IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 5 Ver. II (Sep. - Oct. 2015), PP 29-36
www.iosrjournals.org

Graph Labelings in Konigsberg Bridge Problem

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Abstract:*Graph theory has its origin with the Konigsberg Bridge Problem. A graph labeling is a one to one function that carries a set of elements onto a set of integers called labels. This paper discusses various graph labelings that can be assigned and few other graph labelings that can not be assigned to the Konigsberg bridge problem.*
AMS MSC Classification: 05C78
Key words:*Konigsberg bridge problem; labeling;bijective function.*

I. Introduction

Graph theory is an interesting area for exploration of proof techniques in discrete mathematics which also has a wide range of applications in various areas like Computing Science, Social Science and Natural Science. Graph theory has its origin with the Konigsberg Bridge Problem which has seven bridges linked between two islands such that one cannot walk through each of the bridge exactly once and come back to the starting point. Though Leonard Euler has solved it in 1736, this was the motivation for the introduction of this paper relating with graph labelings.

A graph labeling is a one to one function that carries a set of elements onto a set of integers called labels. If the domain set is the vertex set or the edge set the labelings are called vertex labeling or edge labeling respectively.In this papervarious labelings are assigned to the Konigsberg bridge problem.

II. Preliminaries

Let G(V,E) be a graph with p vertices and q edges.The vertex set and edge set are denoted by V(G) and E(G) respectively.The number of vertices and edges are denoted by |V(G)|,|E(G)| respectively

Definition 2.1

A bijectivefunction f : V(G) {1,2,..,p} is called a prime labeling if for each edge e = uv E(G), gcd (f(u), f(v)) = 1.

Definition 2.2

An injective function f : V(G) {0,1,2,..,p} is called graceful labeling, when ,f(uv) {1,2,..,p}. A graph that admits a graceful labeling is said graceful graph.

Definition 2.3

An injective function f:V(G)→{0,1,..,q-1} is called Harmonious labeling if the induced function f*: E(G) {1, 2…..q} defined as f*(uv) = (f(u) +f(v))(mod q) is bijective. A Graph which admits harmonious labeling is called harmonious graph.

Definition 2.4

A function f :V(G)→{0,1} is called a cordial labeling if the induced function f^*:E(G)→{0,1} defined as f^*(uv) =|f(u)-f(v)| gives the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1, and also the number of edges labeled 0 and the number of edges labeled 1 differ at most by 1.

Definition 2.5

An injective function f :V(G) →{0,1,2,…q} is said to elegant labeling if each edge uv is assigned the label

f(uv)=f(u) + f(v) (mod (q + 1))

the resulting edge labels are distinct and nonzero.

Definition 2.6

An injective function f:V(G)→{0,1,2,..q} is called a mean labeling if
f(uv) =
then the resulting labels are distinct.

Definition 2.7
A function $f^*:E(G) \rightarrow \{1,2,..,q\}$ is called vertex prime labeling if $gcd(f^*(e_1),f^*(e_2),\ldots)=1$ where e_i's are edges.

Definition 2.8
A bijective function f:V(G)∪E(G){1,2,…,p+q}is called an edge magic total labeling if for all edges uv ,

$f(u) + f(v) + f(uv)$ is a constant.

Definition 2.9
A function f: V(G)∪E(G) →{1,2,..p+q} of integers is called a vertex magic total labeling if for every vertex v, , N(u) is the set of vertices adjacent to u, where k is a constant called magic constant or weight.

Definition 2.10
A function f: V(G)∪E(G) →{1,2,..p+q} of integers is called a vertex bimagic total labeling if for every vertex v,, where k_1 or k_2 is a constant called magic constant or weight .

Definition 2.11
A bijective function f : V(G)∪E(G) {1, 2,…,p+q} is called a vertex anti magic total labeling if the vertex-weights are different, thus wt() wt(v) for all v V(G),
wt() f() + ,where N(u) is the set of vertices adjacent to u.

Definition 2.12
A functionf: V(G)∪E(G)→{1,2..p+q=k} is called a vertex irregular total k-labeling if for every pair of distinct vertices u and v.

III. Konigsberg Bridge Problem [Kbp]

Two islands linked to each other with seven bridges. The problem is to start from any one of the four land areas, take a stroll across the seven bridges and get back to the starting point without crossing a bridge second time.This is called Konigsberg Bridge Problem and we abbreviate the problem as KBP.[3]

We represent the vertices and edges of the Konigsberg bridge problem as

$$V = \{v_1,v_2,v_3,v_4\} ;$$
$$E = \{e_1,e_2,e_3,e_4,e_5,e_6,e_7\}$$

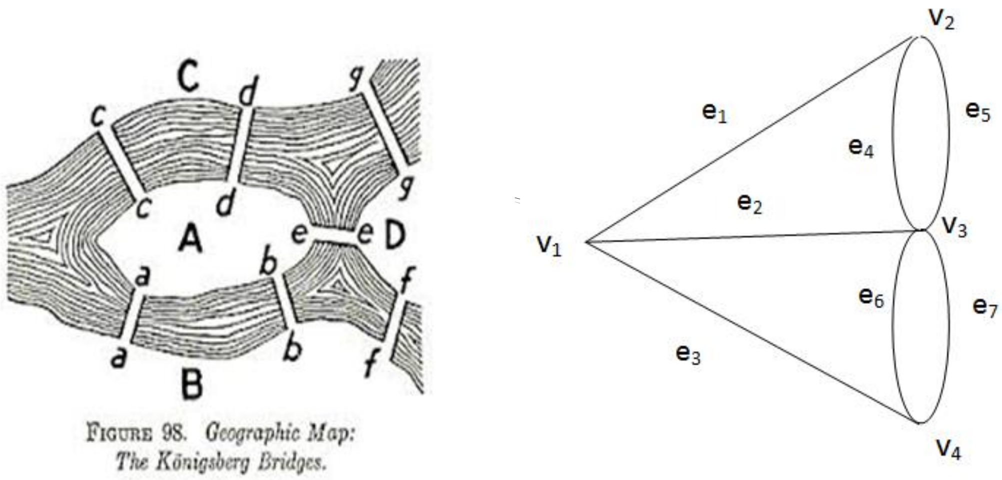

Fig :1 Konigsberg Bridge Problem G.[KBP]

Let v_i be the vertices in the graph where i=1,2,3,4. Let e_i be the edges in the graph where i=1 to 7.

Inthis graph the degree of all vertices is odd.

(ie.,) $d(v_i)$ = odd v_iG.

Let us represent the labels of v_i and e_i as $f(v_i)$ and $f(e_i)$ respectively

IV. Konigsberg Bridge Problem And Various Labelings

Though there exists a large variety of labelings only few labelings can be assigned to the Konigsberg Bridge Problem which are discussed in the below problem.

PROBLEM 1:

The following labelings exists for Konigsberg Bridge Problem.

(i) Prime Labeling.
(ii) Vertex Prime Labeling.
(iii) Vertex Magic Total Labeling.
(iv) Vertex Bimagic Total Labeling.
(v) Vertex Anti Magic Total Labeling.
(vi) Vertex Irregular Total k-Labeling.

Solution:

(i) The KBP has Prime Labeling.

Let us label the graph G as follows

$f(v_1) = 1$, $f(v_2) = 2$, $f(v_3) = 3$, $f(v_4) = 4$.

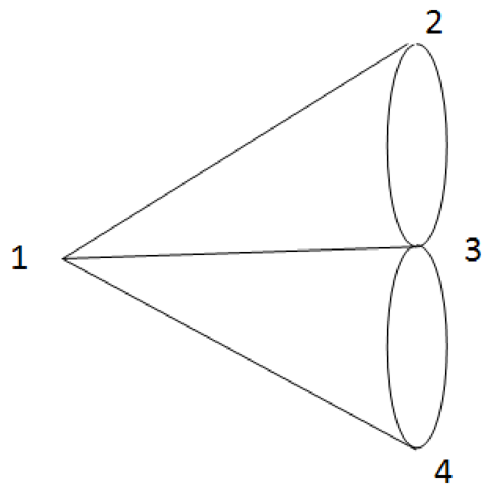

Fig.2 Prime Labeling of KBP

Now we take greatest common divisor between the each pair of the vertices as follows

$gcd(f(v_1),f(v_2)) = gcd(1,2) = 1$
$gcd(f(v_1),f(v_3)) = gcd(1,3) = 1$
$gcd(f(v_1),f(v_4)) = gcd(1,4) = 1$
$gcd(f(v_2),f(v_3)) = gcd(2,3) = 1$
$gcd(f(v_3),f(v_4)) = gcd(3,4) = 1$

By definition of Prime labeling[9]

$gcd(f(u), f(v)) = 1$.

From the above the greatest common divisor between each pair of vertices is 1.
It satisfies the condition for prime labeling.
Hence the graph G has prime labeling.

(ii) The KBP has Vertex prime labeling.

Let us label edges of the graph G as follows

$f(e_1) = 4$, $f(e_2) = 5$, $f(e_3) = 6$, $f(e_4) = 2$,
$f(e_5) = 3$, $f(e_6) = 7$, $f(e_7) = 1$.

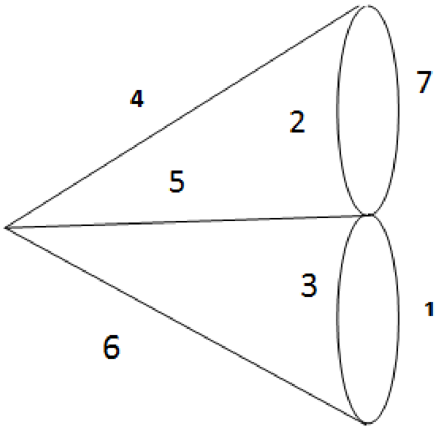

Fig.3Vertex Prime Labelingof KBP

Now we take greatest common divisor between the each labels on its incident edges of the vertices as follows

$$gcd(f^*(e_1), f^*(e_2), f^*(e_3))=gcd(f(v_1v_2),f(v_1v_3),f(v_1v_4)) = gcd(4,5,6) = 1.$$

Similarly,

gcd(4,2,7) = 1.
gcd(6,3,1) = 1.
gcd(7,2,5,3,1) = 1.

By definition of Vertex Prime Labeling
From the above the greatest common divisor is 1, it satisfies the condition for vertex prime labeling.
Hence the graph G has Vertex Prime Labeling.

(iii) The KBP has Vertex magic total labeling.

Let us label the graph G as follows

$f(v_1) = 11$, $f(v_2) = 10$, $f(v_3) = 2$, $f(v_4) = 9$
$f(e_1) = 4$, $f(e_2) = 3$, $f(e_3) = 7$, $f(e_4) = 5$,
$f(e_5) = 6$, $f(e_6) = 1$, $f(e_7) = 8$.

We define the function

$$f : V(G)\cup E(G)\{ 1,2,\ldots,p+q\}$$

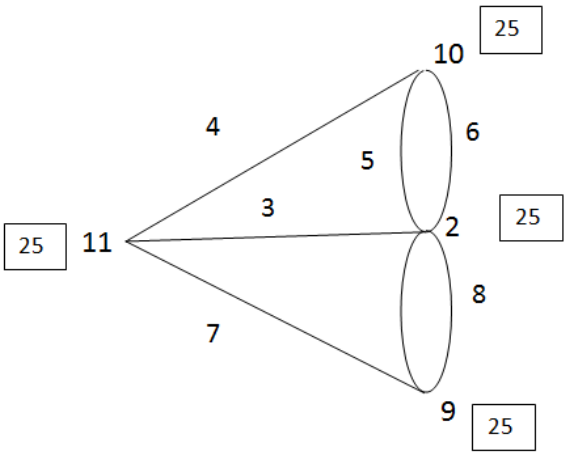

Fig .4Vertex Magic Total Labeling of KBP

Here $w_f(v_i)$ represent the vertex weight (where i =1 to 4) of the sum of values assigned to all edges incident to given vertex v_i together with the value assigned to v_i itself[5]

From fig.4 the weights of the vertices are

Here, $w_f(v_1) = (4+3+7)+(11) =25$

Similarly,

$w_f(v_2) = 25$
$w_f(v_3) = 25$
$w_f(v_4) = 25$

It follows that the vertex weights $w_f(v_i)$ are all distinct
Hence the graph G has Vertex Magic Total Labeling.

(iv) The KBP has Vertex bimagic total labeling.

Let us label the graph G as follows

$f(v_1) = 7$, $f(v_2) = 5$, $f(v_3) = 2$, $f(v_4) = 6$ and $f(e_1) = 10$, $f(e_2) = 1$, $f(e_3) = 11$, $f(e_4) = 8$, $f(e_5) = 9$, $f(e_6) = 4$, $f(e_7) = 3$.

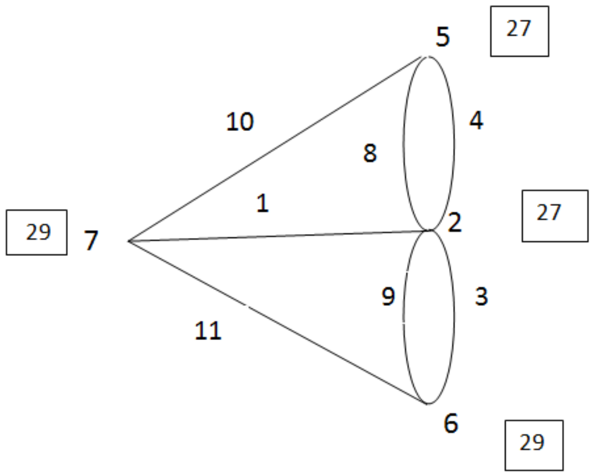

Fig.5Vertex Bimagic Total Labeling of KBP

We define the function

f : V(G)∪E(G){ 1,2,…,p+q}

Here $w_f(v_i)$ represent the vertex weight of the vertex v_i , v_iV

ie.,

where e_i's are edges incident with v_i

From fig.3

$w_f(v_1) = f(v_1)+f(v_1v_2)+ f(v_1v_3)+ f(v_1v_4)$
$w_f(v_1) = 7+10+1+11$
$w_f(v_1) = 29$

Similarly,

$w_f(v_2) = 27$
$w_f(v_3) = 27$
$w_f(v_4) = 29$

From the above two magic constants are seenk_1=29 and k_2=27by the definition of Vertex bimagic total labeling[6].

Hence KBP has Vertex bimagic total labeling.

(v) The KBP has Vertex anti magictotal labeling.

Let us label the graph G as follows

$f(v_1) = 7$, $f(v_2) = 6$, $f(v_3) = 4$, $f(v_4) = 9$
$f(e_1) = 11$, $f(e_2) = 2$, $f(e_3) = 5$, $f(e_4) = 8$,
$f(e_5) = 10$, $f(e_6) = 1$, $f(e_7) = 3$.

We define the function

f : V(G)∪E(G){ 1,2,…,p+q}

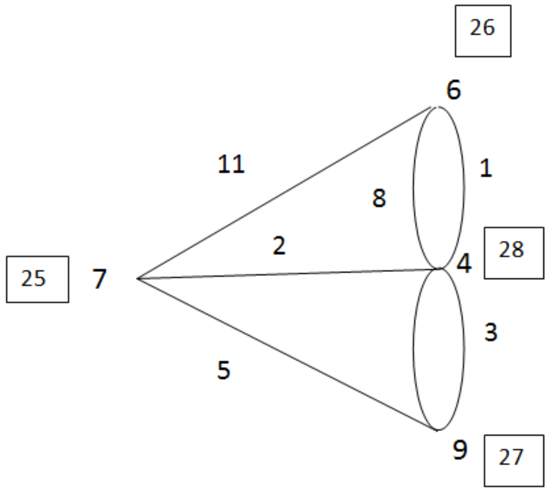

Fig. 6 Vertex Anti Magic Total Labeling of KBP

Here $w_f(v_i)$ represent the vertex weight (where i =1 to 4) of the sum of values assigned to all edges incident to given vertex v_i together with the value assigned to v_i itself[8].
From fig.6the weights of the vertices are where e_i's are edges incident with v_i
Here, $w_f(v_1) = (11+2+5)+(7) = 25$

Similarly,

$w_f(v_2) = 26$
$w_f(v_3) = 28$
$w_f(v_4) = 27$

It follows that the vertex weights $w_f(v_i)$ are all distinct
Hence the graph G has Vertex anti magic total labeling.

(iv) The KBP has Vertex Irregular Total k- Labeling.
Let us label the graph G as follows
$f(v_1) = 11$, $f(v_2) = 8$, $f(v_3) = 9$, $f(v_4) = 10$ and $f(e_1) = 7$, $f(e_2) = 3$, $f(e_3) = 6$, $f(e_4) = 2$, $f(e_5) = 4$, $f(e_6) = 1$, $f(e_7) = 5$.

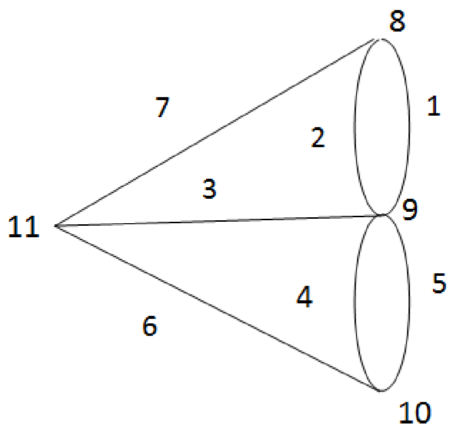

Fig.7Vertex Irregular Total k- Labeling of KBP

We define the function

$f : V(G)\cup E(G) \{ 1,2,\ldots,p+q\}$

By the definition of Vertex Irregular Total k- Labeling[1]

Here,

$11+7+3+6 \neq 8+7+2+1$
$37 \neq 18$

Similarly,

It satisfies the condition for Vertex Irregular Total k- Labeling. Hence the graph G has Vertex Irregular Total k- Labeling.

Now the labelings that cannot be assigned to the Konigsberg Bridge Problem and the reasons are discussed in the below problem.

PROBLEM 2:

The labelings does not exist for KBP.

(i) Graceful Labeling
(ii) Harmonious Labeling
(iii) Edge Magic Total Labeling
(iv) Cordial Labeling.
(v) Elegant Labeling.
(vi)Mean Labeling.

Solution:

(i) Let G be a KBP. Let us label the graph G as follows

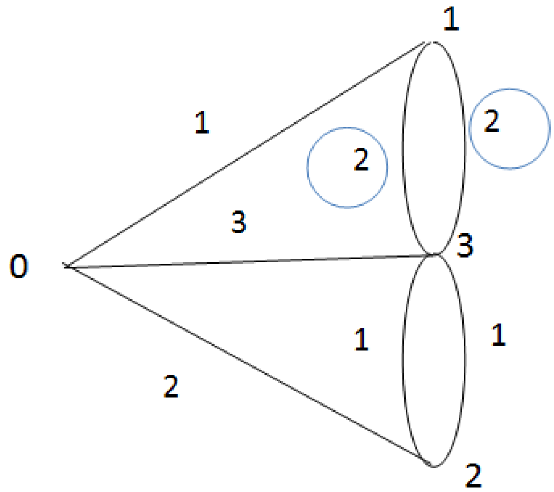

Fig.8 Graceful Labeling

We define the bijective function

f : V(G) {0,1,2,…,q}

The edge labeling depends on the vertex labeling for all the these labelings. In the above graph there exsists parallel edges between two pair of vertices.

So a bijective function cannot exits.

ie., The distinct values does not occur for each edge

So Konigsberg Bridge Problem does not have (i)Graceful Labeling

In Harmonious Labeling the vertex labeling is induced on the edge labeling as bijection. So the distinct values does not occur for each edge.

In Edge Magic Total Labeling ,Cordial Labeling, Elegant Labeling and Mean Labeling the vertex labeling is induced on the edge labeling which also requires simple graph without parallel edges. So here again the distinct values do not occur for each edge.

So Konigsberg Bridge Problem does not have (ii) Harmonious Labeling (iii) Edge Magic Total Labeling (iv) Cordial Labeling (v) Elegant Labelingand (vi)Mean Labeling.

Acknowledgement:

The Authors would wish to thank UGC for having funded for the project MRP-6162/15(SERO/UGC)

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