IOSR Journal of Economics and Finance (IOSR-JEF)
e-ISSN: 2321-5933, p-ISSN: 2321-5925.Volume 13, Issue 4 Ser. VI (Jul. – Aug. 2022), PP 12-18
www.iosrjournals.org

A new model for amortization of debt

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Abstract:*In the paper we present a new flexible model for amortization of debt (loan repayment). First, for a given debt (loan) the amount of total interest is calculated. The calculation can be based on any classical model preferred, such as equal payments or equal principal payments. Then, a debtor can arbitrarily choose the initial repayment parameters -interest rate and payment amount. The model is designed in such a way that any initial choice yields the same given amount of total interest. It enables the debtor to choose the amortization schedule which agrees with his financial possibilities. The model can also be applied for rescheduling of debt (loan) when financial difficulties prevent regular payment.*

Key Words*:Amortization of debt, rescheduling of debt, amortization schedule, interest rate, total interest.*

Date of Submission: 18-08-2022 Date of Acceptance: 02-09-2022

I. Introduction and motivation

In the modern, unstable world every one of us wants to get better and have more. Our aspirations often rise above real possibilities. A wide variety of loans on offer seem to provide an easy way to overcoming the gap. Any loan, however, means debt and it can become a problem over time as many difficulties may occur over the term of the annuity. We are witnessing such situations daily, both on the individual and collective (national and international) level. The problem can be solved in many ways, from delayed payment, reduction of interest and/or debt to debt write-off. Generally speaking, such solutions are unfavourable for creditors who lose their income and capital. We present a compromise model which could be acceptable to both sides: creditors and debtors.

In the literature, there are numerous attempts at determining reasons for loan repayment problems and analysing them. Since many people need loans to complete their education, student loans have been widely studied. In [1] the authors present econometric models for evaluating determinants of regular student loan repayment. A similar analysis is presented in [7] and, using structural equation modelling, in similar analysis in [10]. Several student loan repayment options are studied in [5], where the author studies the conventional or fixed schedule, the income contingent, and the hybrid form of repayment obligation which he advocates as the best loan scheme. A very detailed discussion of evidence on the extent to which students are able to obtain credit and to repay it after is presented in [6]. Other types of loans have also been extensively studied. Theoretical underpinnings of high-frequency repayment are analysed in [3]. Critical factors affecting the repayment of microcredit are examined in [8]. An objective analysis of prepayment risk, based on a study of factors influencing bank customer behaviour and their impact on early loan repayment, is given in [9]. In contrast to the widespread opinion, a large-scale randomized field experiment with a typical urban microfinance institution (MFI) in [2] provides no evidence that lower frequency repayment schedules encourage irresponsible repayment behaviour among first-time borrowers receiving small loans.

In the paper we present a model for amortization of debt with a flexible repayment schedule that respects both creditor's requests and debtor's possibilities. The input data are the present value and the term of annuity. The total interest can be calculated using any preferred existing model or simply agreed on. The debtor chooses the initial interest rate and payment amount (or the portion of the principal in the payment) in agreement with his financial possibilities, independently of the input data. Over the term, they are gradually adopted in such a way that the total interest does not change. Note that it can also be changed (decreased or even increased) depending on the agreement between the debtor and the creditor. We develop a mathematical model for such a payment scheme. Some examples with different amortization schedules and the same final effect are also provided.

II. Definitions and notation

In the paper we consider an ordinary simple annuity. It means that payments are placed at the end of each equal rent period (payment intervals) and that the interest is compounded at the same frequency as the payments are made. We use the following basic terms and notation:

D_0 is the amount of debt (loan, principal) to be paid (the present value of annuity),
n is the number of payments in the term of annuity (the number of rent periods).
For the kth rent period, $k \in \{1, 2, ..., n\}$, we set:
i_k is the interest rate (nominal rate divided by periods per year),
R_k is the payment, P_k is the principal paid, I_k is the interest paid and
D_k is the outstanding balance (the rest of debt) after the payment.

The above terms in each period are connected by the relations (see [4]),

$$I_k = D_{k-1} i_k, \quad R_k = P_k + I_k, \quad D_k = D_{k-1} - P_k, \quad k = 1, 2, 3, ..., n, \tag{1}$$

which are used to create the amortization schedule. Since debt is paid by the last payment ($D_n = 0$), we have $D_0 = P_1 + P_2 + ... + P_n$ and generally,

$$D_{k-1} = P_k + P_{k+1} + ... + P_n, \quad k = 1, 2, 3, ..., n.$$

Total amount of interest is the difference between total payments and principal,

$$I = R_1 + R_2 + ... + R_n - D_0 = I_1 + I_2 + ... + I_n.$$

III. Classical models

Thereare two most commonly used amortization models where a fixed interest rate i for each rent period is assumed.

Model of equal payments. At the end of each period equal amount R has to be paid. It is well known (see [4] and also [11]) that the expressions for R and for total interest I are

$$R = D_0 i \frac{(1+i)^n}{(1+i)^n - 1}, \quad I = nR - D_0 = D_0 \left[\frac{ni(1+i)^n}{(1+i)^n - 1} - 1 \right]. \tag{2}$$

Model of equal principal payments. The payment R_k is placed at the end of each period $k \in \{1, 2, ..., n\}$. Payments are different but the portion of principal P is the same in each payment. For this model we have

$$P = \frac{D_0}{n}, \quad R_k = P\left[1 + (n-k+1)i\right], \quad I = \frac{D_0 i}{2}(n+1). \tag{3}$$

In the model which we present here, for the given D_0 and n, I can be calculated by using (2) or (3), or it can be a matter of agreement between debtor and creditor. We take D_0, n, I as input data for the model. Since the term of annuity can be short or very long, the interest rate is not a true indicator for debtor nor for creditor. For example, if payments take place monthly and $i = 0.5\% = 0.005$ then in the model of equal payments (2) we have

n	60	120	180	240	300	360
I / D_0 (%)	16.00	33.22	51.89	71.94	93.29	115.84

We see that I (or I / D_0) shows the actual load for debtor and income for creditor. For that reason we take I as a basic variable in the model.

IV. New model

Let D_0, n and I be the given debt (loan), term of annuity and total interest, respectively. Suppose that debtor chose the initial interest rate i_1 and principal payment P_1 in the first rent period. Note that he could also choose the initial payment R_1, in which case we have $P_1 = R_1 - D_0 i_1$. Let u and U be fixed increments for interest rates and principal payment in the subsequent periods, respectively. It means that in the kth period we have $i_k = i_{k-1} + u$, $P_k = P_{k-1} + U$ or

$$i_k = i_1 + (k-1)u, \quad P_k = P_1 + (k-1)U, \quad k = 1, 2, ..., n. \tag{4}$$

Our aim is to find u and U such that the total interest will be I. For this purpose we shall use the following expressions,

$$\sum_{k=1}^{n}(k-1) = \frac{n(n-1)}{2}, \quad \sum_{s=1}^{k-2} s = \frac{(k-2)(k-1)}{2},$$
$$\sum_{k=1}^{n}(k-1)^2 = \frac{n(n-1)(2n-1)}{6}, \qquad \sum_{k=1}^{n}(k-1)^3 = \frac{n^2(n-1)^2}{4}.$$

Since debt (loan) is sum of principal payments, using (4) we have

$$\sum_{k=1}^{n} P_k = D_0 \;\Rightarrow\; \sum_{k=1}^{n}\left[P_1 + (k-1)U\right] = D_0 \;\Rightarrow\; nP_1 + \frac{n(n-1)}{2}U = D_0,$$

which yields

$$U = \frac{2}{n-1}\left(\frac{D_0}{n} - P_1\right). \tag{5}$$

By this relation the increment U is given in the terms of initial data. Note that, if we set $P = D_0 / n$, we have

$$U > 0 \Leftrightarrow P_1 < P, \quad U < 0 \Leftrightarrow P_1 > P, \quad U = 0 \Leftrightarrow P_1 = P.$$

Thus, if debtor chooses $P_1 < P$ then the sequence $P_1, P_2, ..., P_n$ is in ascending order, for $P_1 > P$ it is in descending order and for $P_1 = P$ it is stationary. Now, we compute total interest. Using (4) we have

$$\begin{aligned}
I &= \sum_{k=1}^{n} I_k = \sum_{k=1}^{n} D_{k-1} i_k = \sum_{k=1}^{n}\left(P_k + P_{k+1} + ... + P_n\right) i_k \\
&= \sum_{k=1}^{n}\left\{\left[P_1 + (k-1)U\right] + \left[P_1 + kU\right] + ... + \left[P_1 + (n-1)U\right]\right\} i_k \\
&= \sum_{k=1}^{n}\left\{(n-k+1)P_1 + \left[(k-1) + k + ... + (n-1)\right]U\right\} i_k \\
&= \sum_{k=1}^{n}\left\{(n-k+1)P_1 + \left[\frac{n(n-1)}{2} - \frac{(k-2)(k-1)}{2}\right]U\right\} \cdot \left[i_1 + (k-1)u\right],
\end{aligned}$$

and thus,

$$I = \alpha P_1 i_1 + \beta P_1 u + \gamma U i_1 + \delta U u, \tag{6}$$

where

$$\alpha = \sum_{k=1}^{n}(n-k+1) = n\sum_{k=1}^{n}1-\sum_{k=1}^{n}(k-1),$$

$$\beta = \sum_{k=1}^{n}(n-k+1)(k-1) = n\sum_{k=1}^{n}(k-1)-\sum_{k=1}^{n}(k-1)^2,$$

$$\gamma = \sum_{k=1}^{n}\left[\frac{n(n-1)}{2}-\frac{(k-2)(k-1)}{2}\right] = \frac{1}{2}\left[n(n-1)\sum_{k=1}^{n}1-\sum_{k=1}^{n}(k-1)^2+\sum_{k=1}^{n}(k-1)\right],$$

$$\delta = \sum_{k=1}^{n}\left[\frac{n(n-1)}{2}-\frac{(k-2)(k-1)}{2}\right](k-1)$$

$$= \frac{1}{2}\left[n(n-1)\sum_{k=1}^{n}(k-1)-\sum_{k=1}^{n}(k-1)^3+\sum_{k=1}^{n}(k-1)^2\right],$$

which yields

$$\alpha=\frac{n(n+1)}{2},\quad \beta=\frac{n(n^2-1)}{6}=\frac{n-1}{3}\alpha,\quad \gamma=\frac{n(n^2-1)}{3}=2\beta,\quad \delta=\frac{n(3n-2)(n^2-1)}{24}=\frac{3n-2}{4}\beta. \qquad (7)$$

From the relation (6) we have

$$u = \frac{I-(\alpha P_1+\gamma U)i_1}{\beta P_1+\delta U}. \qquad (8)$$

Thus, our model can be summarized as follows.

<u>Flexible repayment model (FRM).</u>

1. Input D_0, n, I .
2. Choose i_1, P_1 .
3. Use (5) to compute U and then use (8) and (7) to compute u .
4. Use (1) and (4) to create amortization schedule.

V. Applications

We apply and explain the above model through the following examples. Suppose that 6000 pennies have to be repaid during 6 periods with interest rate of 5% using the model of equal principal payments. Using (3) we have

$$D_0=6000,\quad n=6,\quad I=1050, \qquad (9)$$

and $P=1000$. Suppose that repayment cannot be realized in this way. We shall apply FRM to show different repayment possibilities. Using the relation (7) for $n=6$ we obtain

$$\alpha=21,\quad \beta=35,\quad \gamma=70,\quad \delta=140. \qquad (10)$$

Example 1. Principal paid of 1000 pennies and interest rate 5% are too high for debtor at the beginning. He chooses the initial values $P_1=500, i_1=0$. The input data is given by (9). Using the relations (5), (8) and (10) we obtain

$$U=200,\quad u=\frac{3}{130}=0.023076923...=2.3076923...\% .$$

We create the repayment schedule by using (1)and (4). We shall express i_k as a fraction to ensure better accuracy.

k	i_k	R_k	I_k	P_k	D_k
0	-	-	-	-	6000
1	0	500.00	0.00	500	5500
2	3/130	826.92	126.92	700	4800
3	6/130	1121.54	221.54	900	3900
4	9/130	1370.00	270.00	1100	2800
5	12/130	1558.46	258.46	1300	1500
6	15/130	1673.08	173.08	1500	0
Σ		7050.00	1050.00	6000	

Example 2. Debtor has a better ability to pay at the beginning. He chooses the initial values $P_1 = 1600,\ i_1 = 0.1$. Using the input data (9) and the relations (5), (8), (10) we obtain

$$U = -240, \quad u = -\frac{9}{320} = -0.028125 = -2.8125\% \ .$$

We create the repayment schedule by using (1)and (4).

k	i_k	R_k	I_k	P_k	D_k
0	-	-	-	-	6000
1	0.100000	2200.00	600.00	1600	4400
2	0.071875	1676.25	316.25	1360	3040
3	0.043750	1253.00	133.00	1120	1920
4	0.015625	910.00	30.00	880	1040
5	- 0.012500	627.00	- 13.00	640	400
6	- 0.040625	383.75	- 16.25	400	0
Σ		7050.00	1050.00	6000	

Example 3. Debtor can even choose negative initial values. In this case he will get one or several payments at the beginning instead of paying them. Let $P_1 = -500,\ i_1 = -0.05$. Using (9) and the relations (5), (8) , (10) we obtain

$$U = 600, \quad u = \frac{3}{76} = 0.0394736842... = 3.94736842...\% \ .$$

We create the repayment schedule by using (1)and (4). We round i_k to 10^{-8}.

k	i_k	R_k	I_k	P_k	D_k
0	-	-	-	-	6000
1	- 0.05	- 800.00	- 300.00	-500	6500
2	- 0.01052632	31.58	- 68.42	100	6400
3	0.02894737	885.26	185.26	700	5700
4	0.06842105	1690.00	390.00	1300	4400
5	0.10789474	2374.74	474.74	1900	2500
6	0.14736842	2868.42	368.42	2500	0
Σ		7050.00	1050.00	6000	

Example 4. One can even play with FRM and it will produce very interesting effects. Suppose that debtor chooses a very large principal paid and a small interest rate. Let $P_1 = 2200$ and $i_1 = 0.01$. Using (9) and the relations (5),(8), (10) we obtain

$$U = -480, \quad u = \frac{33}{350} = 0.094285714... = 9.4285714...\% \ .$$

We create the repayment schedule by using (1)and (4). We shall express i_k again as a fraction. We have $i_1 = 0.01 = 7/700$, $u = 66/700$.

k	i_k	R_k	I_k	P_k	D_k
0	-	-	-	-	6000
1	7/700	2260.00	60.00	2200	3800
2	73/700	2116.29	396.29	1720	2080
3	139/700	1653.03	413.03	1240	840
4	205/700	1006.00	246.00	760	80
5	271/700	310.97	30.97	280	- 200
6	337/700	- 296.29	- 96.29	- 200	0
Σ		7050.00	1050.00	6000	

We see that the last amount is paid to debtor. We give a general explanation for this situation. Let $P = D_0 / n$. If we want to have $P_n \le 0$ then (4) and (5) imply

$$P_n = P_1 + (n-1)U = P_1 + (n-1)\cdot\frac{2}{n-1}(P - P_1) = 2P - P_1 \le 0 \quad \Rightarrow \quad P_1 \ge 2P.$$

Similarly, we can require $P_k \le 0$ for any k.

Example 5. Suppose that debtor and creditor agree to reschedule debt in the way that the term of annuity is increased to 10 periods. Other input data remains as in the relation (9). Thus we have $D_0 = 6000,\ n = 10,\ I = 1050$. Using the relation (7) for $n = 10$ we obtain

$$\alpha = 55, \quad \beta = 165, \quad \gamma = 330, \quad \delta = 1155.$$

Suppose that debtor chooses the initial values as in Example 1, $P_1 = 500,\ i_1 = 0$. Using the relations (5) and (8) we obtain

$$U = \frac{200}{9} = 22.222222..., \quad u = \frac{63}{6490} = 0.0097072419... = 0.97072419...\% .$$

We create the repayment schedule by using (1)and (4). Calculation is performed with 10 correct digits. In the table i_k is rounded to 10^{-8} and the other results to 10^{-2}.

k	i_k	R_k	I_k	P_k	D_k
0	-	-	-	-	6000.00
1	0	500.00	0.00	500.00	5500.00
2	0.00970724	575.61	53.39	522.22	4977.78
3	0.01941448	641.09	96.65	544.44	4433.34
4	0.02912173	695.77	129.10	566.67	3866.67
5	0.03882897	739.03	150.14	588.89	3277.78
6	0.04853621	770.20	159.09	611.11	2666.67
7	0.05824345	788.65	155.32	633.33	2033.34
8	0.06795069	793.72	138.16	655.56	1377.78
9	0.07765794	784.77	106.99	677.78	700.00
10	0.08736518	761.16	61.16	700.00	0
Σ		7050.00	1050.00	6000.00	

Since the interest rate increases slowly while the balance decreases faster, we can see how payments increase in the beginning, then slow down and finally decrease at the end of the term. The other examples can be also solved with $n = 10$ (or with any other n) which will produce very interesting effects. These effects become more evident for a larger n.

VI. Conclusion

Inthe paper we have presented a new, flexible model for debt (loan) repayment (FRM). Nowadays prosperity generally rests on debt. Loans are the basis of business and private investment. Since in the modern world circumstances are changing rapidly, regular debt repayment can become hampered or even impossible. The repayment schedule, which was agreed, frequently needs to be changed. Creditors (mainly banks) often have a rigid attitude towards debt rescheduling. They are afraid of losing their income and therefore they apply additional insurance instruments, which can be harmful to debtors.

Our FRM is an attempt to find a compromise solution. Debt (loan), repayment term and total interest are the input data for the model. Debtor chooses the initial repayment parameters (interest rate and payment) in accordance with his possibilities. The repayment schedule is created respecting his choice and the total interest. In this way, debtor realizes his payments as he can and creditor does not lose his income. We provide some examples to show how different repayment plans produce the same final effect. We see that debtor can even receive payment during the repayment time without affecting the final outcome. Note that, using FRM, loan rescheduling can be done several times, whenever regular repayment becomes questionable.

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MladenMatejaš. “A new model for amortization of debt.” *IOSR Journal of Economics and Finance (IOSR-JEF)*, 13(4), 2022, pp. 12-18.