IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 6 Ver. 1 (Nov. - Dec. 2015), PP 91-98
www.iosrjournals.org

Melting Heat Transfer in MHD Boundary Layer Stagnation-Point Flow towards a Stretching Sheet

M.Subhas Abel*, Jayashree Sanamani
Dept of Mathematics Gulbarga University,Gulbarga 585106,Karnataka,India

Abstract: *An analysis is carried out to study the MHD steady two-dimensional stagnation-point flow and heat transfer from a warm, laminar liquid flow to a melting stretching sheet. The governing partial differential equations are transformed into ordinary differential equations by similarity transformation, before being solved numerically using Runge-Kutta-Fehlberg method. Results for the skin friction coefficient, local Nusselt number, velocity profiles as well as temperature profiles are presented for different values of the governing parameters. Effects of the Magnetic parameter, melting parameter, stretching parameter and prandtl number on the flow and heat transfer characteristics are thoroughly examined.*
Key Words: *Laminar boundary layer; MHD; Runge-Kutta-Fehlberg method;Stagnation point;*
Prandtl Number; Nusselt number.

Nomenclature
a Parameter of the temperature distributed in stretching surface.
B_0 Externally imposed magnetic field in the y-direction.
C_b Drag coefficient.
C_f Local skin-friction coefficient.
c_p Specific heat of the fluid at constant pressure.
f Dimensionless stream function.
M_n Magnetic Parameter
L Characteristic length of the flow dynamics.
Nu_x Nusselt number.
Pr Prandtl number.
T Fluid temperature.
T_∞ Ambient temperature.
T_0 Reference number.
T_w Wall temperature.
u fluid axial velocity.
U_w Velocity of horizontal stretching surface.
U_e Velocity of external flow.
v fluid transverse velocity.
x, y Coordinates along and normal to the vertical stretching surface plate.
X Dimensionless coordinate along the plate.

Greek symbol
υ Kinematic viscosity.
τ_{wx} Local shear stress.
α Thermal diffusivity.
η Non-dimensional transformed variable.
μ Viscosity of the fluid.
σ Fluid electrical conductivity.
ψ Stream function.
ρ Fluid density.
θ Dimensionless temperature.

Subscripts
x local.
w Conditions on the wall.
o Reference.
∞ Ambient or free stream condition.
Superscript' Differentiation with respect to η .

I. Introduction

The study of laminar boundary layer flow and heat transfer over a stretching surface is important and has attracted considerable interest of many researchers because of its large number of applications in industry and technology. Few of these applications are materials manufactured by polymer extrusion, drawing of copper wires, continuous stretching of plastic films, artificial fibers, hot rolling, wire drawing, glass fiber, metal extrusion and metal spinning, cooling of metallic sheets or electronic chips, and many others. In these cases, the final product of desired characteristics depends on the rate of cooling in the process and the rate of stretching. After the pioneering work by Sakiadis [1], a large amount of literature is available on boundary layer flow of Newtonian and non-Newtonian fluids over linear and nonlinear stretching surfaces [2–10].

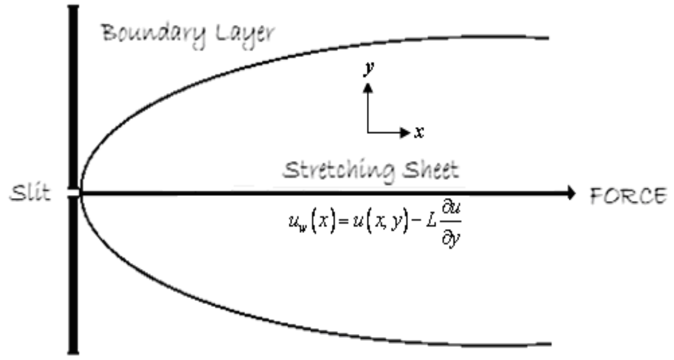
Stagnation point flow, describing the fluid motion near the stagnation region of a circular body, exists for both the cases of a fixed or moving body in a fluid. The two dimensional stagnation flow towards a stationary semi-infinite wall was first studied by Hiemenz**[**11], using a similarity transformation, to reduce the Navier-stokes equations to a set of nonlinear ordinary differential equations. This problem was then extended by Homann [12] to the case of axisymmetric stagnation-point flow.

Melting heat transfer in steady laminar flow over a stationary flat plate has been considered by Epstein and Cho [13], and very recently this problem was extended by Ishak et al. [14] to a melting surface moving parallel to constant stream. Chen et al. [15] studied free convection melting of a solid immersed in a hot dissimilar fluid, while Kazmierczak et al. [16,17], Bakier[18], Gorla et al. [19] and Cheng and Lin[20, 21] studied the effects of mixed convection boundary layer flow about a vertical surface on the phenomenon of melting process in a fluid-saturated porous medium. Recently melting heat transfer in boundary layer stagnation-point flow towards a stretching sheet has investigated by Bachok et al.[27]. The numerical results obtained are then compared with those reported by Wang [20] for M = 0 (melting is absent). More recently a paper by Bachok et al[28]. Investigated the Boundary layer stagnation-point flow and heat transfer over an exponentially stretching sheet in a nano fluid.

The aim of the present paper is to study the similarity solutions of stagnation –point flow and the heat transfer from a warm, laminar liquid flow to a melting stretching sheet with magnetic parameter. The governing partial differential equations are first transformed into a system of ordinary differential equations before being solved numerically. A through literature show that there is not any published paper on melting heat transfer in boundary layer stagnation point flow towards a stretching sheet with magnetic field effect. We believe that the obtained results are new and original. It should be mentioned that in addition to its importance from a fundamental standpoint, the present study finds important application in magma solidification, the melting of permafrost, the thawing of frozen grounds, and the preparation of semi-conductor materials. In addition, in solidifying, casting the existence of a mushy zone separating the liquid phase from the solid phase has been observed. This zone consists of a fine mesh of dendrites growing into the melt region. The heat and fluid flow processes inside the mushy zone play an important role in the solidification phenomenon (Bakier [18]).

II. Problem formulation

Fig.1. Schematic diagram of stretching sheet



Consider a steady stagnation-point flow towards a horizontal linearly stretching sheet with porous parameter melting at a steady rate into a constant property, warm liquid of the same material, as shown in the Fig.1. It is assumed that the velocity of the external flow is $u_e(x)=ax$ and the velocity of the stretching sheet is $u_w(x)=cx$, where a is a positive constant, while c is a positive (stretching rate) constant and x is the coordinate measure along the stretching sheet. It is also assumed that the temperature of melting surface is T_m, while the temperature in the free-stream condition is T_∞, where $T_\infty > T_m$. It is also assumed that the viscous dissipation and the heat generation or absorption is neglected. Under the usual boundary layer approximations, the equations of motion with magnetic parameter and the equation representing temperature distribution in the liquid flow must obey the following equations,

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}+\frac{\sigma B_0^2\left(u_e-u\right)}{\rho}, \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \qquad (3)$$

Where y is the Cartesian coordinates measured normal to the stretching surface, u and v are the velocity components along the x- and y-axis, ν and α are the kinematic viscosity and thermal diffusivity of the fluid, respectively. Following Hiemenz[11]and Epstein and Cho[14], we assume that the boundary conditions of Eqs.(1)-(3) are the following

$$\begin{aligned} &u=u_w(x), \qquad T=T_m \quad at \quad y=0 \\ &u\to u_e(x), \qquad T\to T_\infty \quad as \quad y\to\infty \end{aligned} \qquad (4a)$$

and

$$k\left(\frac{\partial T}{\partial y}\right)_{y=0}=\rho[\lambda+c_s(T_m-T_0)]v(x,0) \qquad (4b)$$

Where ρ is the fluid density k is the thermal conductivity, λ is the latent heat of the fluid c_s is the heat capacity of the solid surface. Eq. (4b) states that the heat conducted to the melting surface is equal to the heat of melting plus the sensible heat required to raise the solid temperature T_0 to its melting temperature T_m (Epstein and Cho[14] or Cheng and Lin [22]).

Following the classical work of Hiemenz [11], we introduce now a similarity transformation to recast the governing partial differential equations into a set of ordinary differential equations:

$$\psi=(a\nu)^{1/2}xf(\eta), \qquad \theta(\eta)=\frac{T-T_m}{T_\infty-T_m}, \qquad \eta=\left(a/\nu\right)^{1/2}y, \qquad (5)$$

Where ψ, the stream function, which is defined in the usual form as $u=\frac{\partial\psi}{\partial y}$ *and* $v=-\frac{\partial\psi}{\partial x}$, which automatically satisfies the continuity equation (1).

By using this definition, we obtain

$$u=axf'(\eta), \qquad v=-(a\nu)^{1/2}f(\eta), \qquad (6)$$

Substituting Eqs.(5) and (6) into Eqs. (2)And (3) produces, the following nonlinear ordinary differential equations:

$$f'''+ff''-f'^2+M_n(1-f')+\varepsilon^2=0, \qquad (7)$$

$$\theta''+Prf\theta'=0, \qquad (8)$$

Where primes denote differentiation with respect to η and Pr is the Prandtl number and $M_n=\frac{\sigma B_0^2}{\rho a}$ is the Magnetic parameter.

The boundary conditions (4) become

$$\begin{aligned} &f'(0)=0, \quad \theta(0)=1, \quad Prf(0)+M\theta'(0)=0, \\ &f'(\infty)=\varepsilon, \quad \theta(\infty)=0, \end{aligned} \qquad (9)$$

Where $\varepsilon=c/a$ is the velocity ratio ($\varepsilon>0$) parameter and M is the dimensionless melting parameter which is defined as

$$M=\frac{c_p(T_\infty-T_m)}{\lambda+c_s(T_m-T_0)}, \qquad (10)$$

Where c_p is the heat capacity of the fluid at constant pressure, and λ is the latent heat of the fluid and the melting parameter M is a combination of the Stefan number $c_f(T_\infty-T_m)/\lambda$ *and* $c_s(T_m-T_0)/\lambda$ for the liquid and solid phases, respectively. It is worth mentioning that for M=0 (melting is absent), Eq. (7) reduces to the classical equation first derived by Hiemenz [11].

The physical quantities of interest are the skin friction coefficient c_f and the local Nusselt number Nu_x or melting rate (local heat flux) at the stretching surface defined as

$$c_f=\frac{\tau_w}{\rho u_e^2}, \qquad Nu_x=\frac{xq_w}{k(T_\infty-T_m)}, \qquad (11)$$

Where τ_w *and* q_w are the surface shear stress and the surface heat flux which are given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \qquad q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad (12)$$

Where μ being the dynamic viscosity of the fluid. Using variables (5), we get

$$\mathrm{Re}_x^{1/2}\, c_f = f''(0), \qquad \mathrm{Re}_x^{-1/2}\, Nu_x = -\theta'(0) \tag{13}$$

Where $\mathrm{Re}_x = u_e(x)x / y$ is the local Reynolds number.

Combining Eqs. (4b) and (13), we can obtain the melting velocity at the stretching surface v(x, 0),which is given by following equation:

$$v(x,0) = -\frac{\alpha}{x} M Nu_x . \tag{14}$$

This velocity is proportional to $x^{-1}$and it shows that the melting process is faster near the stagnation-point of the stretching sheet.

Numerical method

Numerical solutions to the ordinary differential equation (7) and (8) with the boundary equation (9) are obtained using the Runge-Kutta-Fehlberg method with shooting technique. The dual solutions are obtained by setting different initial guesses for the values of skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$,where all the profiles satisfies the far field boundary conditions (9) asymptotically but with the different shapes. These methods have been successfully used by the present authors to solve various problems related to boundary layer flow and heat transfer (see Bachok et al. [23, 24] and Ishak et al. [25, 26]).

III. Results and Discussion

For the stretching sheet, ε is positive. Table (1) shows the comparison of $f''(0)$ with those of the previous studies, i.e mainly Bachok et al. [27], which shows a favourable agreement and thus gives us a confidence that the numerical results obtained by us are accurate. Moreover, the values of $f''(0)$ for $M \neq 0$ with magnetic parameter $M_n \neq 0$ are also included in table 1.

Variations of the skin friction coefficient $f''(0)$ with ε and M when Pr=1 M_n = 0.5 are shown in fig. (2). The result presented in fig (2), indicates that as melting parameter M increases, the skin friction coefficient $f''(0)$ decreases. The value of $f''(0)$ when $\varepsilon = 0.5\, to\, 1$ for all values of M are considered.

Fig (3) - (8) display the variation of velocity and temperature profiles within the boundary layer for the different values of , M, M_n Pr and ε .

Fig (3) and (4) depicts the effect of Magnetic parameter M_n over velocity profile and temperature profile, it is noticed from fig (3) that velocity decreases with the increase of magnetic parameter M_n , and from Fig (4) it is noticed that temperature profile increases with increase of Magnetic parameter M_n which offers resistance to the flow resulting in decrease of velocity in the boundary layer, which agrees, with results of various author's for instance the results of Dulal Pal [29].

Fig (5) and (6) shows that the boundary layer thickness for these solutions. This result is realised physically that the melting phenomenon acts as a blowing boundary condition at the stretching surface. Consequently, more intense melting parameter (M) tends to thicken the boundary layer. The dimensionless temperature profiles $\theta(\eta)$ increases monotonically at the stretching surface.

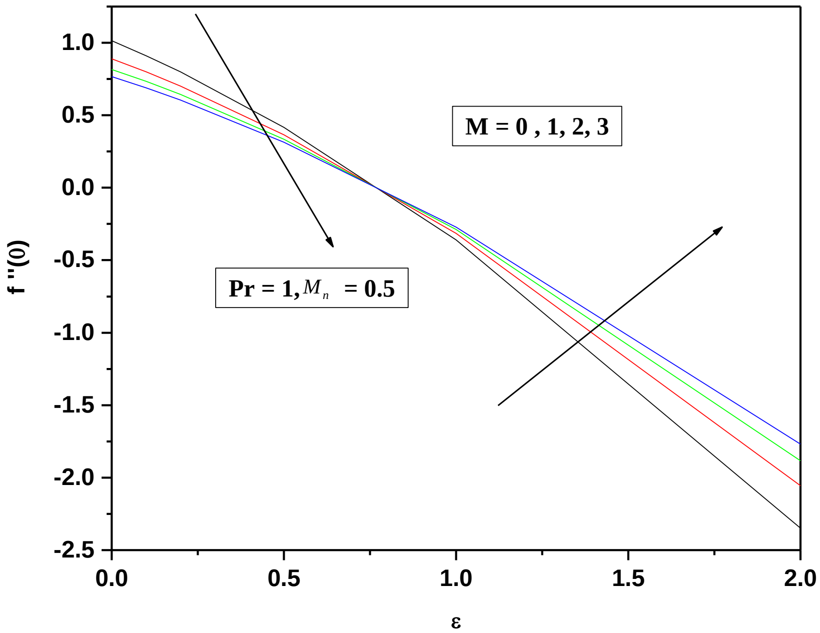
Fig (7) and (8) represents the effect of Pr on the velocity and temperature profiles. It is observed that the magnitude of the velocity gradient at the surface is higher for higher values of Pr . Thus the skin friction increases with Pr and, consequently increases the heat transfer rate at the surface.

IV. Conclusions

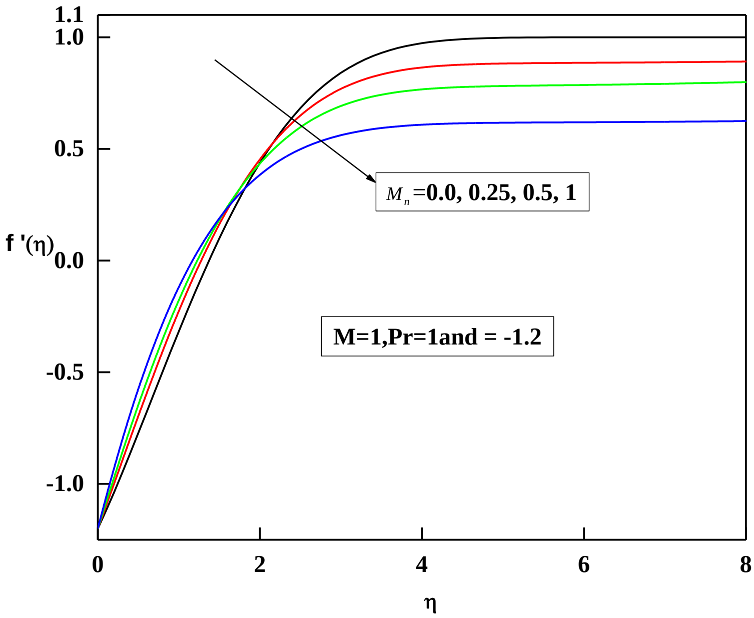
We have numerically studied the similarity solution for the MHD steady stagnation-point flow towards a horizontal stretching sheet with melting phenomena at a steady rate into a constant property, warm liquid of the same material. The governing partial differential equations are converted into ordinary differential equations by similarity transformation, before being solved numerically using Runge-Kutta-Fehlberg method. Results for the skin friction coefficient, local Nusselt number, velocity profiles as well as temperature profiles are presented for different values of the governing parameters. Effects of the Magnetic parameter, melting parameter, velocity ratio parameter and prandtl number on the flow and heat transfer characteristics are thoroughly examined.

Table.1

	N. Bachok et al.	Present Work			
	M=0	**M=0**	**M=1**	**M=2**	**M=3**
0	**1.2325877**	**1.013007**	**0.877660**	**0.814243**	**0.774780**
0.1	**1.1465610**	**0.909889**	**0.786875**	**0.729473**	**0.693834**
0.2	**1.0511300**	**0.798269**	**0.689189**	**0.638476**	**0.607054**
0.5	**0.7132949**	**0.416234**	**0.357798**	**0.330882**	**0.314290**
1	**0**	**-0.360226**	**-0.308268**	**-0.284526**	**-0.269963**
2	**-1.8873066**	**-2.350145**	**-1.997338**	**-1.838173**	**-1.741245**

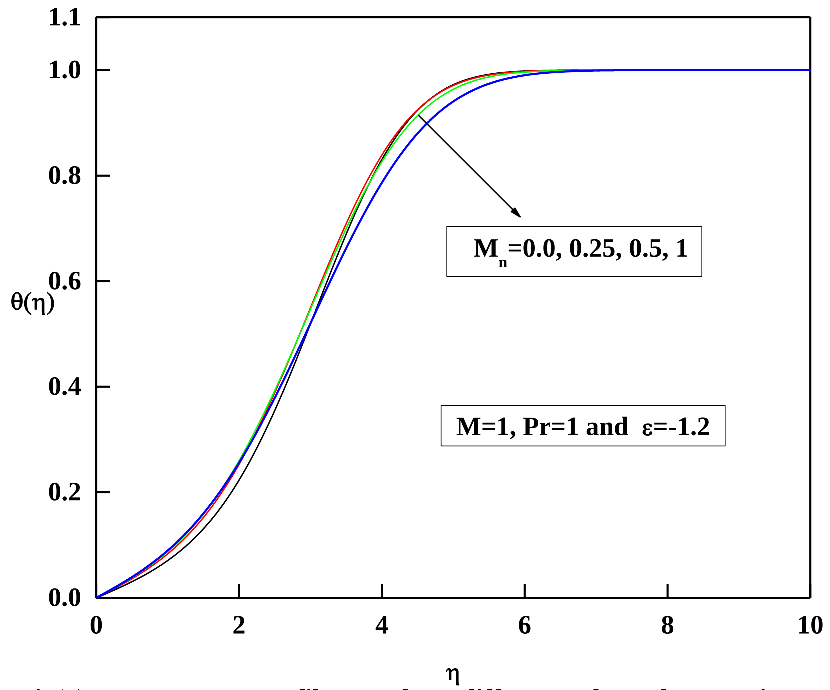


Fig(2) Variation of skin friction coefficient f ''(0) with ε for various values of M

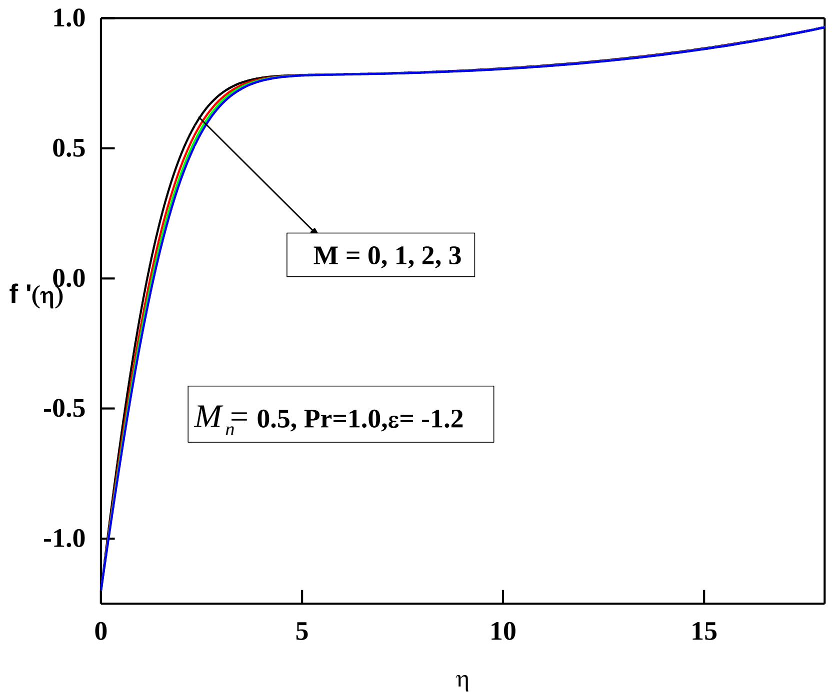


M_n

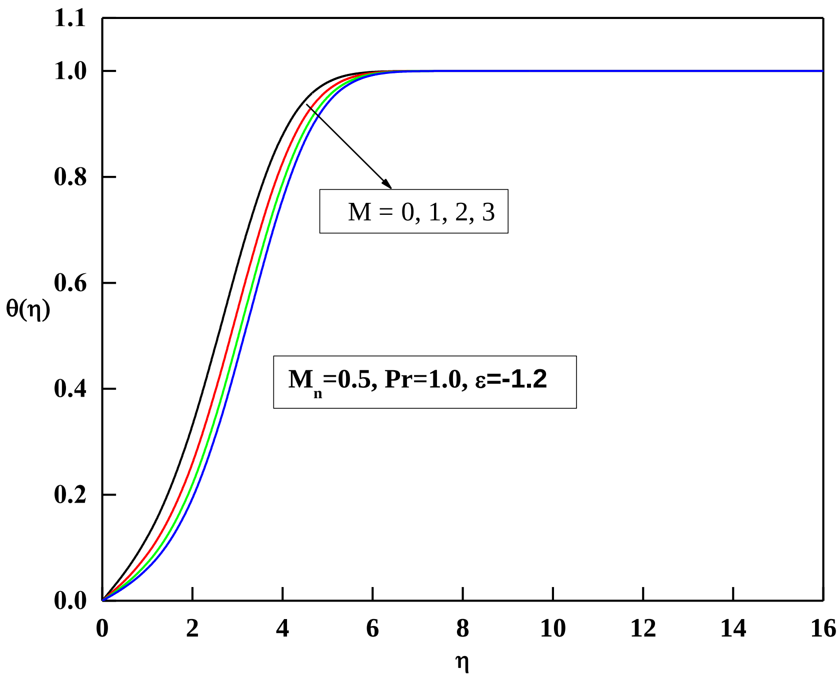
Fig(3).velocity profile f (η) versus η for different values of magnetic parameter M_n



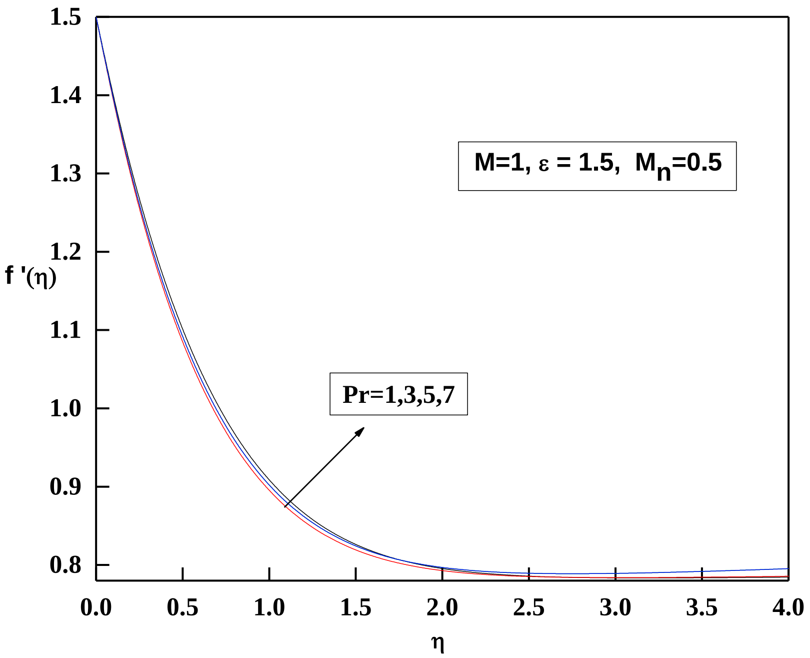
Fig(4). Temperature profiles θ(η) for η different values of Magnetic parameter M_n.



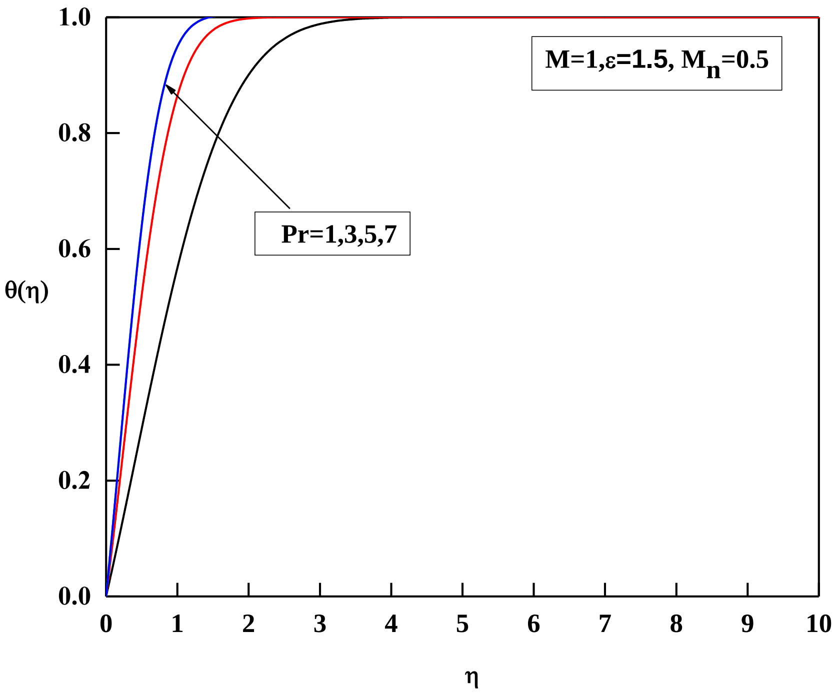
Fig(5). Velocity profiles f '(η) versus η for different values of Melting parameter M



fig(6).Temperature profile θ(η) versus η for different values of Melting parameter M.



Fig(7). Velocity profiles f '(η) versus η for different values of Prandtl number *Pr*



Fig(8). Temperature profiles θ(η) *versus*η for different values of Prantdl number Pr.

References

[1] B.C. Sakiadis, Boundary layer behavior on continuous solid surfaces: I Boundary layer equations for two dimensional and axisymmetric flow, AIChE J. 7 (1961) 26–28.

[2] I.C. Liu, Flow and heat transfer of an electrically conducting fluid of second grade over a stretching sheet subject to a transverse magnetic field, Int. J. Heat Mass Transfer 47 (2004) 4427–4437.

[3] S.K. Khan, M. Subhas Abel, M. Sonth Ravi, Viscoelastic MHD flow heat and mass transfer over a porous stretching sheet with dissipation of energy and stress work, Int. J. Heat Mass Transfer 40 (2003) 47–57.

[4] R. Cortell, Effects of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet, Phys. Lett. A 357 (2006) 298–305.

[5] B.S. Dandapat, B. Santra, K. Vajravelu, The effects of variable fluid properties And thermo capillarity on the flow of a thin film on an unsteady stretching sheet, Int. J. Heat Mass Transfer 50 (2007) 991–996.

[6] S. Nadeem, A. Hussain, M. Khan, HAM solutions for boundary layer flow in the region of the stagnation point towards a stretching sheet, Commun. Nonlinear Sci. Numer. Simulat. 15 (2010) 475–481.

[7] N. Bachok, A. Ishak, Flow and heat transfer over a stretching cylinder with prescribed surface heat flux Malays, J. Math. Sci. 4 (2010) 159–169.

[8] N. Bachok, A. Ishak, R. Nazar, Flow and heat transfer over an unsteady stretching sheet in a micropolar fluid with prescribed surface heat flux, Int. J. Math.Mod.Meth. Appl. Sci. 4 (2010) 167–176.

[9] N. Bachok, A. Ishak, I. Pop, On the stagnation point flow towards a stretching sheet with homogeneous-heterogeneous reactions effects, Commun. Nonlinear Sci. Numer.Simulat. 16 (2011) 4296–4302.

[10] N. Bachok, A. Ishak, R. Nazar, Flow and heat transfer over an unsteady stretching sheet in a micropolar fluid, Meccanica 46 (2011) 935–942.

[11] K.Hiemenz, dinglerspoytech. J. 326 (1911) 321.

[12] F.Homann, Z. Angew. Math.Mech. 16 (1936) 153.

[13] M.Sajid, T. Hayat, T. Javed,NonlinearDyn. 51(2008) 259.

[14] E.M. Epstein, D.H. Cho, J. Heat Transfer 98 (1976) 531.

[15] A.Ishak, R. Nazar, N. Bachok, I. Pop, Heat Mass Transfer 46 (2010) 463.

[16] M.M, Chen,R. Farhadieh, J.L. Baker, Int. J. Heat Mass Transfer 29 (1986)1087.

[17] M. Kazmierczak, D. Poulikakos, I. Pop, Numer. Heat Transfer 10 (1986) 571.

[18] M. Kazmierczak, D. Poulikakos, D. Sadowski, Int. Comm. Heat Mass Transfer 14 (1987) 507.

[19] A.Y.Bakier, Transp. Porous Media 29 (1997) 127.

[20] R.S.R. Gorla,M.A. Mansour, I.A. Hassanien, A.Y. Bakier, Transp. Porous Media 36 (1999) 245.

[21] W.T. Cheng, C.H. Lin, Int. J. Eng. Sci. 44 (2006) 1023.

[22] W.T. Cheng, C.H. Lin, Int. J. Heat Mass Transfer 50 (2007) 3026.

[23] N. Bachok, A. Ishak, I. Pop, Transp. Porous Media 82 (2010) 363.

[24] N. Bachok, A. Ishak, I. Pop, Int. J. Thermal Sci. 49 (2010) 1663.

[25] A. Ishak, R. Nazar, I. Pop, Phys. Lett, A 372 (2008) 559.

[26] A. Ishak, R. Nazar, N. Bachok, I. Pop, Physica A 389 (2010) 40.

[27] N.Bachok, A. Ishak, I. Pop, Phys. Lett. 374 (2010) 4075.

[28] N.Bachok, A. Ishak, I. Pop, Int. J. Heat Mass Transfer xxx (2012) xxx.

[29] Dulal Pal, Hiranmo Int.l Communications in Heat and Mass Transfer 38 (2011) 463–467