Area calculation of the level region which is enclosed by the graphical representation of three continuous functions.

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Abstract: *This proof was conceived in order to introduce a different way of calculating a level region. By the end of the paper I will calculate the area of a level region using continuous functions.*
Keywords: *calculate, area, enclosed, level, region*

I. Introduction

As known the area for the level region (Ω) that we examine is given by a formula, thus a figure is required **"necessarily"**. Enough of my students have asked me if there was a way to avoid signing the functions and drawing a figure to calculate the area. For that reason, I thought of the work that follows, which calculates the area for the region in question without determining the sings of the functions or drawing a figure.

II. Area calculation of the level region (Ω)

In the international literature, to my knowledge, the previous area is given by the formula:

$$E(\Omega) = \int_{\rho_1}^{\rho_2} \left| f_3(x) - f_1(x) \right| dx + \int_{\rho_2}^{\rho_3} \left| f_3(x) - f_2(x) \right| dx \qquad (1)$$

where p_1, p_2, p_3 are the roots of the equations: $f_1(x) = f_3(x)$, $f_1(x) = f_2(x)$, $f_2(x) = f_3(x)$ respectively and for the calculation of $E(\Omega)$, a figure is required necessarily (see Fig 1).
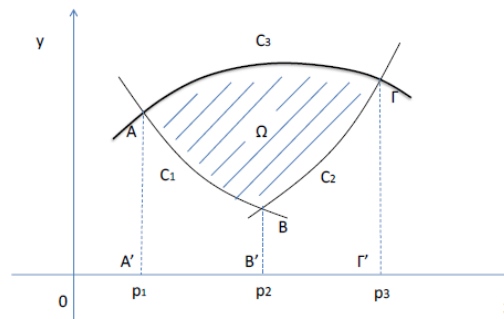

Fig 1

This work calculates the area that we described without requiring to draw and without finding any sign. Specifically, there will be no need in finding the sign for the functions $f_3 - f_1, f_3 - f_2$ and the drawing of $C_{f_1}, C_{f_2}, C_{f_3}$ for the calculation of $E(\Omega)$.

III. Calculating the level region (Ω) using continuous functions

The area of the region (Ω) is given by the formula:

$E(\Omega) = \left| \int_{p_1}^{p_2} f_1 x + \int_{p_2}^{p_3} f_2(x)\, dx + \int_{p_3}^{p_1} f_3(x)\, dx \right| \quad (1)$

because if

- (Ω_1) is the region which is enclosed by C_1, the straight lines x=p_1, x=p_2 and the axis x'x
- (Ω_2) is the region area which is enclosed by C_2, the straight lines x=p_2, x=p_3 and the axis x'x and
- (Ω_3) is the region which is enclosed by C_3, the straight lines x=p_1, x=p_3 and the axis x'x

as it is evident from Figure (1) we have:

$$E(\Omega) = \left| E(\Omega_1) + E(\Omega_2) - E(\Omega_3) \right| = \left| \int_{p_1}^{p_2} f_1(x) + \int_{p_2}^{p_3} f_2(x)\, dx - \int_{p_1}^{p_3} f_3(x)\, dx \right| =$$

$$= \left| \int_{p_1}^{p_2} f_2(x)\, dx + \int_{p_2}^{p_3} f_1(x) + \int_{p_3}^{p_1} f_3(x)\, dx \right| \quad \textbf{(2)}$$

It is not required for $f_1(x) > 0$, $f_2(x) > 0$, $f_3(x) > 0$ to apply, as there will always be a $c \in \Re$ that :
$f_1(x) + c > 0$, $f_2(x) + c > 0$, $f_3(x) + c > 0$
and the position of $f_1(x), f_2(x), f_3(x)$ in the formula that the previous functions take.

3.1. Application mechanism for the formula 2

In order to remember the formula (2) we write in turn the p_1, p_2, p_3 and under every root, we denote the equation that it derives from (see fig 2) .
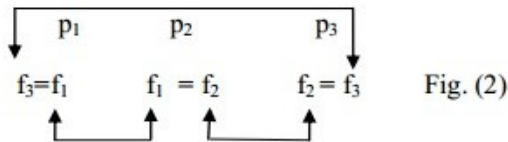
Fig. (2)

The area is equal with the absolute value of the sum of the three integrals with the
- First integral from p_1 to p_2 of the common function f_1 (see fig (2))
- Second integral from p_2 to p_3 of the common function f_2 (see fig (2))
- Third integral from p_3 to p_1 of the common function f_3 (see fig (2))

Thus we don't need to draw C_1, C_2, C_3 and we don't need to find the sign of the differences $f_2 - f_1$, $f_2 - f_3$ which was required until now by the formula (1).

3.2. Application

Calculate the area of level region (Ω) which is enclosed by the graphs of the functions
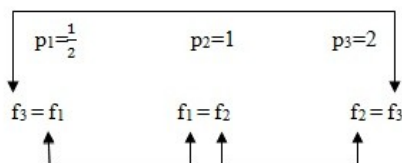$f_1(x) = \ln \frac{1}{x} = -\ln x$
$f_2(x) = \ln x$
$f_3(x) = \ln 2$
Solution
$f_3(x) = f_1(x) \Leftrightarrow \ln \frac{1}{2} = \ln x \Leftrightarrow x = \frac{1}{2}$
$f_1(x) = f_2(x) \Leftrightarrow \ln \frac{1}{x} = \ln x \Leftrightarrow x = 1$
$f_2(x) = f_3(x) \Leftrightarrow \ln x = \ln 2 \Leftrightarrow x = 2$
Therefore, $p_1 = \frac{1}{2}$, $p_2 = 1$, $p_3 = 2$
We draw the design that I mentioned in the memory rule

We have, without designing, the region (Ω) :
$E(\Omega) = \int_{\frac{1}{2}}^{1} \ln \frac{1}{x}\, dx + \int_{1}^{2} \ln x\, dx + \int_{2}^{\frac{1}{2}} \ln 2\, dx = ... =$

IV.   Conclusion

In this paper a different approach on area calculation of the level region was introduced. The method commonly used until now required the use of graphs and signs for the functions that were involved in the process whereas with the new way of calculating the area only continuous functions are used. It was proven to be easier and more accessible. By using this method, hopefully the calculation of such an area will be simpler and quicker.

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