simple and compound interest accounts are the standard most common ways of interest calculation (see [1], [2], [3], [4]). These are discrete accounts (the interest is merged with the principle after each fixed time interval – accounting period). If this merging process takes place constantly, at each moment, then we have continuous interest account which is usually connected with the natural growth of living beings.

The challenge to generalize these different accounts to one general model was the motivation for this work. In the paper we present such model which covers all of them.

II. Standard interest accounts and notation

In the standard interest accounts the interest is a function of three variables: principal, interest rate and time (the number of accounting periods). We shall use the following notation.

C_0 is the initial value of capital (the principal),

n is the number of accounting periods.

p is decimal value of the interest rate (the decursive one) for one period,

I is the total amount of interest,

$C_n = C_0 + I$ is the final value of capital after n periods (the principal merged with the interest).

The standard interest accounts are given by the following relations (see [1], [2], [3], [4]).

$$\text{Simple account:} \qquad C_n = C_0\left(1+np\right), \qquad I = C_0 pn. \tag{1}$$

$$\text{Compound account:} \qquad C_n = C_0(1+p)^n, \qquad I = C_0\left[(1+p)^n - 1\right]. \tag{2}$$

$$\text{Continuous account:} \qquad C_n = C_0 e^{np}, \qquad I = C_0\left(e^{np} - 1\right). \tag{3}$$

If the interest rate is variable, p_k for the kth period, $k \in \{1, 2, ..., n\}$, then term np is replaced with $p_1 + p_2 + ... + p_n$ in the relations (1) and (3) while $(1+p)^n$ is replaced with $(1+p_1)(1+p_2)\cdot ... \cdot(1+p_n)$ in the relation (2).

III. General model

In the general approach we consider the value of capital as a function of time, $C(t)$. If this value is known in one or more moments (values $K_1, K_2, ...$ in the moments $t = s_1, s_2, ...$) then we require $C(s_1) = K_1$, $C(s_2) = K_2, ...$. It can be the present moment or any moment in the past or future when this value is known or assumed. Generally, since the value of capital can vary over time in any direction, $C(t)$ may be any arbitrary function (increasing or/and decreasing, constant, positive or/and negative etc.). These variations we can measure by absolute and relative changes.

For the given time interval $[t_1, t_2]$ we have the following measures: the absolute change $\Delta C(t_1,t_2)$, the average absolute change per time unit $\Delta\bar{C}(t_1,t_2)$, the relative change $\delta C(t_1,t_2)$ and the average relative change per time unit $\delta\bar{C}(t_1,t_2)$. They are defined by

$$\Delta C(t_1,t_2) = C(t_2) - C(t_1), \qquad \Delta\bar{C}(t_1,t_2) = \frac{C(t_2)-C(t_1)}{t_2-t_1},$$

$$\delta C(t_1,t_2) = \frac{\Delta C(t_1,t_2)}{C(t_1)} = \frac{C(t_2)-C(t_1)}{C(t_1)}, \qquad \delta\bar{C}(t_1,t_2) = \frac{\Delta\bar{C}(t_1,t_2)}{C(t_1)} = \frac{\dfrac{C(t_2)-C(t_1)}{t_2-t_1}}{C(t_1)}.$$

Since $C(t)$ is the value of capital, the above expressions have the following meanings: $\Delta C(t_1,t_2)$ is the total amount of interest from the moment t_1 to t_2, $\Delta\bar{C}(t_1,t_2)$ is the average interest per time unit, $\delta C(t_1,t_2)$ is the interest coefficient, and $\delta\bar{C}(t_1,t_2)$ is average interest rate per time unit. Thus, using $C(t)$, we can state the general definition for the average interest rate per time unit on the given time interval $[t_1, t_2]$,

$$p(t_1,t_2) = \frac{\dfrac{C(t_2)-C(t_1)}{t_2-t_1}}{C(t_1)}. \tag{4}$$

From the relation (4) we can generate the standard interest accounts by assuming that $p(t_1,t_2)$ has constant value on the appropriate chosen interval $[t_1, t_2]$. In this way for $[t_1, t_2]=[0, t]$ we have

$$p(0, t) = \frac{\dfrac{C(t)-C(0)}{t}}{C(0)} = p \quad \Rightarrow \quad C(t) = C(0)\cdot(1+tp),$$

that is the simple account (1). For $[t_1, t_2]=[t, t+1]$ we obtain

$$p(t, t+1) = \frac{\dfrac{C(t+1)-C(t)}{1}}{C(t)} = p \quad \Rightarrow \quad C(t+1) = C(t)\cdot(1+p),$$

and hence, $t=0$: $C(1)=C(0)\cdot(1+p)$, $t=1$: $C(2)=C(1)\cdot(1+p)=C(0)\cdot(1+p)^2$, . . . , $t=n-1$: $C(n)=C(n-1)\cdot(1+p)=C(0)\cdot(1+p)^n$, that is the compound account (2). Finally, if we reduce the interval $[t_1, t_2]$ to the single point, $[t_1, t_2]=[t, t]=\left[t, \lim_{h\to 0}(t+h)\right]$, we have

$$p(t, t) = \frac{\lim\limits_{h\to 0}\dfrac{C(t+h)-C(t)}{h}}{C(t)} = \frac{C'(t)}{C(t)} = p \quad \Rightarrow \quad C(t) = C(0)\cdot e^{tp}, \tag{5}$$

that is the continuous account (3). Thus, all the standard interest accounts are generated from the same relation (4). Many other nonstandard accounts can be obtained from (4). We provide some examples in the next section.

IV. Examples

Since the classical interest accounts originate from the same relation (4), they must be somehow connected. It means that we could replace one of them with another one, under certain conditions, with the same effect. We explain that through the following example.

Example 1. Suppose that a principle doubled for ten years. What was the average interest rate per year if the interest is calculated by using: (a) simple, (b) compound, (c) continuous account?

If we insert $C_{10} = 2C_0$ in the relations (1), (2) and (3), we obtain

(a) $1+10p=2 \;\Rightarrow\; p=0.1=10\%$,
(b) $(1+p)^{10}=2 \;\Rightarrow\; p=0.07177... \approx 7.18\%$,
(c) $e^{10p}=2 \;\Rightarrow\; p=0.1\ln 2=0.06931... \approx 6.93\%$.

We see that the same effect is achieved by using different accounts with different interest rates. This example can be generalized. If p_1, p_2 and p_3 are the interest rates for simple, compound and continuous account, respectively, then the same amount of interest on the time interval $[0, t]$ will be obtained under following conditions.

If p_1 is given then $p_2(t)=\sqrt[t]{1+tp_1}-1, \quad p_3(t)=\dfrac{\ln(1+tp_1)}{t}$.
If p_2 is given then $p_1(t)=\dfrac{(1+p_2)^t-1}{t}, \quad p_3=\ln(1+p_2)$.
If p_3 is given then $p_1(t)=\dfrac{e^{tp_3}-1}{t}, \quad p_2=e^{p_3}-1$.

We can see that the constant p_2 yields the constant p_3 and vice versa while the others are time dependent. On the other hand, if the interest rate is the same for these three accounts we can have a big differences. It is shown in the next example.

Example 2. If an our ancient relative put for us 1 penny, with 1% of interest per year, before (a) 100, (b) 1000, (c) 10000 years, what amount we have at disposal today if (i) simple, (ii) compound, (iii) continuous interest account were applied ?

We use the relations (1), (2) and (3) with $C_0=1,\ p=0.01$ and (a) $n=100$, (b) $n=1000$, (c) $n=10000$. We obtain C_n as follows.

(a): (i) 2, (ii) 2.70, (c) 2.72; (b): (i) 11, (ii) 20959.16, (iii) 22026.47;
(c): (i) 101, (ii) $1.635828711...\cdot 10^{43}$, (iii) $2.688117142...\cdot 10^{43}$.

Finally, let us see some other nonstandard interest accounts which can be easily generated from the relations (4)and (5). Suppose that the monitoring of capital begins at the moment $t=a$ and ends at $t=b$. Thus, $[a, b]$ becomes domain of the function $C(t)$. For the general discrete model (4) we can make any partition,

$$a=t_1<t_2<t_3< \,...\, <t_n=b,$$

and then define

$$C(t_{i+1})=\left[1+(t_{i+1}-t_i)\cdot p(t_i, t_{i+1})\right]\cdot C(t_i), \qquad i=1,2,...,n-1.$$

where $p(x,y)$ can be any arbitrary chosen function defined on $[a, b]\times[a, b]$. This relation, together with an initial condition, defines the values in the partition nodes. Apart from them we have again freedom of choice, so we can adapt definition in accordance to the nature and properties of the considered situation. For example, if we want to ensure continuity of $C(t)$, we can set

$$C(t)=\left[1+(t-t_i)\cdot p(t_i, t_{i+1})\right]\cdot C(t_i), \quad t\in[t_i, t_{i+1}], \quad i=1,2,...,n-1,$$

where the interest rate is constant on each interval $[t_i, t_{i+1}]$, or

$$C(t)=\left[1+(t-t_i)\cdot p(t_i, t)\right]\cdot C(t_i), \quad t\in[t_i, t_{i+1}], \quad i=1,2,...,n-1,$$

where it is variable one, etc., etc.

Example 3. Suppose that we are monitoring the value of capital on the interval $[0, 10]$ with partition $0 < 2 < 5 < 6 < 10$. What is the values of capital in the partition nodes if $p(x, y) = (x + y)/1000$ and we have: (a) $C(0) = 10000$, (b) $C(5) = 10000$?

We have

$$\begin{aligned}
C(2) &= [1 + (2-0)\cdot(0+2)/1000]\cdot C(0) = 1.004\cdot C(0),\\
C(5) &= [1 + (5-2)\cdot(2+5)/1000]\cdot C(2) = 1.021\cdot C(2),\\
C(6) &= [1 + (6-5)\cdot(5+6)/1000]\cdot C(5) = 1.011\cdot C(5),\\
C(10) &= [1 + (10-6)\cdot(6+10)/1000]\cdot C(6) = 1.064\cdot C(6).
\end{aligned}$$

Hence we obtain: (a) $C(2) = 10040,\ C(5) = 10250.84,\ C(6) = 10363.60,\ C(10) = 11026.87$,
(b) $C(5) = 10000 = 1.021\cdot C(2) \Rightarrow C(2) = 9794.32 = 1.004\cdot C(0) \Rightarrow C(0) = 9755.30$,
$C(6) = 10110,\ C(10) = 10757.04$.

For the general continuous model (5), $p(t, t)$ is in fact an arbitrary chosen function of one variable $p(t)$ on the monitoring interval $[a, b]$. The function $C(t)$ is defined by differential equation with an initial condition in any point $t_0 \in [a, b]$,

$$\frac{C'(t)}{C(t)} = p(t),\ C(t_0) = K_0 \ \Rightarrow\ C(t) = K_0 \exp\left(\int_{t_0}^{t} p(\tau)d\tau\right),\ t \in [a, b].$$

Example 4. What is the value of capital $C(t)$ if: (a) $p(t) = At$, (b) $p(t) = A/t$, (c) $p(t) = -A/t^2$, where $A > 0$ is the given constant.

We have: (a) $C(t) = Ke^{\frac{A}{2}t^2}$, (b) $C(t) = Kt^A$, (c) $C(t) = Ke^{\frac{A}{t}}$. The functions of such type represents the natural growth (functions like (a), (b) and (3) with positive p) or decay (functions like (c)and (3) with negative p) of living beings and populations, so they have applications in biology, medicine and social science.

V. Conclusion

The simple, compound and continuous interest accounts, which are commonly used in practice, are generalized into one model. Thus, the standard models become only the special cases of the general one. Many other nonstandard models also come out from the general one. It enables us to choose the most appropriate model for the considered situation. Such different features are shownthrough examples.

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