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Generalized Double Star Closed Sets in Interior Minimal Spaces

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Abstract: *The aim of this paper is to introduce and investigate Generalized Double Star Closed set in Interior Minimal spaces. Several properties of these new notions are investigated.*
Keywords: *$\mathcal{M}$-g** Separationspace, Maximal $\mathcal{M}$-g**closed, Maximal $\mathcal{M}$-g**open, Minimal $\mathcal{M}$-g**closed, Minimal $\mathcal{M}$-g** open*

I. Introduction

Norman Levine [1] introduced the concepts of generalized closed sets in topological spaces.Closed sets are fundamental objects in a topological space. For example, one can define the topology on a set by using either the axioms for the closed sets or the Kuratowski closure axioms. By definition, a subset A of a topological space X is called generalized closed if clA⊆U whenever A⊆U and U is open. This notion has been studied by many topologists because generalized closed sets are not only natural generalization of closed sets but also suggest several new properties of topological spaces. Nakaoka and Oda[2,3,4] have introduced minimal open sets and maximal open sets, which are subclasses of open sets. Later on many authors concentrated in this direction and defined many different types of minimal and maximal open sets. Inspired with these developments we further study a new type of closed and open sets namely Minimal $\mathcal{M}$-g**Open sets, Maximal $\mathcal{M}$-g** Closed sets. In this paper a space X means a Minimal space (X, $\mathcal{M}$). For any subset A of X its $\mathcal{M}$-interior and $\mathcal{M}$-closure are denoted respectively by the symbols $\mathcal{M}$-int(A) and $\mathcal{M}$-cl(A).

II. Preliminaries

Definition 1A is said to be $\mathcal{M}$-g^{**}closed ($\mathcal{M}$-generalized double star closed) if $\mathcal{M}$-cl(A) ⊆ U whenever A ⊆ U and U is *$\mathcal{M}$-g*open.*

Definition2A is said to be *$\mathcal{M}$-g^{**}*open($\mathcal{M}$-generalized double star open) if $\mathcal{M}$-cl(A) ⊆ U whenever A ⊆ U and U is *$\mathcal{M}$-g*closed.*

Theorem3 Every$\mathcal{M}$-closed set is $\mathcal{M}$-g**closed.
ProofLet A be an $\mathcal{M}$-closed subset of X.Let A ⊂ U and U be $\mathcal{M}$-g*open.Since A is $\mathcal{M}$-closed, then $\mathcal{M}$-cl(A) =A. So $\mathcal{M}$-cl(A) ⊂ U.Thus A is $\mathcal{M}$-g**closed.
Converse is not true

Theorem4 If A is $\mathcal{M}$-g**closed and A ⊂ B ⊂$\mathcal{M}$-cl(A), then B is $\mathcal{M}$-g**closed.
ProofSince B ⊂$\mathcal{M}$-cl(A), $\mathcal{M}$-cl(B) ⊂$\mathcal{M}$-cl(A). Let B ⊂ U and U be $\mathcal{M}$-g*open.Since A ⊂ B, A is $\mathcal{M}$-g**closed and U is $\mathcal{M}$-g*open. Gives $\mathcal{M}$-cl(A) ⊂ U. Since B ⊂$\mathcal{M}$-cl(A), $\mathcal{M}$-cl(B) ⊂$\mathcal{M}$-cl(A). $\mathcal{M}$-cl(B) ⊂ U.Hence B is $\mathcal{M}$-g**closed.

Theorem5 Every$\mathcal{M}$-open set is *$\mathcal{M}$-g**open.*
ProofLet A be a $\mathcal{M}$-open set of X.Then A^c is $\mathcal{M}$-closed set. By Theorem3, A^c is $\mathcal{M}$-g**closed, then A is $\mathcal{M}$-g**open.

Converse is not true
Theorem6 If A is *$\mathcal{M}$-g**open* and $\mathcal{M}$-int(A) ⊂ B ⊂ A. Then B is $\mathcal{M}$-g**open.
ProofA is *$\mathcal{M}$-g**open.* Hence A^c is *$\mathcal{M}$-g**closed.* Also $\mathcal{M}$-Int(A) ⊂ B ⊂ A gives$[\mathcal{M}\text{-Int}(A)]^c \supset B^c \supset A^c$. So $A^c \subset B^c \subset \mathcal{M}$-cl($A^c$). Thus B^c is $\mathcal{M}$-g**closed and hence B is $\mathcal{M}$-g**open.

Theorem7 IfA and B are *$\mathcal{M}$-g**closed* sets in X and $\mathcal{M}$-cl(A∪B) = $\mathcal{M}$-cl(A) ∪$\mathcal{M}$-cl(B), then A∪B is also *$\mathcal{M}$-g**closed* set in X.

ProofSuppose that $U$is $\mathcal{M}$-*g*open* and $A\cup B\subseteq U$. Then $A\subseteq U$ and $B\subseteq U$.Since A and B are $\mathcal{M}$-*g**closed* subsets in X,$\mathcal{M}$-cl(A)$\subseteq U$ and **M**–cl(B)$\subseteq U$.Hence $\mathcal{M}$-(A) $\cup\mathcal{M}$-(B)$\subseteq U$. Given$\mathcal{M}$-cl($A\cup B$) = $\mathcal{M}$-$cl(A)$ $\cup\mathcal{M}$-$cl(B)$ $\subseteq U$. Therefore $A\cup B$ is also$\mathcal{M}$-*g**closed* set in X.

Definition8A Interior minimal space (X, $\mathcal{M}$) is said to be a$\mathcal{M}$-g**T_0space if for every pair of points x ≠ y in X either there exists$\mathcal{M}$-g***open* set U such that x $\in U$, y $\notin U$ *or* y $\in U$, x $\notin U$.

Theorem9 Every$\mathcal{M}$-*T_0space* is $\mathcal{M}$-*g**T_0*space but not conversely.
Proof is obvious since every $\mathcal{M}$-open set is $\mathcal{M}$-g**open.

Definition10 An Interior minimal space (X, **M**) is said to be a $\mathcal{M}$-g**T_1space if for every pair of points x ≠ y in X there exists$\mathcal{M}$-g***open* sets U and V such that x $\in U$, y $\notin U$ and y $\in V$, x $\notin V$.

Theorem11 Every$\mathcal{M}$-T_1space is $\mathcal{M}$-g**T_1space. *Proof* is obvious since every $\mathcal{M}$-open set is $\mathcal{M}$-g**open.

Definition12 An Interior minimal space(X, $\mathcal{M}$) is said to be a $\mathcal{M}$-g**T_2 space if for every pair of distinct points x, y in X there exists disjoint$\mathcal{M}$-g**open sets U and V in X such that U∩V = ϕ.

Theorem13 Every$\mathcal{M}$-T_2space is $\mathcal{M}$-g**T_2space. *Proof* is obvious since every $\mathcal{M}$-open set is $\mathcal{M}$-g**open.

Theorem14 Every$\mathcal{M}$-g**T_2space is $\mathcal{M}$-g**T_1space but not conversely.
Proof is obvious from the definitions.

Theorem15 Any singleton set in an Interior minimal space is either $\mathcal{M}$-semiclosed or $\mathcal{M}$-g**open
ProofLet {x} be a singleton set in an Interior minimal space X.If {x} is $\mathcal{M}$-semiclosed, then proof is over.If {x} is not a $\mathcal{M}$-semiclosed set,
Then $\{x\}^c$ is not $\mathcal{M}$-semiopen.Therefore X is the only $\mathcal{M}$-semiopen set containing $\{x\}^c$ and$\{x\}^c$ is $\mathcal{M}$-g**closed. Hence {x} is $\mathcal{M}$-g**open. Thus {x} is $\mathcal{M}$-semiclosed or $\mathcal{M}$-g*s*open.

Definition16 An Interior minimal space X is called $\mathcal{M}$-g**$T1_{/3}$space if every $\mathcal{M}$-gclosed set in X is $\mathcal{M}$-g**closed in X.

Theorem17 An Interior minimal space X is $\mathcal{M}$-g**$T_{1/3}$ if and only if every $\mathcal{M}$-gopen set is $\mathcal{M}$-g**open.
ProofLet X be $\mathcal{M}$-g**$T_{1/3.}$ Let A be $\mathcal{M}$-gopen set.To prove, A is $\mathcal{M}$-g**open.Since A is $\mathcal{M}$-gopen then A^c is $\mathcal{M}$-gclosed.Thus A^c is $\mathcal{M}$-g**closed. [X is $\mathcal{M}$-g**$T_{1/3}$] givesA is $\mathcal{M}$-g**open.*Conversely*,Suppose every $\mathcal{M}$-gopen is $\mathcal{M}$-g**open.Then every $\mathcal{M}$-gclosed set is $\mathcal{M}$-g**closed.Hence X is $\mathcal{M}$-g**$T_{1/3.}$

Definition18 An Interior minimal space X is called $\mathcal{M}$-g**$T_{1/2}$space if every $\mathcal{M}$-g**closed set is $\mathcal{M}$-closed.

Theorem19 An Interior minimal space X is $\mathcal{M}$-g**$T_{1/2}$ if and only if every $\mathcal{M}$-g**open set is $\mathcal{M}$-open.
ProofLet X be $\mathcal{M}$-g**$T_{1/2}$ and let A be $\mathcal{M}$-g**open.Then A^c is $\mathcal{M}$-g**closed and so A^c is $\mathcal{M}$-closed.Thus A is $\mathcal{M}$-open.
Conversely,Suppose every $\mathcal{M}$-g**open set is $\mathcal{M}$-open.Let A be any $\mathcal{M}$-g**closed set.Then A^c is $\mathcal{M}$-g**open gives A^c is $\mathcal{M}$-open and A is $\mathcal{M}$-closed.Thus every $\mathcal{M}$-g**closed set is $\mathcal{M}$-closed set.Hence X is $\mathcal{M}$-g**$T_{1/2}$.

3. Minimal $\mathcal{M}$-g**open sets and Maximal $\mathcal{M}$-g**closed sets

We now introduce Minimal $\mathcal{M}$-g**open sets and Maximal $\mathcal{M}$-g**closed sets in Interior minimal spaces as follows.

Definition1A proper nonempty $\mathcal{M}$-g**open subset U of X is said to be a Minimal $\mathcal{M}$-g**open set if any $\mathcal{M}$-g**open set contained in U is ϕ or U.

*Remark*Minimal$\mathcal{M}$-open set and Minimal $\mathcal{M}$-g**open set are independent to each other.

Theorem2
(i) Let U be a Minimal $\mathcal{M}$-g**open set and W be a$\mathcal{M}$-g**open set. Then U ∩ W = ϕ or U⊂W.
(ii) Let U and V be Minimal $\mathcal{M}$-g**open sets.

Then U ∩ V = ϕ or U = V.

Proof

(i) Let U be a Minimal ***M***-g**open set and W be a***M***-g**open set.If U ∩ W = ϕ, then there is nothing to prove. If U ∩ W ≠ ϕ. Then U ∩ W ⊂ U. Since U is a Minimal ***M***-g**open set, we have U ∩ W = U. Therefore U ⊂ W.

(ii) Let U and V be Minimal **M**-g**open sets. If U ∩ V ≠ ϕ, then U⊂V and V⊂U by (i). Therefore U = V.

Theorem3 Let U be a Minimal ***M***-g**open set. If x ∈ U, then U ⊂ W for some ***M***-g**open set W containing x.

ProofLet U be a Minimal ***M***-g**open set and x∈U.Let W be any other ***M***-g**open set. U ∩ W =ϕ or U⊂W. If U∩W ≠ ϕ⇒x ∈ U ⇒ x ∈ W. Since W is arbitrary, there exists a***M***-g**open set W containing x such that U ⊂ W.

Theorem4 Let U be a Minimal ***M***-g**open set.Then U = ∩ {WW is a **M**-g**open set of X containing x} for any element x of U.

ProofBy Theorem3, and U is a Minimal ***M***-g**open set containing x, then U ⊂ W for some ***M***-g**open set W containing x.We have U ⊂ ∩ {WW is a ***M***-g**open set of X containing x} ⊂ U. Thus U = ∩ {WW is a ***M***-g**open set of X containing x} ⊂ U.

Theorem5 Let V be a nonempty finite ***M***-g**open set. Then there exists at least one (finite) Minimal ***M***-g**open set U such that U ⊂ V.

ProofLet V be a nonempty finite ***M***-g**open set. If V is a Minimal ***M***-g**open set, we may set U = V. If V is not a Minimal ***M***-g**open set, then there exists (finite) ***M***-g**open set V_1 such that ϕ ≠ V_1⊂V. If V_1 is a Minimal ***M***-g**open set, we may set U = V_1. If V_1 is not a Minimal ***M***-g**open set, then there exists (finite)***M***-g**open set V_2 such that ϕ ≠ V_2 ⊂ V_1. Continuing this process, we have a sequence of ***M***-g**open sets V⊃ V_1⊃ V_2⊃ V_3⊃ .....⊃V_k⊃ ......Since V is a finite set, this process repeats only finitely. Then finally we get a Minimal ***M***-g**open set U = V_n for some positive integer n. Hence there exists atleast one Minimal ***M***-g**open set U such that U⊂V.

[An Interior minimal space X is said to be ***M***-locally finite space if each of its elements is contained in a finite ***M***-g**open set.]

Corollary6 Let X be a***M***-locally finite space and V be a nonempty ***M***-g**open set. Then there exists at least one (finite) Minimal ***M***-g**open set U such that U ⊂ V.

ProofLet X be a ***M***-locally finite space and V be a nonempty ***M***-g**open set.Let x ∈ V.Since X is ***M***-locally finite space, we have a finite ***M***-open set V_x such that x ∈V_x.Then V∩V_x is a finite ***M***-g**open set. By Theorem5, there exists at least one (finite) Minimal ***M***-g**open set U such that U⊂V∩Vx.That is U⊂V∩V_x⊂V. Hence there exists at least one (finite) Minimal ***M***-g**open set U such that U⊂V.

Corollary7 Let V be a finite Minimal ***M***-open set. Then there exists at least one (finite) Minimal ***M***-g**open set U such that U⊂V.

ProofLet V be a finite Minimal ***M***-open set. Then V is a nonempty finite ***M***-g**open set. By Theorem5, there exists at least one (finite) Minimal ***M***-g**open set U such that U ⊂ V.

Theorem8 Let U; U_λ be Minimal ***M***-*g**open* sets for any element λ∈Γ. If U⊂$\cup_{\lambda\in\Gamma}U_\lambda$, then there exists an element λ ∈Γ such that U = U_λ.

ProofLet U⊂$\cup_{\lambda\in\Gamma}U_\lambda$. Then U ∩($\cup_{\lambda\in\Gamma}U_\lambda$) = U. That is $\cup_{\lambda\in\Gamma}$(U ∩ U_λ) = U.Also by Theorem2 (ii), U ∩ U_λ = ϕ or U = U_λ for any λ∈Γ.Then there exists an element λ∈Γ such that U = U_λ.

Theorem9 Let U; U_λ be Minimal ***M***-g**open sets for any λ∈Γ. If U = U_λ for any λ∈Γ, then ($\cup_{\lambda\in\Gamma}U_\lambda$) ∩ U = ϕ.

ProofSuppose that ($\cup_{\lambda\in\Gamma}U_\lambda$) ∩ U≠ ϕ for any λ∈Γ such that U ≠ U_λ. That is $\cup_{\lambda\in\Gamma}$(U_λ ∩ U) ≠ ϕ. Then there exists an element λ∈Γ such that U ∩ U_λ≠ ϕ.By Theorem2(ii), we have U = U_λ, which contradicts the fact that U ≠ U_λ for any λ∈Γ.Hence ($\cup_{\lambda\in\Gamma}U_\lambda$)∩U = ϕ.

We now introduce Maximal *M*-gclosed sets in Interior Minimal spaces as follows.**

Definition10A proper nonempty ***M***-g**closed set F⊂X is said to be Maximal ***M***-g**closed setif any ***M***-g**closed set containing F is either X or F.

*Remark*Every Maximal $\mathcal{M}$-closed sets are Maximal $\mathcal{M}$-g**closed sets.

Theorem11A proper nonempty subset F of X is Maximal $\mathcal{M}$-g**closed set if and only if X\F is a Minimal $\mathcal{M}$-g**open set.

ProofLet F be a Maximal $\mathcal{M}$-g**closed set. Suppose X\F is not a Minimal $\mathcal{M}$-g**open set. Then there exists a$\mathcal{M}$-g**open set U ≠ X\F such that $\phi \neq U \subset$ X\F. That is $F \subset$ X\U and X\U is a$\mathcal{M}$-g**closed set gives a contradiction to F is a Maximal $\mathcal{M}$-g**closed set.So X\F is a Minimal $\mathcal{M}$-g**open set.

Conversely, Let X\F be a Minimal $\mathcal{M}$-g**open set. Suppose F is not a Maximal $\mathcal{M}$-g**closed set. Then there exists a$\mathcal{M}$-g**closed set E ≠ F such that $F \subset E \neq X$. That is $\phi \neq$ X\E ⊂ X\F and X\E is a$\mathcal{M}$-g**open set gives a contradiction to X\F is a Minimal $\mathcal{M}$-g**open set. Thus F is a Maximal $\mathcal{M}$-g**closed set.

Theorem12

(i) Let F be a Maximal $\mathcal{M}$-g**closed set and W be a$\mathcal{M}$-g**closed set. Then $F \cup W = X$ or $W \subset F$.

(ii) Let F and S be Maximal $\mathcal{M}$-g**closed sets. Then $F \cup S = X$ or $F = S$.

Proof(i)Let F be a Maximal $\mathcal{M}$-g**closed set and W be a $\mathcal{M}$-g**closed set. If $F \cup W = X$, then there is nothing to prove.Suppose $F \cup W \neq X$. Then $F \subset F \cup W$.Therefore $F \cup W = F$ implies $W \subset F$.

(ii) Let F and S be Maximal **M**-g**closed sets.If $F \cup S \neq X$, then we have $F \subset S$ and $S \subset F$ by (i).Therefore F = S.

Theorem13 Let F_α, F_β, F_δ be Maximal $\mathcal{M}$-g**closed sets such that $F_\alpha \neq F_\beta$. If $F_\alpha \cap F_\beta F_\delta$, then either $F_\alpha = F_\delta$ or $F_\beta = F_\delta$

ProofGiven that $F_\alpha \cap F_\beta \subset F_\delta$. If $F_\alpha = F_\delta$ then there is nothing to prove. If $F_\alpha \neq F_\delta$ then we have to prove $F_\beta = F_\delta$.Now $F_\beta \cap F_\delta = F_\beta \cap (F_\delta \cap X)$

$= F_\beta \cap (F_\delta \cap (F_\alpha \cup F_\beta))$(by Theorem12(ii))

$= F_\beta \cap ((F_\delta \cap F_\alpha) \cup (F_\delta \cap F_\beta))$

$= (F_\beta \cap F_\delta \cap F_\alpha) \cup (F_\beta \cap F_\delta \cap F_\beta)$

$= (F_\alpha \cap F_\beta) \cup (F_\delta \cap F_\beta)$ (by $F_\alpha \cap F_\beta F_\delta$)

$= (F_\alpha \cup F_\delta) \cap F_\beta$

$= X \cap F_\beta$

(Since F_α and F_δ are Maximal **M**-g**closed sets by Theorem12(ii), $F_\alpha \cup F_\delta = X$)= F_β.

That is$F_\beta \cap F_\delta = F_\beta$ implies $F_\beta \subset F_\delta$.

Since F_β and F_δ are Maximal **M**-g**closed sets, we have $F_\beta = F_\delta$

Therefore $F_\beta = F_\delta$

Theorem14 Let F_α, F_β and F_δ be different Maximal $\mathcal{M}$-g**closed sets to each other. Then $(F_\alpha \cap F_\beta) \not\subset (F_\alpha \cap F_\delta)$.

ProofLet $(F_\alpha \cap F_\beta) \subset (F_\alpha \cap F_\delta) \Rightarrow (F_\alpha \cap F_\beta) \cup (F_\delta \cap F_\beta) \subset (F_\alpha \cap F_\delta) \cup (F_\delta \cap F_\beta)$gives$(F_\alpha \cup F_\delta) \cap F_\beta \subset F_\delta \cap (F_\alpha \cup F_\beta)$.Since by Theorem12(ii), $F_\alpha \cup F_\delta = X$ and $F_\alpha \cup F_\beta = X$ gives$X \cap F_\beta \subset F_\delta \cap X$.That is $F_\beta \subset F_\delta$. From the definition of Maximal $\mathcal{M}$-g**closed set, we get $F_\beta = F_\delta$ which gives a contradiction to the fact that F_α, F_β and F_δ are different to each other. Thus$(F_\alpha \cap F_\beta) \not\subset (F_\alpha \cap F_\delta)$.

Theorem15 Let F be a Maximal $\mathcal{M}$-g**closed set and x be an element of F. Then F = ∪{ SS is a $\mathcal{M}$-g**closed set containing x such that $F \cup S \neq X$}.

ProofBy Theorem13, and fact that F is a $\mathcal{M}$*-g**closed* set containing x, we have F ⊂∪ {SS is a $\mathcal{M}$*-g**closed* set containing x such that $F \cup S \neq X$} ⊂ F.So we have F = ∪ {SS is a $\mathcal{M}$-g**closed set containing x such that $F \cup S \neq X$}.

Theorem16 Let F be a Maximal $\mathcal{M}$-g**closed set. If x is an element of X\F. Then X\F ⊂ E for any $\mathcal{M}$-g**closed set E containing x.

ProofLet F be a Maximal $\mathcal{M}$-g**closed set and x is in X\F. $E \not\subset F$ for any $\mathcal{M}$-g**closed set E containing x.Then $E \cup F = X$ (by Theorem12 (ii)). Thus X\F⊂ E.

Minimal $\mathcal{M}$-gclosed set and Maximal $\mathcal{M}$-g**open set**

We now introduce Minimal $\mathcal{M}$-g**closed sets and Maximal $\mathcal{M}$-g**open sets in Interior Minimal spaces as follows.

Definition17A proper nonempty $\mathcal{M}$-g**closed subset F of X is said to be a Minimal $\mathcal{M}$-g**closed set if any $\mathcal{M}$-g**closed set contained in F is ϕ or F.

*Remark*Minimal$\mathcal{M}$-closed and Minimal $\mathcal{M}$-g**closed set are independent to each other.

Definition18A proper nonempty $\mathcal{M}$-g**open U⊂X is said to be a Maximal $\mathcal{M}$-g**open setif any $\mathcal{M}$-g**open set containing U is either X or U.
*Remark*Maximal$\mathcal{M}$-open set and Maximal $\mathcal{M}$-g**open set are independent to each other.

Theorem19A proper nonempty subset U of X is Maximal $\mathcal{M}$-g**open set if and only ifX\U is a Minimal $\mathcal{M}$-g**closed set.
ProofLet U be a Maximal $\mathcal{M}$-g**open set. Suppose X\U is not a Minimal $\mathcal{M}$-g**closed set. Then there exists $\mathcal{M}$-g**closed set V ≠ X\U such that ϕ ≠ V ⊂ X\U. That is U⊂ X\V and X\V is a$\mathcal{M}$-g**open set gives a contradiction to U is a Minimal $\mathcal{M}$-g**closed set.
Conversely, Let X\U be a Minimal $\mathcal{M}$-g**closed set.Suppose U is not a Maximal $\mathcal{M}$-g**open set. Then there exists $\mathcal{M}$-g**open set E ≠ U such that U⊂ E ≠ X. That is ϕ ≠ X\E ⊂ X\U and X\E is a$\mathcal{M}$-g**closed set which is a contradiction for X\U is a Minimal $\mathcal{M}$-g**closed set.Therefore U is a Maximal $\mathcal{M}$-g**closed set.

Theorem20

(i) Let U be a Minimal **M-**g**closed set and W be a**M-**g**closed set. Then U ∩ W = ϕ or U ⊂ W.
(ii) Let U and V be Minimal **M-**g**closed sets.
Then U ∩ V = ϕ or U = V.
Proof
(i) Let U be a Minimal $\mathcal{M}$-g**closed set and W be a$\mathcal{M}$-g**closed set. If U ∩ W =ϕ, then there is nothing to prove.If U ∩ W ≠ϕ. Then U ∩ W ⊂ U. Since U is a Minimal $\mathcal{M}$-g**closed set, we have U ∩ W = U.Therefore U ⊂ W.
(ii) Let U and V be Minimal $\mathcal{M}$-g**closed sets. If U∩V ≠ ϕ, then U⊂V and V⊂U by (i). Therefore U = V.

Theorem21 Let V be a nonempty finite $\mathcal{M}$-g**closed set. Then there exists at least one (finite) Minimal $\mathcal{M}$-g**closed set U such that U⊂V.
ProofLet V be a nonempty finite $\mathcal{M}$-g**closed set. If V is a minimal $\mathcal{M}$-g**closed set, we may set U = V.If V is not a Minimal $\mathcal{M}$-g**closed set, then there exists (finite) $\mathcal{M}$-g**closed set V_1 such that ϕ ≠ V_1⊂V. If V_1 is a Minimal $\mathcal{M}$-g**closed set, we may set U = V_1.If V_1 is not a minimal $\mathcal{M}$-g**closed set, then there exists (finite)$\mathcal{M}$-g**closed set V_2 such that ϕ ≠ V_2⊂ V_1.

Continuing this process, we have a sequence of $\mathcal{M}$-g**closed sets $V \supset V_1 \supset V_2 \supset V_3 \supset \ldots\ldots \supset V_k \supset$Since V is a finite set, this process repeats only finitely. Then finally we get a Minimal $\mathcal{M}$-g**closed set U = V_n for some positive integer n.

Corollary22 Let X be a $\mathcal{M}$-locally finite space and V be a nonempty $\mathcal{M}$-g**closed set. Then there exists at least one (finite) Minimal $\mathcal{M}$-g**closed set U such that U⊂ V.
ProofLet X be a $\mathcal{M}$-locally finite space and V be a nonempty $\mathcal{M}$-g**closed set. Let x ∈ V.Since X is $\mathcal{M}$-locally finite space, we have a finite $\mathcal{M}$-open set V_x such that x ∈V_x. Then V∩V_x is a finite $\mathcal{M}$-g**closed set. By Theorem21, there exists at least one (finite) Minimal $\mathcal{M}$-g**closed set U such that U⊂ V∩V_x. That is U⊂ V∩V_x⊂V. Hence there exists at least one (finite) Minimal $\mathcal{M}$-g**closed set U such that U⊂V.

Corollary23 Let V be a finite Minimal $\mathcal{M}$-open set. Then there exists at least one (finite) Minimal $\mathcal{M}$-g**closed set U such that U⊂V.
ProofLet V be a finite Minimal $\mathcal{M}$-open set. Then V is a nonempty finite $\mathcal{M}$-g**closed set. By Theorem21,there exists at least one (finite) Minimal $\mathcal{M}$-g**closed set U such that U⊂V.

Theorem24 Let U; U_λ be Minimal $\mathcal{M}$-g**closed sets for any element λ∈Γ. If $U \subset \cup_{\lambda\in\Gamma} U_\lambda$, then there exists an element λ ∈Γ such that U = U_λ.
ProofLet $U \subset \cup_{\lambda\in\Gamma} U_\lambda$. Then $U \cap (\cup_{\lambda\in\Gamma} U_\lambda) = U$. That is $\cup_{\lambda\in\Gamma}(U \cap U_\lambda) = U$. Also by Theorem20(ii), $U \cap U_\lambda = \phi$ or $U = U_\lambda$ for any λ∈Γ. This implies that there exists an element λ∈Γ such that U = U_λ.

Theorem25 Let U; U_λ be Minimal $\mathcal{M}$-g**closed sets for any λ∈Γ. If U = U_λ for any λ∈Γ, then $(\cup_{\lambda\in\Gamma} U_\lambda) \cap U = \phi$.
ProofSuppose that $(\cup_{\lambda\in\Gamma} U_\lambda) \cap U \neq \phi$. That is $\cup_{\lambda\in\Gamma}(U_\lambda \cap U) \neq \phi$. Then there exists an element λ∈Γ such that $U \cap U_\lambda \neq \phi$. By Theorem20(ii), we have U = U_λ, which contradicts the fact that U ≠ U_λ for any λ∈Γ. Hence $(\cup_{\lambda\in\Gamma} U_\lambda) \cap U = \phi$.

Theorem26A proper nonempty subset F of X is Maximal $\mathcal{M}$-g**open set if and only ifX\F is a Minimal $\mathcal{M}$-g**closed set.

ProofLet F be a Maximal $\mathcal{M}$-g**open set. Suppose X\F is not a Minimal $\mathcal{M}$-g**open set. Then there exists $\mathcal{M}$-g**open set U $\neq$ X\F such that $\phi \neq U \subset X\backslash F$. That is F$\subset$ X\U and X\U is a$\mathcal{M}$-g**open set which is a contradiction for F is a Minimal $\mathcal{M}$-g**closed set.

Conversely, Let X\F be a Minimal $\mathcal{M}$-g**closed set. Suppose F is not a Maximal $\mathcal{M}$-g**open set. Then there exists $\mathcal{M}$-g**open set E $\neq$ F such that F$\subset$ E $\neq$ X. That is $\phi \neq X\backslash E \subset X\backslash F$ and X\E is a$\mathcal{M}$-g**open set which is a contradiction for X\F is a Minimal $\mathcal{M}$-g**closed set.
Thus F is a Maximal $\mathcal{M}$-g**open set.

Theorem27

(i) Let F be a Maximal $\mathcal{M}$-g**open set and W be a$\mathcal{M}$-g**open set. Then $F\cup W = X$ or $W\subset F$.

(ii) Let F and S be Maximal $\mathcal{M}$-g**open sets.

Then $F\cup S=X$ or F = S.

Proof

(i) Let F be a Maximal $\mathcal{M}$-g**open set and W be a$\mathcal{M}$-g**open set.If $F\cup W = X$, then there is nothing to prove. Suppose $F\cup W \neq X$. Then $F\subset F\cup W$.Therefore $F\cup W = F$ gives$W\subset F$.

(ii) Let F and S be Maximal $\mathcal{M}$-g**open sets.If $F\cup S \neq X$, then we have $F\subset S$ and $S\subset F$ by (i). Therefore F = S.

Theorem28 Let F_α, F_β, F_δ be Maximal $\mathcal{M}$-g**open sets such that $F_\alpha \neq F_\beta$. If $F_\alpha\cap F_\beta\subset F_\delta$, then either $F_\alpha = F_\delta$ or $F_\beta = F_\delta$

ProofGiven that $F_\alpha\cap F_\beta\subset F_\delta$.
If $F_\alpha = F_\delta$ then there is nothing to prove. If $F_\alpha \neq F_\delta$ then we have to prove $F_\beta = F_\delta$.
Now $F_\beta\cap F_\delta = F_\beta\cap (F_\delta\cap X) = F_\beta\cap (F_\delta\cap (F_\alpha\cup F_\beta)$(by Theorem27(ii))
$= F_\beta\cap ((F_\delta\cap F_\alpha) \cup (F_\delta\cap F_\beta)) = (F_\beta\cap F_\delta\cap F_\alpha) \cup (F_\beta\cap F_\delta\cap F_\beta) = (F_\alpha\cap F_\beta) \cup (F_\delta\cap F_\beta)$ (by $F_\alpha\cap F_\beta\subset F_\delta$) $= (F_\alpha\cup F_\delta) \cap F_\beta = X \cap F_\beta$. (Since F_α and F_δ are Maximal $\mathcal{M}$-g**open sets by Theorem27 (ii), $F_\alpha\cup F_\delta = X$)$= F_\beta$.That is $F_\beta\cap F_\delta= F_\beta$.Thus $F_\beta\subset F_\delta$. Since F_β and F_δ are Maximal $\mathcal{M}$-g**open sets, we have $F_\beta = F_\delta$. Therefore $F_\beta = F_\delta$.

Theorem29 Let F_α, F_β and $F_\delta$be different Maximal $\mathcal{M}$-g**open sets to each other. Then $(F_\alpha\cap F_\beta) \not\subset (F_\alpha\cap F_\delta)$.

ProofLet $(F_\alpha\cap F_\beta)\subset (F_\alpha\cap F_\delta) \Rightarrow (F_\alpha\cap F_\beta) \cup (F_\delta\cap F_\beta)\subset (F_\alpha\cap F_\delta) \cup (F_\delta\cap F_\beta) \Rightarrow (F_\alpha\cup F_\delta) \cap F_\beta\subset F_\delta\cap (F_\alpha\cup F_\beta)$.(Since by Theorem27(ii), $F_\alpha\cup F_\delta = X$ and $F_\alpha\cup F_\beta = X \Rightarrow X \cap F_\beta\subset F_\delta\cap X \Rightarrow F_\beta\subset F_\delta$.
From the definition of Maximal $\mathcal{M}$-g**open set it follows that $F_\beta = F_\delta$, which is a contradiction to the fact that F_α, F_β and F_δ are different to each other. Therefore $(F_\alpha\cap F_\beta) \not\subset (F_\alpha\cap F_\delta)$.

Theorem30 Let F be a Maximal $\mathcal{M}$-g**open set. If x is an element of X \ F. Then X\F$\subset$ E for any$\mathcal{M}$-g**open set E containing x.

ProofLet F be a Maximal $\mathcal{M}$-g**open set and $x \in X\backslash F$. $E \not\subset F$ for any $\mathcal{M}$-g**open set E containing x. Then $E \cup F = X$ by Theorem27(ii).Therefore X\F$\subset$ E.

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