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A Class of Continuous Mappings in Ideal Topological Spaces

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Abstract: *In this paper, we introduce the notions of I_{rwg}-continuous maps and I_{rwg}-irresolute maps in ideal topological spaces. We investigate some of their properties.*
Key words: *I_{rwg}- closed set, I_{rwg}-continuous maps, I_{rwg}-irresoluteness.*

I. Introduction

In 1990, T.R.Hamlett and D.Jankovic[1], introduced the concept of ideals in topological spaces and after that [2,3, 4, 5, 6] several authors turned their attention towards generalizations of various concepts of topology by considering ideal topological spaces.

A non-empty collection I of subsets on a topological space (X, τ) is called a topological ideal if it satisfies the following two conditions:

(i) If A∈ I and B⊂ A implies B∈I (heredity)

(ii) If A∈ I and B ∈I, then A∪B ∈I (finite additivity)

By a space (X, τ), we always mean a topological space (X, τ). If A⊂X, cl(A) and int(A) will, respectively, denote the closure and interior of A in (X, τ). Let (X,τ ,I) be an ideal topological space and A⊂X . $A^*(I, \tau) = \{x \in X / U \cap A \notin I$ for every $U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau : x \in U\}$, is called the local function of A with respect to I [7] . For every topological space (X, τ,I) there exists a topology τ^* finer than τ defined as $\tau^* = \{U \subseteq X : cl^*(X-U) = X-U\}$. A Kuratowski closure operator $cl^*(.)$ for topology $\tau^*(I, \tau)$,$cl^*(A) = A \cup A^*$. Clearly, if I = {∅}, then $cl^*(A) = cl(A)$ for every subset A of X. In this paper, we define I_{rwg}- continuous mappings, I_{rwg} -irresoluteness in ideal spaces and discuss their properties and characterizations.

II. Preliminaries

Definition 2.1:[8]A function f: (X,τ) → (Y, σ) is g -continuous if $f^{-1}(V)$ is g – closed in (X, τ) for every closed set V of (Y, σ).
Definition 2.2: A function f: (X,τ, I) → (Y, σ) is I-rg -continuous if $f^{-1}(V)$ is I-rg – open in (X, τ, I) for every open set V in (Y, σ).
Definition 2.3: A function f: (X,τ, I) → (Y,Ω,J) is said to be weakly I – continuous if for each x∈ X and each open set V in Y containing f(x), there exists an open set U containing x such that $f(U) \subset cl^*(V)$.
Definition 2.4[10]:A functionf: (X,τ) → (Y, σ) is said to be I_{s*g}- continuous if for every U ∈σ, $f^{-1}(U)$ is I_{s*g}-open in (X,τ, I).
Definition 2.5: A functionf: (X,τ, I) → (Y, σ) is * -continuous if $f^{-1}(A)$ is * – closed in X forevery closed set A in Y.
Lemma 2.6[9]: Let (X,τ, I) be an ideal topological space and A⊂ X. If $A \subset A^*$, then$A^* = cl(A^*) = cl(A) = cl^*(A)$.
Lemma 2.7: If U is open and A is I_{rwg}- open, then U ∩ A is I_{rwg}- open.

III. Regular Weakly Generalized Continuous Mappings In Ideal Topological Spaces

Definition 3.1: A function f: (X,τ, I) → (Y, σ) is said to be I_{rwg}-continuous if $f^{-1}(V)$ is I_{rwg}-closed in (X,τ, I) for every closed set V in (Y, σ).
Theorem 3.2:A function f: (X,τ, I) → (Y, σ) is I_{rwg}-continuous if and only if $f^{-1}(V)$ is I_{rwg}-open in (X,τ, I) for every open set V in (Y, σ) .
Proof: Let V be an open set in (Y, σ) and f: (X,τ, I) → (Y, σ) be I_{rwg}-continuous. Then V^c is closed in (Y, σ) and $f^{-1}(V^c)$ is I_{rwg}- closed in (X,τ, I). But $f^{-1}(V^c) = (f^{-1}(V))^c$ and so $f^{-1}(V)$ is I_{rwg}-open in (X,τ, I).
Conversely, suppose that $f^{-1}(V)$ is I_{rwg}-open in (X,τ, I) for each open set V in (Y, σ). Let F be a closed set in (Y, σ) . Then F^c is open in (Y,σ) and by hypothesis $f^{-1}(F^c)$ is I_{rwg}- open in (X,τ, I). Since $f^{-1}(F^c) = (f^{-1}(F))^c$, we have $f^{-1}(F^c)$ is I_{rwg}-closed in (X,τ, I) and so f is I_{rwg}-continuous.
Theorem 3.3: Every *- continuous function is I_{rwg}-continuous.
Proof: Let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is *-closed in (X,τ, I) because f is * - continuous in X. Since every * - closed set is I_{rwg} – closed, $f^{-1}(V)$ is I_{rwg} – closed in (X,τ, I). Therefore f is I_{rwg}-continuous.
The converse of the above theorem need not be true as seen from the following example.

Example 3.4: X = {1,2,3} , τ = { ∅, X, {1},{2,3}} , I = { ∅ , {3}} and σ = { ∅ , X , {1,3}}.
The identity map f: (X,τ, I) → (Y, σ) is I_{rwg}-continuous but not *- continuous.
Theorem 3.5: Every continuous function is I_{rwg}-continuous.
Proof: Let f be a continuous function and V be a closed set in (Y, σ) .Then $f^{-1}(V)$ is closed in (X,τ, I).Since every closed set is * -closed and hence I_{rwg} – closed, $f^{-1}(V)$ is I_{rwg} – closed in (X,τ, I).Therefore,f is I_{rwg}-continuous.
The converse of the above theorem need not be true as seen from the following example.
Example 3.6: Let X = {1,2,3} , τ = { ∅, X, {1},{2,3}} , I = { ∅ , {3}} and σ = { ∅ , X , {1}}.
Define f: (X,τ, I) → (Y, σ) as f(1) = 2, f(2) = 1, f(3) = 3. Then f is I_{rwg}-continuous but not continuous.
Remark 3.7: The above relationships are shown in the following diagram:
Continuity ⟹ *- Continuity ⟹I_{rwg}-continuity
Definition 3.8:[10] An ideal topological space (X,τ, I) is said to be T-dense if every subset of X is *-dense in itself.
Theorem 3.9: Let (X,τ, I) be T-dense. Then for a function f: (X,τ, I) → (Y, σ) the following statements are equivalent:

1. f is I_{rwg}-continuous.
2. For each x ∈ X and each open set V in Y with f(x) ∈ V, there exists an I_{rwg} -open set U containing x such that f(U) ⊂ V.
3. For each x ∈ X and each open set V in Y with f(x) ∈ V, $f^{-1}(V)$ is an I_{rwg} -open neighborhood of x.

Proof: (1) ⇒ (2) Let x ∈ X and let V be an open set in Y such that f(x) ∈ V. Since f is I_{rwg}-continuous, $f^{-1}(V)$ is an I_{rwg} -open in X. By taking U = $f^{-1}(V)$, we have x ∈ U and f(U) ⊂ V.
(2) ⇒ (3) Let V be an open set in Y and let f(x) ∈ V.Then by (2), there exists an I_{rwg} - open set U containing x such that f(U) ⊂ V. So x ∈ U ⊂ $f^{-1}(V)$.Hence $f^{-1}(V)$ is an I_{rwg} -open neighborhood of x.
(3) ⇒ (1) Let V be an open set in Y and let f(x) ∈ V. Then by (3) $f^{-1}(V)$ is an I_{rwg} -open neighborhood of x. Thus for each x ∈ $f^{-1}(V)$, there exists an I_{rwg} -open set U_x containing x such that x ∈U_x⊂ $f^{-1}(V)$. Hence $f^{-1}(V)$ = $\bigcup_{x\in f^{-1}(V)} U_x$ and so $f^{-1}(V)$ is an I_{rwg} -open in X.
Theorem 3.10: Let f: (X,τ, I) → (Y, σ, J) be a function and {U_α : α ∈∇ } be an open cover of a T-dense space X. If the restriction f | U_α is I_{rwg}-continuous for each α ∈∇, f is I_{rwg}-continuou**s.**
Proof: Suppose V is an arbitrary open set in (Y, σ, J). Then for each α ∈∇,we have $(f \mid U_\alpha)^{-1}(V) = f^{-1}(V) \cap U_\alpha$. Because f | U_α is I_{rwg}-continuous, therefore $f^{-1}(V) \cap U_\alpha$ is I_{rwg} - open set in X for each α ∈∇.Since for each α ∈∇, U_α is open in x, by [9 Theorem 5] $f^{-1}(V) \cap U_\alpha$ is I_{rwg} -open set in X. Now since X is T-dense, $\bigcup_{\alpha\in\nabla} f^{-1}(V) \cap U_\alpha$ = $f^{-1}(V)$ is I_{rwg}- open in X. This implies f is I_{rwg}-continuous.
Theorem 3.11: If (X,τ, I) is T-dense space and f: (X,τ, I) → (Y, σ) is I_{rwg}-continuous, then graph function g: X → X × Y, defined by g(x) = (x ,f(x)) for each x ∈ X, is I_{rwg}-continuous.
Proof: Let x ∈ X and W be any open set in X × Y containing g(x) = (x ,f(x)). Then there exists a basic open set U × V such that g(x) ⊂ U × V ⊂W. Since f is I_{rwg}-continuous, there exists an I_{rwg}- open set U_1 in X containing x such that f(U_1) ⊂ V. By Lemma 2.7 $U_1 \cap U$ is I_{rwg}- open in X and we have x ∈ $U_1 \cap U$ ⊂ U and g($U_1 \cap U$) ⊂ U × V ⊂ W. Since X is T-dense, therefore Theorem 3.9, g is I_{rwg}-continuous.
Definition 3.12: For a function f: (X,τ, I) → (Y, σ), the subset { (x,f(x)) : x ∈ X } ⊂ X ×Y is called the graph of f and is denoted by G(f).
Theorem 3.13: Let f: (X,τ, I) → (Y, σ)be a function and g: X → X × Y be the graph function of f. If g is I_{rwg}-continuous, then f is I_{rwg}-continuous.
Proof: Suppose that g is I_{rwg}-continuous. Let x ∈ X and V be any open set of Y containing f(x). Then X × V is open in X × Y and I_{rwg}-continuity of g, then exists U ∈ I_{rwg}- open of X containing x such that g(U) ⊂ X ×V. Therefore we obtain f(U) ⊂ V. Hence f is I_{rwg}-continuous.
Remark 3.14: The composition of two I_{rwg}-continuous maps need not be I_{rwg}-continuous as seen from the following example.
Example 3.15: Let X = Y = Z = {1,2,3,4} , τ = { ∅, X, {1},{1,2,3}} and σ = { ∅ , Y , {3}}
Ω = { ∅ , Z , {2,4}} , I = { ∅ , {1},{2},{1,2}} , J ={ ∅ , {3}}. f: (X,τ, I) → (Y, σ,J) and
g: (Y, σ,J) → (Z, Ω) be identity maps. Then the maps f and g are I_{rwg}-continuous but g o f is not I_{rwg}-continuous.

IV. Regular Weakly Generalized Irresoluteness In Ideal Topological Spaces

Definition 4.1 : A function f: $(X, \tau, I) \rightarrow (Y, \sigma, J)$ is said to be I_{rwg}-irresolute if $f^{-1}(V)$ is I_{rwg}-closed in (X, τ, I) for every I_{rwg}-closed set V in (Y, σ, J).

Theorem4.2: Every I_{rwg}-irresolute function is I_{rwg}-continuous.

Proof: Suppose f: $(X, \tau, I) \rightarrow (Y, \sigma, J)$ is I_{rwg}-irresolute. Let V be a closed in Y which is I_{rwg}-closed then $f^{-1}(V)$ is I_{rwg}-closed in X. Hence f is I_{rwg}-continuous.

Converse of the theorem is not true as seen from the following example.

Example 4.3: Let X = Y= {a,b,c}, τ = {∅, X, {a}, {b},{a,b}},I = { ∅,{c}}, σ = {∅,{a,b}},J = {∅,{b}}. The identity function is I_{rwg}-continuous but not I_{rwg}-irresolute. Since the I_{rwg}-closed sets in Y are the power set of Y and I_{rwg}-closed sets of X are {∅, X,{c},{a,b},{a,b},{b,c}}.

Theorem 4.4: A function f: $(X, \tau, I) \rightarrow (Y, \sigma, J)$ is I_{rwg}-irresolute if and only if the inverse mage of every I_{rwg}-open in (Y, σ, J) is I_{rwg}-open in (X, τ, I).

Theorem 4.5: If function f: $(X, \tau, I) \rightarrow (Y, \sigma, J)$ is I_{rwg}- irresolute and g: $(Y, \sigma, J) \rightarrow (Z, \eta)$ is *-continuous then $g \circ f$: $(X, \tau, I) \rightarrow (Z, \eta)$ is I_{rwg}-continuous.

Proof: Let V be any closed set of (Z, η). Then $g^{-1}(V)$ is * - closed in (Y, σ, J) .Therefore $f^{-1}(g^{-1}(V) = (g \circ f)^{-1}(V)$ is I_{rwg}- closed in (X, τ, I), since every *- closed set is I_{rwg}- closed.

Hence $g \circ f$ is I_{rwg}-continuous.

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