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Similarity Measure Using Interval Valued Vague Sets in Multiple Criteria Decision Making

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Abstract*: In real life, a person may observe that an object belongs and not belongs to a set to certain degree, but it is possible that he is not sure about it. In other words, there may be some hesitation or uncertainty about the membership and non-membership degree of an object belonging to a set. In fuzzy set theory there is no means to incorporate that hesitation in membership degree. A possible solution is to use vague sets and the concept of vague set was proposed by Gau and Buehrer [1993]. Distance measure between vague sets is one of the most important technologies in various application fields of vague sets. But these methods are unsuitable to deal with the similarity measures of IFSs. In this paper we have extended the work of Zeshui Xu [2007] and also proposed a method to develop some similarity measure of interval valued vague sets and define the positive and negative ideal of interval valued vague sets, and apply the similarity measures to multiple attribute decision making based on vague information. A numerical example is also given to elaborate our technique.*
Keywords: *Vague sets; Fuzzy sets; Membership function; Distance measure; Interval Valued Vague Sets; Similarity Measure; Decision Making*

I. Introduction:

Atanassov (1986) defined the notion of intuitionstic fuzzy sets, which is a generalization of notion of Zadeh's fuzzy set which was later on called vague set the concept given by Gau and Buehrer (1993), which is characterized by a membership function and a non membership function. The concept of vague set is the generalization of the fuzzy set which introduced by Zadeh (1965), and has also been found to be very highly useful to deal with vagueness. In less than two decades since its first appearance, the IFS theory has been investigated by many authors (Atanassov & Georgiev , 1993; Bustince et al.,2000; De et al ., 2001; Deschrijver & Kerre, 2003; Grzegorzewski, 2004; Mondal & Samanta, 2001,2002; Szmidt & Kacprzyk, 2000,2001), and has been applied in different fields, including decision making (Atanassov, Pasi & Yager, 2005; Chen and Tan.1994; Hong & Choi, 2000; Szmidt & Kacprzyk, 2002; Xu & Ronald, 2006), logic programming (Atanassov & Georgiev, 1993), medical diagnosis (De et al., 2001), etc Gau & Buehrer (1993) defined the concept of vague set. Bustine & Burillo (1996) showed that the notion of vague set coincides with that of IFS. De et al., 2000 defined concentrated IFS, dilated IFS, normalization of IFS, and made some characterization. Atanassov & Georgiev (1993) presented a logic programming system which uses a theory of IFSs to model various forms of uncertainty. Bustince et al (2000) proposed some definitions of distances between IFSs and compared them with the approach used for fuzzy sets. Szmidt & Kacprzyk (2001) introduced a measure of entropy for IFS. Mondal and Samanta (2001) defined the topology of interval valued IFSs. Mondal and Samanta (2002) established an intuitionistic fuzzy topological space. Deschrijver & Kerre (2003) presented an intuitionistic fuzzy version of triangular compositions and investigated some properties of these compositions, such as containment, convertibilitry, monotonicity, interaction with union and intersection. Many methods have been proposed for measuring the degree of similarity between fuzzy sets.

In real life, a person may observe that an object belongs and not belongs to a set to certain degree, but it is possible that he is not sure about it. In other words, there may be some hesitation or uncertainty about the membership and non-membership degree of an object belonging to a set. In fuzzy set theory there is no means to incorporate that hesitation in membership degree. A possible solution is to use vague sets and the concept of vague set was proposed by Gau and Buehrer [1993]. But these methods are unsuitable to deal with the similarity measures of IFSs.

In this paper we have extended the work of Zeshui Xu [2007] and also proposed a method to develop some similarity measure of interval valued vague sets and define the positive and negative ideal of interval valued vague sets, and apply the similarity measures to multiple attribute decision making based on vague information.

II. Some Definitions:

2.1: Definition: An interval valued fuzzy sets $\tilde{A}$ over a universe of discourse X is defined by a function $T_{\tilde{A}}: X \rightarrow D([0,1]),$ where D ([0, 1]) is the set of all intervals within [0, 1] i.e. for all $x \in X$, $T_{\tilde{A}}(x)$ is an interval $[\mu_1, \mu_2]$ and $0 \leq \mu_1 \leq \mu_2 \leq 1.$

2.2: Definition: Vague set. A vague set $\tilde{A}$ in the universe of discourse X is characterized by two membership functions as:

(1) truth membership function

$$\mu_{\tilde{A}}: X \rightarrow [0,1] \text{ and}$$

(2) false membership function

$$\nu_{\tilde{A}}: X \rightarrow [0,1].$$

The grade of membership for any element *x* in the vague set is bounded by a sub interval [$\mu_{\tilde{A}}(x)$,1- $\nu_{\tilde{A}}(x)$] of [0,1] where the grade $\mu_{\tilde{A}}(x)$ is called the lower bound of membership grade of *x* derived from favourable evidence for *x* and $\nu_{\tilde{A}}(x)$ is the lower bound of membership grade on the negation of *x* derived from the evidence against *x,* where $\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq$ 1. The interval, [$\mu_{\tilde{A}}(x)$,1- $\nu_{\tilde{A}}(x)$] is called the vague value of x in $\tilde{A}$. In the extreme case of equality where $\mu_{\tilde{A}}(x)$ = 1- $\nu_{\tilde{A}}(x)$, the vague set reduces to the fuzzy set with interval value of the membership grade reducing to a single value $\mu_{\tilde{A}}(x)$. In general, however,

$$\mu_{\tilde{A}}(x) \leq \text{Exact membership grade of } x \leq 1 - \nu_{\tilde{A}}(x).$$

2.3: Definition: An interval valued vague sets $\tilde{A}^V$ over a universe of discourse X is defined as an object of the form $\tilde{A}^V = <[x_i, T_{\tilde{A}^V}(x_i), F_{\tilde{A}^V}(x_i)]>, x_i \in X,$ where $T_{\tilde{A}^V}: X \rightarrow D([0,1]),$ and $F_{\tilde{A}^V}: X \rightarrow D([0,1]),$ are called "Truth membership function" and "False membership function" respectively and where D ([0, 1]) is the set of all intervals within [0, 1], or in other word an interval valued vague set can be represented by $\tilde{A}^V = <[(x_i), [\mu_1, \mu_2], [\nu_1, \nu_2]]>, x_i \in X,$ where $0 \leq \mu_1 \leq \mu_2 \leq 1.$ and $0 \leq \nu_2 \leq \nu_1 \leq 1.$ For each interval valued vague set $\tilde{A}^V$, $\pi_{1\tilde{A}^V}(x_i) = 1 - \mu_1(x_i) - \nu_{1\tilde{A}^V}(x_i)$ and are called degree of hesitancy of x_i in $\tilde{A}^V$ respectively.

III. Similarity Measures:

Let X = { $x_1, x_2, ...x_n$ } be a universe of discourse, and $\omega = (\omega_1, \omega_2, ...\omega_n)^T$ be the weight vector of the elements x_i (i=1, 2 …….n), where $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1.$

A vague set $\tilde{A}^V = \left\{ \left\langle x_i, T_{\tilde{A}^V}(x_i), F_{\tilde{A}^V}(x_i) \right\rangle | \, x_i \in X \right\}$

which is characterized by a truth membership function $T_{\tilde{A}^V}$ and a false membership function $F_{\tilde{A}^V}$, where

$$T_{\tilde{A}^V}: X \rightarrow [0,1], x_j \in X \rightarrow T_{\tilde{A}^V}(x_j) \in [0,1],$$

$$F_{\tilde{A}^V}: X \rightarrow [0,1], x_j \in X \rightarrow F_{\tilde{A}^V}(x_j) \in [0,1],$$

With the condition

$$0 \leq T_{\tilde{A}^V}(x_i) + F_{\tilde{A}^V}(x_i) \leq 1, \text{ for all } x_i \in X$$

For each vague set in X, if

$\pi_{\tilde{A}^V}(x_i) = 1 - T_{\tilde{A}^V}(x_i) - F_{\tilde{A}^V}(x_i) = 0,$ then vague set $\tilde{A}^V$ is reduced to a fuzzy set $\tilde{A}$

Chen et al. (1995) examined the similarity measures of fuzzy sets, which are based on the geometric model, set theoretic approach, and matching function. In this paper, we extend the work of Chen et al. (1995); to investigate similarity measures of interval valued vague set.For convenience, let Φ(X) be the set of all interval valued vague sets of X. Below, we introduce the concept of similarity measure between two interval valued vague sets.

Definition 3.1 Let S be a mapping $S:\Phi(X)^2 \rightarrow [0,1],$ then the degree of similarity between $\tilde{A}^V \in \phi(x)$ and $\tilde{B}^V \in \phi(x)$ is defined as $S(\tilde{A}^V, \tilde{B}^V),$ which satisfies the following properties:

(1) $0 \le S(\tilde{A}^V, \tilde{B}^V) \le 1;$

(2) $S(\tilde{A}^V, \tilde{B}^V) = 1$, iff $\tilde{A}^V = \tilde{B}^V$;

(3) $S(\tilde{A}^V, \tilde{B}^V) = S(\tilde{B}^V, \tilde{A}^V);$

(4) If $S(\tilde{A}^V, \tilde{B}^V) = 0$ and $S(\tilde{A}^V, \tilde{C}^V)$ then $S(\tilde{B}^V, \tilde{C}^V) = 0.$

Let $\tilde{A}^V \in \phi(x)$ and $\tilde{B}^V \in \phi(x)$, where $\tilde{A}^V$ and $\tilde{B}^V$ are vague sets then based on above, Szmidt & Kacprzyk (1996 & 2000) proposed the following distances:

(1) The Hamming distance:

$$d(\tilde{A}^V,\tilde{B}^V) = \frac{1}{2}\sum_{i=1}^{n}\left(\left|\mu_{\tilde{A}^V}(x_i) - \mu_{\tilde{B}^V}(x_i)\right| + \left|\nu_{\tilde{A}^V}(x_i) - \nu_{\tilde{B}^V}(x_i)\right| + \left|\pi_{\tilde{A}^V}(x_i) - \pi_{\tilde{B}^V}(x_i)\right|\right). \quad \cdots\cdots\cdots(1)$$

(2) The normalized Hamming distance:

$$d(\tilde{A}^V,\tilde{B}^V) = \frac{1}{2n}\sum_{i=1}^{n}\left(\left|\mu_{\tilde{A}^V}(x_i) - \mu_{\tilde{B}^V}(x_i)\right| + \left|\nu_{\tilde{A}^V}(x_i) - \nu_{\tilde{B}^V}(x_i)\right| + \left|\pi_{\tilde{A}^V}(x_i) - \pi_{\tilde{B}^V}(x_i)\right|\right). \quad \cdots\cdots\cdots(2)$$

(3) The Euclidean distance:

$$d(\tilde{A}^V,\tilde{B}^V) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left(\mu_{\tilde{A}^V}(x_i) - \mu_{\tilde{B}^V}(x_i)\right)^2 + \left(\nu_{\tilde{A}^V}(x_i) - \nu_{\tilde{B}^V}(x_i)\right)^2 + \left(\pi_{\tilde{A}^V}(x_i) - \pi_{\tilde{B}^V}(x_i)\right)^2}. \quad \cdots(3)$$

(4) The normalized Euclidean distance:

$$d(\tilde{A}^V,\tilde{B}^V) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left(\mu_{\tilde{A}^V}(x_i) - \mu_{\tilde{B}^V}(x_i)\right)^2 + \left(\nu_{\tilde{A}^V}(x_i) - \nu_{\tilde{B}^V}(x_i)\right)^2 + \left(\pi_{\tilde{A}^V}(x_i) - \pi_{\tilde{B}^V}(x_i)\right)^2}. \quad ..(4)$$

Let $\tilde{A}^V \in \phi(x)$ and $\tilde{B}^V \in \phi(x)$, where $\tilde{A}^V$ and $\tilde{B}^V$ are vague sets then based on above, Zeshui Xu (2007) proposed the following distances:

(1) $$d(\tilde{A}^V,\tilde{B}^V) = \left[\frac{1}{2}\sum_{i=1}^{n}\left(\left|\mu_{\tilde{A}^V}(x_i) - \mu_{\tilde{B}^V}(x_i)\right|^\alpha + \left|\nu_{\tilde{A}^V}(x_i) - \nu_{\tilde{B}^V}(x_i)\right|^\alpha + \left|\pi_{\tilde{A}^V}(x_i) - \pi_{\tilde{B}^V}(x_i)\right|^\alpha\right)\right]^{1/\alpha}. \quad \cdots(5)$$

(2) $$d(\tilde{A}^V,\tilde{B}^V) = \left[\frac{1}{2n}\sum_{i=1}^{n}\left(\left|\mu_{\tilde{A}^V}(x_i) - \mu_{\tilde{B}^V}(x_i)\right|^\alpha + \left|\nu_{\tilde{A}^V}(x_i) - \nu_{\tilde{B}^V}(x_i)\right|^\alpha + \left|\pi_{\tilde{A}^V}(x_i) - \pi_{\tilde{B}^V}(x_i)\right|^\alpha\right)\right]^{1/\alpha}. \quad ..(6)$$

Where $\alpha > 0.$

Particular case:

(i) If $\alpha = 1$ then the above equations reduce to the equations (1) and (2) respectively. (Kacprzyk, in 1996)

(ii) If $\alpha = 2$ then these results reduce to equations (3) and (4) respectively. (Szmidt & Kacprzyk in 2000)

Based on geometrical distance model and using interval valued vague sets, we generalized the above equations, (1)-(6) distances as follow:

(1) $$d(\tilde{A}^V,\tilde{B}^V) = \left[\frac{1}{2}\sum_{i=1}^{n}\begin{pmatrix}\left|\mu_{1\tilde{A}^V}(x_i) - \mu_{1\tilde{B}^V}(x_i)\right|^\alpha + \left|\mu_{2\tilde{A}^V}(x_i) - \mu_{2\tilde{B}^V}(x_i)\right|^\alpha + \left|\nu_{1\tilde{A}^V}(x_i) - \nu_{1\tilde{B}^V}(x_i)\right|^\alpha \\ + \left|\nu_{2\tilde{A}^V}(x_i) - \nu_{2\tilde{B}^V}(x_i)\right|^\alpha + \left|\pi_{1\tilde{A}^V}(x_i) - \pi_{1\tilde{B}^V}(x_i)\right|^\alpha + \left|\pi_{2\tilde{A}^V}(x_i) - \pi_{2\tilde{B}^V}(x_i)\right|^\alpha\end{pmatrix}\right]^{1/\alpha}. \quad ----(7)$$

(2) $$d(\tilde{A}^V,\tilde{B}^V) = \left[\frac{1}{2n}\sum_{i=1}^{n}\begin{pmatrix}\left|\mu_{1\tilde{A}^V}(x_i) - \mu_{1\tilde{B}^V}(x_i)\right|^\alpha + \left|\mu_{2\tilde{A}^V}(x_i) - \mu_{2\tilde{B}^V}(x_i)\right|^\alpha + \left|\nu_{1\tilde{A}^V}(x_i) - \nu_{1\tilde{B}^V}(x_i)\right|^\alpha \\ + \left|\nu_{2\tilde{A}^V}(x_i) - \nu_{2\tilde{B}^V}(x_i)\right|^\alpha + \left|\pi_{1\tilde{A}^V}(x_i) - \pi_{1\tilde{B}^V}(x_i)\right|^\alpha + \left|\pi_{2\tilde{A}^V}(x_i) - \pi_{2\tilde{B}^V}(x_i)\right|^\alpha\end{pmatrix}\right]^{1/\alpha}.$$

…(8)

In many situations, the weight of the elements $x_i \in X$ should be taken into account, for example, in multiple attribute decision making the considering attributes usually have different importance, and thus need to be assigned with different weights. So we further extend (12) and define the weight distance as follow:

$$d(\tilde{A}^V,\tilde{B}^V)=\left[\frac{1}{2}\sum_{i=1}^{n} w_i \left(\begin{array}{l} \left|\mu_{1\tilde{A}^V}(x_i)-\mu_{1\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\mu_{2\tilde{A}^V}(x_i)-\mu_{2\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\nu_{1\tilde{A}^V}(x_i)-\nu_{1\tilde{B}^V}(x_i)\right|^{\alpha} \\ +\left|\nu_{2\tilde{A}^V}(x_i)-\nu_{2\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\pi_{1\tilde{A}^V}(x_i)-\pi_{1\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\pi_{2\tilde{A}^V}(x_i)-\pi_{2\tilde{B}^V}(x_i)\right|^{\alpha} \end{array}\right)\right]^{1/\alpha} \quad \text{.....(9)}$$

Where $w=(w_1,\ w_2..........w_n)^T$ is the weight vector of $x_i(i=1,\ 2,.........n)$, and $\alpha>0$. If $w=(1/n,\ 1/n..........1/n)^T$, Then (9) reduces to (8).

Based on (8), we define the similarity measure between the interval valued vague sets $\tilde{A}^V$ and $\tilde{B}^V$ as follow:

$$S(\tilde{A}^V,\tilde{B}^V)=1-\left[\frac{1}{2n}\sum_{i=1}^{n} \left(\begin{array}{l} \left|\mu_{1\tilde{A}^V}(x_i)-\mu_{1\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\mu_{2\tilde{A}^V}(x_i)-\mu_{2\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\nu_{1\tilde{A}^V}(x_i)-\nu_{1\tilde{B}^V}(x_i)\right|^{\alpha} \\ +\left|\nu_{2\tilde{A}^V}(x_i)-\nu_{2\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\pi_{1\tilde{A}^V}(x_i)-\pi_{1\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\pi_{2\tilde{A}^V}(x_i)-\pi_{2\tilde{B}^V}(x_i)\right|^{\alpha} \end{array}\right)\right]^{1/\alpha} \quad \text{...(10)}$$

Where $\alpha>0$ and $S(\tilde{A}^V,\tilde{B}^V)$ is the degree of similarity of $\tilde{A}^V$ and $\tilde{B}^V$.

If we take the weight of each elements $x_i \in X$ into account, then

$$S(\tilde{A}^V,\tilde{B}^V)=1-\left[\frac{1}{2}\sum_{i=1}^{n} w_i \left(\begin{array}{l} \left|\mu_{1\tilde{A}^V}(x_i)-\mu_{1\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\mu_{2\tilde{A}^V}(x_i)-\mu_{2\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\nu_{1\tilde{A}^V}(x_i)-\nu_{1\tilde{B}^V}(x_i)\right|^{\alpha} \\ +\left|\nu_{2\tilde{A}^V}(x_i)-\nu_{2\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\pi_{1\tilde{A}^V}(x_i)-\pi_{1\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\pi_{2\tilde{A}^V}(x_i)-\pi_{2\tilde{B}^V}(x_i)\right|^{\alpha} \end{array}\right)\right]^{1/\alpha} \quad \text{...(11)}$$

If each elements has the same importance, i.e. $w=(1/n,\ 1/n..........1/n)^T$, then (11) reduces to (10). By (11) it can easily be known that $S(\tilde{A}^V,\tilde{B}^V)$ satisfies all the properties of definition 3.1.

Similarly, we define another similarity measure of $\tilde{A}^V$ and $\tilde{B}^V$ as:

$$S(\tilde{A}^V,\tilde{B}^V)=1-\left[\frac{\sum_{i=1}^{n}\left(\begin{array}{l} \left|\mu_{1\tilde{A}^V}(x_i)-\mu_{1\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\mu_{2\tilde{A}^V}(x_i)-\mu_{2\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\nu_{1\tilde{A}^V}(x_i)-\nu_{1\tilde{B}^V}(x_i)\right|^{\alpha} \\ +\left|\nu_{2\tilde{A}^V}(x_i)-\nu_{2\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\pi_{1\tilde{A}^V}(x_i)-\pi_{1\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\pi_{2\tilde{A}^V}(x_i)-\pi_{2\tilde{B}^V}(x_i)\right|^{\alpha} \end{array}\right)}{\sum_{i=1}^{n}\left(\begin{array}{l} \left|\mu_{1\tilde{A}^V}(x_i)+\mu_{1\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\mu_{2\tilde{A}^V}(x_i)+\mu_{2\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\nu_{1\tilde{A}^V}(x_i)+\nu_{1\tilde{B}^V}(x_i)\right|^{\alpha} \\ +\left|\nu_{2\tilde{A}^V}(x_i)+\nu_{2\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\pi_{1\tilde{A}^V}(x_i)+\pi_{1\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\pi_{2\tilde{A}^V}(x_i)+\pi_{2\tilde{B}^V}(x_i)\right|^{\alpha} \end{array}\right)}\right]^{1/\alpha} \quad \text{....(12)}$$

If we take the weight of each element $x_i \in X$ into account, then

$$S(\tilde{A}^V,\tilde{B}^V)=1-\left[\frac{\sum_{i=1}^{n} w_i\left(\begin{array}{l} \left|\mu_{1\tilde{A}^V}(x_i)-\mu_{1\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\mu_{2\tilde{A}^V}(x_i)-\mu_{2\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\nu_{1\tilde{A}^V}(x_i)-\nu_{1\tilde{B}^V}(x_i)\right|^{\alpha} \\ +\left|\nu_{2\tilde{A}^V}(x_i)-\nu_{2\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\pi_{1\tilde{A}^V}(x_i)-\pi_{1\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\pi_{2\tilde{A}^V}(x_i)-\pi_{2\tilde{B}^V}(x_i)\right|^{\alpha} \end{array}\right)}{\sum_{i=1}^{n} w_i\left(\begin{array}{l} \left|\mu_{1\tilde{A}^V}(x_i)+\mu_{1\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\mu_{2\tilde{A}^V}(x_i)+\mu_{2\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\nu_{1\tilde{A}^V}(x_i)+\nu_{1\tilde{B}^V}(x_i)\right|^{\alpha} \\ +\left|\nu_{2\tilde{A}^V}(x_i)+\nu_{2\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\pi_{1\tilde{A}^V}(x_i)+\pi_{1\tilde{B}^V}(x_i)\right|^{\alpha}+\left|\pi_{2\tilde{A}^V}(x_i)+\pi_{2\tilde{B}^V}(x_i)\right|^{\alpha} \end{array}\right)}\right]^{1/\alpha} \quad \text{----(13)}$$

This has also been proved that all the properties of definition 3.1 are satisfied, if each element has the same importance, and then (13) reduces to (12).

3.2 Similarity measures based on the Interval Valued Vague Set theoretic approach:-

Let $\tilde{A}^V \in \phi(x)$ and $\tilde{B}^V \in \phi(x)$, where $\tilde{A}^V$ and $\tilde{B}^V$ are interval valued vague sets, then we define a similarity measure $\tilde{A}^V$ and $\tilde{B}^V$ from the point of set theoretic as:

$$S(\tilde{A}^V,\tilde{B}^V)=\frac{\sum_{i=1}^{n}\begin{pmatrix}\min\left(\mu_{1\tilde{A}^V}(x_i),\ \mu_{1\tilde{B}^V}(x_i)\right)+\min\left(\mu_{2\tilde{A}^V}(x_i),\ \mu_{2\tilde{B}^V}(x_i)\right)+\min\left(\nu_{1\tilde{A}^V}(x_i),\ \nu_{1\tilde{B}^V}(x_i)\right)\\+\min\left(\nu_{2\tilde{A}^V}(x_i),\ \nu_{2\tilde{B}^V}(x_i)\right)+\min\left(\pi_{1\tilde{A}^V}(x_i),\ \pi_{1\tilde{B}^V}(x_i)\right)+\min\left(\pi_{2\tilde{A}^V}(x_i),\ \pi_{2\tilde{B}^V}(x_i)\right)\end{pmatrix}}{\sum_{i=1}^{n}\begin{pmatrix}\max\left(\mu_{1\tilde{A}^V}(x_i),\ \mu_{1\tilde{B}^V}(x_i)\right)+\max\left(\mu_{2\tilde{A}^V}(x_i),\ \mu_{2\tilde{B}^V}(x_i)\right)+\max\left(\nu_{1\tilde{A}^V}(x_i),\ \nu_{1\tilde{B}^V}(x_i)\right)\\+\max\left(\nu_{2\tilde{A}^V}(x_i),\ \nu_{2\tilde{B}^V}(x_i)\right)+\max\left(\pi_{1\tilde{A}^V}(x_i),\ \pi_{1\tilde{B}^V}(x_i)\right)+\max\left(\pi_{2\tilde{A}^V}(x_i),\ \pi_{2\tilde{B}^V}(x_i)\right)\end{pmatrix}} \quad \ldots\ldots.(14)$$

If we take the weight of each element $x_i \in X$ into account, then

$$S(\tilde{A}^V,\tilde{B}^V)=\frac{\sum_{i=1}^{n}w_i\begin{pmatrix}\min\left(\mu_{1\tilde{A}^V}(x_i),\ \mu_{1\tilde{B}^V}(x_i)\right)+\min\left(\mu_{2\tilde{A}^V}(x_i),\ \mu_{2\tilde{B}^V}(x_i)\right)+\min\left(\nu_{1\tilde{A}^V}(x_i),\ \nu_{1\tilde{B}^V}(x_i)\right)\\+\min\left(\nu_{2\tilde{A}^V}(x_i),\ \nu_{2\tilde{B}^V}(x_i)\right)+\min\left(\pi_{1\tilde{A}^V}(x_i),\ \pi_{1\tilde{B}^V}(x_i)\right)+\min\left(\pi_{2\tilde{A}^V}(x_i),\ \pi_{2\tilde{B}^V}(x_i)\right)\end{pmatrix}}{\sum_{i=1}^{n}w_i\begin{pmatrix}\max\left(\mu_{1\tilde{A}^V}(x_i),\ \mu_{1\tilde{B}^V}(x_i)\right)+\max\left(\mu_{2\tilde{A}^V}(x_i),\ \mu_{2\tilde{B}^V}(x_i)\right)+\max\left(\nu_{1\tilde{A}^V}(x_i),\ \nu_{1\tilde{B}^V}(x_i)\right)\\+\max\left(\nu_{2\tilde{A}^V}(x_i),\ \nu_{2\tilde{B}^V}(x_i)\right)+\max\left(\pi_{1\tilde{A}^V}(x_i),\ \pi_{1\tilde{B}^V}(x_i)\right)+\max\left(\pi_{2\tilde{A}^V}(x_i),\ \pi_{2\tilde{B}^V}(x_i)\right)\end{pmatrix}} \quad \ldots.(15)$$

Particularly, if each element has the same importance, then (15) is reduced to (14), clearly this also satisfies all the properties of definition 3.1.

IV. Applying The Similarity Measure To Multiple Attribute Decision Making Under Vague Environment:

In the following section, we have applied the above similarity measures to multiple attribute decision making based on interval valued vague sets.

For a multiple attribute decision making problem, let $A=\{A_1, A_2,........A_k\}$ be a set of alternatives, and let $C=\{C_1, C_2,........C_n\}$ be a set of attributes and $w=(w_1, w_2..........w_n)^T$ be the weight vector of attributes, with the condition $w_i \geq 0$ and $\sum_{i=1}^{n} w_i = 1.$ Assume that the characteristics of the alternative $\tilde{A}_j^V$ are represented by the interval valued vague sets as follows:

$$\tilde{A}_j^V =<\left[\left\{C_i,\ [\mu_{1\tilde{A}_j^V}(C_i),\ \mu_{2\tilde{A}_j^V}(C_i)],\ [\nu_{1\tilde{A}_j^V}(C_i),\ \nu_{2\tilde{A}_j^V}(C_i)]\right\}\right]>,\ \ j=1,2...........k, \quad (16)$$

where $\mu_{1\tilde{A}_j^V}(C_i)$ and $\mu_{2\tilde{A}_j^V}(C_i)$ are the lower and upper bond of the degree of truth membership i.e. indicates the range of degree that the alternative $\tilde{A}_j^V$ satisfies the attribute C_i, similarly $\nu_{1\tilde{A}_j^V}(C_i)$ and $\nu_{2\tilde{A}_j^V}(C_i)$ are the lower and upper bond of the degree of false membership i.e. indicates the range of degree that the alternative $\tilde{A}_j^V$ does not satisfies the attribute C_i, and $\mu_{1\tilde{A}_j^V}(C_i),\mu_{2\tilde{A}_j^V}(C_i),\nu_{1\tilde{A}_j^V}(C_i),\nu_{2\tilde{A}_j^V}(C_i)\in[0.1]$, $\leq \mu_{m\tilde{A}_j^V}(C_i)+\nu_{m\tilde{A}_j^V}(C_i)\leq 1$, for m={1,2}.

Let $\pi_{1\tilde{A}_j^V}(C_i)=1-\mu_{1\tilde{A}_j^V}(C_i)-\nu_{1\tilde{A}_j^V}(C_i)$, and $\pi_{2\tilde{A}_j^V}(C_i)=1-\mu_{2\tilde{A}_j^V}(C_i)-\nu_{2\tilde{A}_j^V}(C_i)$, for all $C_i \in C$, then we define the positive and negative ideals for interval valued vague set as follows:

$$\tilde{A}^{V+} =<\left[\left\{C_i,\ [\mu_{1\tilde{A}^{V+}}(C_i),\ \mu_{2\tilde{A}^{V+}}(C_i)],\ [\nu_{1\tilde{A}^{V+}}(C_i),\ \nu_{2\tilde{A}^{V+}}(C_i)]\right\}: C_i \in C\right]> \quad (17)$$

$$\tilde{A}^{V-} =<\left[\left\{C_i,\ [\mu_{1\tilde{A}^{V-}}(C_i),\ \mu_{2\tilde{A}^{V-}}(C_i)],\ [\nu_{1\tilde{A}^{V-}}(C_i),\ \nu_{2\tilde{A}^{V-}}(C_i)]\right\}: C_i \in C\right]>, \quad (18)$$

Where, $\mu_{1\tilde{A}^{V+}}(C_i)=\max_j\left\{\mu_{1\tilde{A}_j^V}(C_i)\right\}$, $\mu_{2\tilde{A}^{V+}}(C_i)=\max_j\left\{\mu_{2\tilde{A}_j^V}(C_i)\right\}$, and

$\nu_{1\tilde{A}^{V+}}(C_i)=\min_j\left\{\nu_{1\tilde{A}_j^V}(C_i)\right\}$, $\nu_{2\tilde{A}^{V+}}(C_i)=\min_j\left\{\nu_{2\tilde{A}_j^V}(C_i)\right\}$.

Let the hesitation part for both the ideals be defined as follows

$\pi_{1\tilde{A}^{V+}}(C_i)=1-\mu_{1\tilde{A}^{V+}}(C_i)-\nu_{1\tilde{A}^{V+}}(C_i)$, $\pi_{2\tilde{A}^{V+}}(C_i)=1-\mu_{2\tilde{A}^{V+}}(C_i)-\nu_{2\tilde{A}^{V+}}(C_i)$, and

$\pi_{1\tilde{A}^{V-}}(C_i)=1-\mu_{1\tilde{A}^{V-}}(C_i)-\nu_{1\tilde{A}^{V-}}(C_i),\ \ \pi_{2\tilde{A}^{V-}}(C_i)=1-\mu_{2\tilde{A}^{V-}}(C_i)-\nu_{2\tilde{A}^{V-}}(C_i).$

Then based on (11), we define the degree of similarity measure for the positive ideal interval valued vague set $\tilde{A}^{V+}$ and alternative $\tilde{A}_j^V$ and degree of similarity for the negative ideal interval valued vague set $\tilde{A}^{V-}$ and alternative $\tilde{A}_j^V$ respectively as follow:

$$s_1(\tilde{A}^{V+},\tilde{A}_j^V)=1-\left[\frac{1}{2}\sum_{i=1}^{n} w_i\left(\begin{array}{l}\left|\mu_{1\tilde{A}^{V+}}(x_i)-\mu_{1\tilde{A}_j^V}(x_i)\right|^{\alpha}+\left|\mu_{2\tilde{A}^{V+}}(x_i)-\mu_{2\tilde{A}_j^V}(x_i)\right|^{\alpha}+\left|\nu_{1\tilde{A}^{V+}}(x_i)-\nu_{1\tilde{A}_j^V}(x_i)\right|^{\alpha}\\+\left|\nu_{2\tilde{A}^{V+}}(x_i)-\nu_{2\tilde{A}_j^V}(x_i)\right|^{\alpha}+\left|\pi_{1\tilde{A}^{V+}}(x_i)-\pi_{1\tilde{A}_j^V}(x_i)\right|^{\alpha}+\left|\pi_{2\tilde{A}^{V+}}(x_i)-\pi_{2\tilde{A}_j^V}(x_i)\right|^{\alpha}\end{array}\right)\right]^{1/\alpha} \quad \text{.....(19)}$$

and

$$S_1(\tilde{A}^{V-},\tilde{A}_j^V)=1-\left[\frac{1}{2}\sum_{i=1}^{n} w_i\left(\begin{array}{l}\left|\mu_{1\tilde{A}^{V-}}(x_i)-\mu_{1\tilde{A}_j^V}(x_i)\right|^{\alpha}+\left|\mu_{2\tilde{A}^{V-}}(x_i)-\mu_{2\tilde{A}_j^V}(x_i)\right|^{\alpha}+\left|\nu_{1\tilde{A}^{V-}}(x_i)-\nu_{1\tilde{A}_j^V}(x_i)\right|^{\alpha}\\+\left|\nu_{2\tilde{A}^{V-}}(x_i)-\nu_{2\tilde{A}_j^V}(x_i)\right|^{\alpha}+\left|\pi_{1\tilde{A}^{V-}}(x_i)-\pi_{1\tilde{A}_j^V}(x_i)\right|^{\alpha}+\left|\pi_{2\tilde{A}^{V-}}(x_i)-\pi_{2\tilde{A}_j^V}(x_i)\right|^{\alpha}\end{array}\right)\right]^{1/\alpha} \quad \text{.....(20)}$$

Based on (19) and (20), we define the percentage of similarity measure d_j corresponding to the alternative $\tilde{A}_j^V$ as follows:

$$d_j=\left(\frac{s_1(\tilde{A}^{V+},\tilde{A}_j^V)}{s_1(\tilde{A}^{V+},\tilde{A}_j^V)+s_1(\tilde{A}^{V-},\tilde{A}_j^V)}\right)*100,\quad j=1,2..........n. \qquad (21)$$

Clearly, the bigger the value of d_j, the better the alternative $\tilde{A}_j^V$.

Similarity, based on (13) and (15) we can define the degree of similarity of the positive ideal interval valued vague set $\tilde{A}^{V+}$ and alternative $\tilde{A}_j^V$, and the degree of similarity of the negative ideal interval valued vague set and alternative $\tilde{A}_j^V$, respectively, as follow:

(1) Based on (13), we define the following:

$$S_2(\tilde{A}^{V+},\tilde{A}_j^V)=1-\left[\frac{\sum_{i=1}^{n} w_i\left(\begin{array}{l}\left|\mu_{1\tilde{A}^{V+}}(C_i)-\mu_{1\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\mu_{2\tilde{A}^{V+}}(C_i)-\mu_{2\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\nu_{1\tilde{A}^{V+}}(C_i)-\nu_{1\tilde{A}_j^V}(C_i)\right|^{\alpha}\\+\left|\nu_{2\tilde{A}^{V+}}(C_i)-\nu_{2\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\pi_{1\tilde{A}^{V+}}(C_i)-\pi_{1\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\pi_{2\tilde{A}^{V+}}(C_i)-\pi_{2\tilde{A}_j^V}(C_i)\right|^{\alpha}\end{array}\right)}{\sum_{i=1}^{n} w_i\left(\begin{array}{l}\left|\mu_{1\tilde{A}^{V+}}(C_i)+\mu_{1\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\mu_{2\tilde{A}^{V+}}(C_i)+\mu_{2\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\nu_{1\tilde{A}^{V+}}(C_i)+\nu_{1\tilde{A}_j^V}(C_i)\right|^{\alpha}\\+\left|\nu_{2\tilde{A}^{V+}}(C_i)+\nu_{2\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\pi_{1\tilde{A}^{V+}}(C_i)+\pi_{1\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\pi_{2\tilde{A}^{V+}}(C_i)+\pi_{2\tilde{A}_j^V}(C_i)\right|^{\alpha}\end{array}\right)}\right]^{1/\alpha} \quad \text{.....(22)}$$

$$S_2(\tilde{A}^{V-},\tilde{A}_j^V)=1-\left[\frac{\sum_{i=1}^{n} w_i\left(\begin{array}{l}\left|\mu_{1\tilde{A}^{V-}}(C_i)-\mu_{1\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\mu_{2\tilde{A}^{V-}}(C_i)-\mu_{2\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\nu_{1\tilde{A}^{V-}}(C_i)-\nu_{1\tilde{A}_j^V}(C_i)\right|^{\alpha}\\+\left|\nu_{2\tilde{A}^{V-}}(C_i)-\nu_{2\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\pi_{1\tilde{A}^{V-}}(C_i)-\pi_{1\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\pi_{2\tilde{A}^{V-}}(C_i)-\pi_{2\tilde{A}_j^V}(C_i)\right|^{\alpha}\end{array}\right)}{\sum_{i=1}^{n} w_i\left(\begin{array}{l}\left|\mu_{1\tilde{A}^{V-}}(C_i)+\mu_{1\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\mu_{2\tilde{A}^{V-}}(C_i)+\mu_{2\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\nu_{1\tilde{A}^{V-}}(C_i)+\nu_{1\tilde{A}_j^V}(C_i)\right|^{\alpha}\\+\left|\nu_{2\tilde{A}^{V-}}(C_i)+\nu_{2\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\pi_{1\tilde{A}^{V-}}(C_i)+\pi_{1\tilde{A}_j^V}(C_i)\right|^{\alpha}+\left|\pi_{2\tilde{A}^{V-}}(C_i)+\pi_{2\tilde{A}_j^V}(C_i)\right|^{\alpha}\end{array}\right)}\right]^{1/\alpha} \quad \textbf{.....(23)}$$

(2) Based on (15), we define the following:

$$S_3(\tilde{A}^{V+},\tilde{A}_j^V)=\frac{\sum_{i=1}^{n} w_i\left(\begin{array}{l}\min\left(\mu_{1\tilde{A}^{V+}}(C_i),\ \mu_{1\tilde{A}_j^V}(C_i)\right)+\min\left(\mu_{2\tilde{A}^{V+}}(C_i),\ \mu_{2\tilde{A}_j^V}(C_i)\right)+\min\left(\nu_{1\tilde{A}^{V+}}(C_i),\ \nu_{1\tilde{A}_j^V}(C_i)\right)\\+\min\left(\nu_{2\tilde{A}^{V+}}(C_i),\nu_{2\tilde{A}_j^V}(C_i)\right)+\min\left(\pi_{1\tilde{A}^{V+}}(C_i),\ \pi_{1\tilde{A}_j^V}(C_i)\right)+\min\left(\pi_{2\tilde{A}^{V+}}(C_i),\ \pi_{2\tilde{A}_j^V}(C_i)\right)\end{array}\right)}{\sum_{i=1}^{n} w_i\left(\begin{array}{l}\max\left(\mu_{1\tilde{A}^{V+}}(C_i),\ \mu_{1\tilde{A}_j^V}(C_i)\right)+\max\left(\mu_{2\tilde{A}^{V+}}(C_i),\ \mu_{2\tilde{A}_j^V}(C_i)\right)+\max\left(\nu_{1\tilde{A}^{V+}}(C_i),\ \nu_{1\tilde{A}_j^V}(C_i)\right)\\+\max\left(\nu_{2\tilde{A}^{V+}}(C_i),\nu_{2\tilde{A}_j^V}(C_i)\right)+\max\left(\pi_{1\tilde{A}^{V+}}(C_i),\ \pi_{1\tilde{A}_j^V}(C_i)\right)+\max\left(\pi_{2\tilde{A}^{V+}}(C_i),\ \pi_{2\tilde{A}_j^V}(C_i)\right)\end{array}\right)}\quad \ldots\ldots(24)$$

$$S_3(\tilde{A}^{V-},\tilde{A}_j^V)=\frac{\sum_{i=1}^{n} w_i\left(\begin{array}{l}\min\left(\mu_{1\tilde{A}^{V-}}(C_i),\ \mu_{1\tilde{A}_j^V}(C_i)\right)+\min\left(\mu_{2\tilde{A}^{V-}}(C_i),\ \mu_{2\tilde{A}_j^V}(C_i)\right)+\min\left(\nu_{1\tilde{A}^{V-}}(C_i),\ \nu_{1\tilde{A}_j^V}(C_i)\right)\\+\min\left(\nu_{2\tilde{A}^{V-}}(C_i),\nu_{2\tilde{A}_j^V}(C_i)\right)+\min\left(\pi_{1\tilde{A}^{V-}}(C_i),\ \pi_{1\tilde{A}_j^V}(C_i)\right)+\min\left(\pi_{2\tilde{A}^{V-}}(C_i),\ \pi_{2\tilde{A}_j^V}(C_i)\right)\end{array}\right)}{\sum_{i=1}^{n} w_i\left(\begin{array}{l}\max\left(\mu_{1\tilde{A}^{V-}}(C_i),\ \mu_{1\tilde{A}_j^V}(C_i)\right)+\max\left(\mu_{2\tilde{A}^{V-}}(C_i),\ \mu_{2\tilde{A}_j^V}(C_i)\right)+\max\left(\nu_{1\tilde{A}^{V-}}(C_i),\ \nu_{1\tilde{A}_j^V}(C_i)\right)\\+\max\left(\nu_{2\tilde{A}^{V-}}(C_i),\nu_{2\tilde{A}_j^V}(C_i)\right)+\max\left(\pi_{1\tilde{A}^{V-}}(C_i),\ \pi_{1\tilde{A}_j^V}(C_i)\right)+\max\left(\pi_{2\tilde{A}^{V-}}(C_i),\ \pi_{2\tilde{A}_j^V}(C_i)\right)\end{array}\right)}\quad \ldots\ldots(25)$$

Now using (21) to calculate the percentage similarity measure d_j corresponding to the alternative $\tilde{A}_j^V$.

V. Numerical Example:

Similarity measure of interval valued vague set has been illustrated in the following numerical example. Suppose that a high technology company needs to hire an engineer. Now there are eight candidates $A=\{A_1,\ A_2,........,A_8\}$ and six benefit criteria are considered:

(1) Personality (C_1)

(2) Past experience (C_2)

(3) Education level (C_3)

(4) Self-confidence (C_4)

(5) Emotional steadiness (C_5)

(6) Oral communication skill (C_6).

The weight vector of these criteria $C_i(i=1,\ 2........,6)$ is $w=(0.12,\ 0.3,\ 0.07,\ 0.25,\ 0.06,\ 0.2).^T$

Assuming that the characteristics of the alternatives $A_j(j=1, 2, 3, 4, 5, 6, 7, 8)$ are represented by interval valued vague sets as follow in table 1

Table 1

Candidates →	$\tilde{A}_1^V$	$\tilde{A}_2^V$	$\tilde{A}_3^V$	$\tilde{A}_4^V$	$\tilde{A}_5^V$	$\tilde{A}_6^V$	$\tilde{A}_7^V$	$\tilde{A}_8^V$	(w)	$\tilde{A}^{V+}$	$\tilde{A}^{V-}$
Personality	C_1	C_1	C_1	C_1	C_1	C_1	C_1	C_1	C_1	C_1	C_1
$\mu_{1\tilde{A}_j^V}(C_1)$	0.2	0.3	0.6	0.4	0.8	0.3	0.5	0	0.12	0.8	0
$\mu_{2\tilde{A}_j^V}(C_1)$	0.25	0.35	0.7	0.45	0.9	0.35	0.6	0.2	0.12	0.9	0.2
$\nu_{1\tilde{A}_j^V}(C_1)$	0.6	0	0.2	0.1	0.1	0.3	0.4	0.8	0.12	0	0.8
$\nu_{2\tilde{A}_j^V}(C_1)$	0.5	0	0.1	0	0.05	0.2	0.3	0.7	0.12	0	0.7
$\pi_{1\tilde{A}_j^V}(C_1)$	0.2	0.7	0.2	0.5	0.1	0.4	0.1	0.2	0.12	0.2	0.2
$\pi_{2\tilde{A}_j^V}(C_1)$	0.25	0.65	0.2	0.55	0.05	0.45	0.1	0.1	0.12	0.1	0.1
Past experience	C_2	C_2	C_2	C_2	C_2	C_2	C_2	C_2	C_2	C_2	C_2
$\mu_{1\tilde{A}_j^V}(C_2)$	0.5	0.3	0.7	0.1	0.2	0.4	0.6	0.5	0.3	0.7	0.1

$\mu_{2\tilde{A}_j^V}(C_2)$	0.53	0.4	0.8	0.3	0.4	0.5	0.65	0.6	0.3	0.8	0.3
$\nu_{1\tilde{A}_j^V}(C_2)$	0.2	0.6	0.1	0.1	0.5	0.4	0.3	0.1	0.3	0.1	0.6
$\nu_{2\tilde{A}_j^V}(C_2)$	0.15	0.5	0	0	0.2	0.3	0.2	0.05	0.3	0	0.5
$\pi_{1\tilde{A}_j^V}(C_2)$	0.3	0.1	0.2	0.8	0.3	0.2	0.1	0.4	0.3	0.2	0.3
$\pi_{2\tilde{A}_j^V}(C_2)$	0.32	0.1	0.2	0.7	0.4	0.2	0.15	0.35	0.3	0.2	0.2
Education level	C_3	C_3	C_3	C_3	C_3	C_3	C_3	C_3	C_3	C_3	C_3
$\mu_{1\tilde{A}_j^V}(C_3)$	0.1	0.5	0.5	0.5	0.4	0.2	0.3	0.3	0.07	0.5	0.1
$\mu_{2\tilde{A}_j^V}(C_3)$	0.2	0.6	0.55	0.6	0.45	0.3	0.35	0.4	0.07	0.6	0.2
$\nu_{1\tilde{A}_j^V}(C_3)$	0.4	0.1	0.4	0.2	0.4	0.6	0.3	0.4	0.07	0.1	0.6
$\nu_{2\tilde{A}_j^V}(C_3)$	0.3	0.05	0.3	0.1	0.3	0.5	0.2	0.3	0.07	0.05	0.5
$\pi_{1\tilde{A}_j^V}(C_3)$	0.5	0.4	0.1	0.3	0.2	0.2	0.4	0.3	0.07	0.4	0.3
$\pi_{2\tilde{A}_j^V}(C_3)$	0.5	0.35	0.15	0.3	0.25	0.2	0.45	0.3	0.07	0.35	0.3
Self-confidence	C_4	C_4	C_4	C_4	C_4	C_4	C_4	C_4	C_4	C_4	C_4
$\mu_{1\tilde{A}_j^V}(C_4)$	0.2	0.3	0.1	0.5	0.6	0.2	0.4	0.7	0.25	0.7	0.1
$\mu_{2\tilde{A}_j^V}(C_4)$	0.4	0.5	0.25	0.55	0.65	0.3	0.5	0.75	0.25	0.75	0.25
$\nu_{1\tilde{A}_j^V}(C_4)$	0.05	0.3	0.6	0.4	0.3	0.6	0.1	0.2	0.25	0.05	0.6
$\nu_{2\tilde{A}_j^V}(C_4)$	0	0.2	0.5	0.3	0.2	0.5	0.05	0.1	0.25	0	0.5
$\pi_{1\tilde{A}_j^V}(C_4)$	0.75	0.4	0.3	0.1	0.1	0.2	0.5	0.1	0.25	0.25	0.3
$\pi_{2\tilde{A}_j^V}(C_4)$	0.6	0.3	0.25	0.15	0.15	0.2	0.45	0.15	0.25	0.25	0.25
Emotional steadiness	C_5	C_5	C_5	C_5	C_5	C_5	C_5	C_5	C_5	C_5	C_5
$\mu_{1\tilde{A}_j^V}(C_5)$	0.5	0.2	0.7	0.3	0.4	0.5	0.1	0	0.06	0.7	0
$\mu_{2\tilde{A}_j^V}(C_5)$	0.65	0.3	0.75	0.35	0.45	0.55	0.2	0.25	0.06	0.75	0.2
$\nu_{1\tilde{A}_j^V}(C_5)$	0.3	0.05	0.2	0.6	0.2	0.4	0.1	0.6	0.06	0.05	0.6
$\nu_{2\tilde{A}_j^V}(C_5)$	0.2	0.01	0.1	0.5	0.1	0.3	0.05	0.5	0.06	0	0.5
$\pi_{1\tilde{A}_j^V}(C_5)$	0.2	0.75	0.1	0.1	0.4	0.1	0.8	0.4	0.06	0.25	0.4
$\pi_{2\tilde{A}_j^V}(C_5)$	0.15	0.69	0.15	0.15	0.45	0.15	0.75	0.25	0.06	0.25	0.3
Communi-cation skill	C_6	C_6	C_6	C_6	C_6	C_6	C_6	C_6	C_6	C_6	C_6
$\mu_{1\tilde{A}_j^V}(C_6)$	0.5	0.1	0	0.2	0.4	0.1	0.1	0	0.2	0.5	0
$\mu_{2\tilde{A}_j^V}(C_6)$	0.6	0.3	0.2	0.25	0.5	0.2	0.3	0.1	0.2	0.6	0.1
$\nu_{1\tilde{A}_j^V}(C_6)$	0.4	0.05	0.2	0.7	0.1	0.3	0.4	0.8	0.2	0.05	0.8
$\nu_{2\tilde{A}_j^V}(C_6)$	0.2	0.01	0.1	0.5	0.05	0.25	0.3	0.6	0.2	0.01	0.6

$\pi_{1\tilde{A}_j^V}(C_6)$	0.1	0.85	0.8	0.1	0.5	0.6	0.5	0.2	0.2	0.45	0.2
$\pi_{2\tilde{A}_j^V}(C_6)$	0.2	0.69	0.7	0.25	0.45	0.55	0.4	0.3	0.2	0.39	0.3

Using (17) and (18), we first calculate the Positive ideal interval valued vague sets $\tilde{A}^{V+}$ and Negative ideal interval valued vague sets $\tilde{A}^{V-}$ shown in table 1.

(a) Similarity measures based on the Interval Valued Vague Set theoretic approach for different value of α :

Now by using (24) and (25) the similarity measure shown in table 2

Table: 2	$\alpha=2$	$\alpha=3$		$\alpha=2$	$\alpha=3$
$s_3(\tilde{A}^{V+},\tilde{A}_1^V)$	0.484505	0.459321	$s_3(\tilde{A}^{V-},\tilde{A}_1^V)$	0.418188	0.543805
$s_3(\tilde{A}^{V+},\tilde{A}_2^V)$	0.436008	0.436008	$s_3(\tilde{A}^{V-},\tilde{A}_2^V)$	0.420001	0.420001
$s_3(\tilde{A}^{V+},\tilde{A}_3^V)$	0.564639	0.564639	$s_3(\tilde{A}^{V-},\tilde{A}_3^V)$	0.417686	0.417686
$s_3(\tilde{A}^{V+},\tilde{A}_4^V)$	0.381215	0.381215	$s_3(\tilde{A}^{V-},\tilde{A}_4^V)$	0.430615	0.430615
$s_3(\tilde{A}^{V+},\tilde{A}_5^V)$	0.590457	0.590457	$s_3(\tilde{A}^{V-},\tilde{A}_5^V)$	0.397868	0.397868
$s_3(\tilde{A}^{V+},\tilde{A}_6^V)$	0.411433	0.411433	$s_3(\tilde{A}^{V-},\tilde{A}_6^V)$	0.573255	0.573255
$s_3(\tilde{A}^{V+},\tilde{A}_7^V)$	0.549187	0.549187	$s_3(\tilde{A}^{V-},\tilde{A}_7^V)$	0.408699	0.408699
$s_3(\tilde{A}^{V+},\tilde{A}_8^V)$	0.459321	0.459321	$s_3(\tilde{A}^{V-},\tilde{A}_8^V)$	0.543805	0.543805

Now using (21) Relative Similarity Measure (RSM) and Percentage Similarity Measure (PSM) shown in table 3

Table:3	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
RSM $(\alpha=2)$	0.536733	0.50935	0.574799	0.469575	0.597432	0.417831	0.573332	0.45788964
RSM $(\alpha=3)$	0.45789	0.5093	0.57479	0.4695	0.59743	0.4178	0.57333	0.45788964

Thus for $(\alpha=2,\ d_5>d_3>d_7>d_1>d_2>d_4>d_8>d_6)$ and $(\alpha=3, d_5>d_3>d_7>d_2>d_6>d_1\sim d_8>d_4)$

VI. Conclusion:

In this paper we have proposed a method for similarity measure of interval valued vague sets and extended the work of Szmidt, Kacprzyk and Zeshui Xu etc. We applied these similarity measure results to multiple attribute decision making. A numerical example of an engineer, hire by a high technology company has been taken to illustrate the application of these developments. We have considered the eight candidates $A=\{A_1, A_2,........, A_8\}$ and six benefit criteria for the illustration of this technique. Finally we knew that, candidate A_5 was the best one who had obtained by all the similarity measures. It is not possible that the belongingness of an element in a set is a single value, but it is an interval and same for not belongingness of element in the set. In this paper we have proposed a method for the development of some similarity measure for interval valued vague sets and define the positive and negative ideal of interval valued vague sets, and applied the similarity measures to multiple attribute decision making based on vague information.

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