IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 2 Ver. I (Mar. - Apr. 2016), PP 33-35
www.iosrjournals.org

The Stability Analysis of Dynamic Model of Unilateral Fish

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Abstract: *By improving the classical Lotka-Volterra model, a reasonable dynamic model is established for the unilateral fish which cannot survive independently. We combine the established model with the stability theory of differential equations to obtain the equilibrium point of the dynamic model about fish mutualism, and analyze the locally stability of the equilibrium point.By constructing Lyapunov function further, we try to analyze the global asymptotical stability of the equilibrium point, and give the corresponding explanations in the view of the evolution of the shoals of fish.*

Keywords: *global asymptotical stability, dynamic model, the unilateral fish cannot survive independently*

I. Introduction

In 1940s, Lotka and Volterra established the theoretical basis of the interspecific competition, and set up the famous Lotka-Volterra model. The survival of a single population under natural conditions is rarely seen, and most of the populations live in a common environment with other populations.[1] Chen Lansun and other ecological mathematician use the method of constructing the Lyapunov function to discuss the problem of the global stability of the positive equilibrium point of the Lotka-Volterra cooperation model.[2]

In the fish ecology system, the relationships between fishes are very complex. Predator-prey, parasites and parasitic, mutual competition and mutual coexistence are four common scenarios. Can the Lotka-Volterra model be used to construct other models about the interaction between fishes? Suppose there are two fish populations, they are A and B, and they meet the following situation: fish A can live alone, but fish B cannot live without A. For the above phenomenon, Will there be a Lotka-Volterra model to discuss the stability of the model of fish school?

Model And Stability Analysis

We take into account a phenomenon that the A fish can survive alone, while fish B can not live alone without A. In the face of this phenomenon, we set up a model of Lotka-Volterra cooperation which belongs to this type. It is efficient to predict the development trend of this kind of fish by establishing the dynamic model. By improving the classical Lotka-Volterra model, a reasonable dynamic model is established for the unilateral fish which cannot survive independently.[3][4]

Assuming that A and B are two schools of fish, they are living in the natural environment of the sea. Without the effects of human beings. Meanwhile we do not consider the effect of self feeding on the number of fish. And when they are in the ocean, the number of fishes obey the law of Logistic.

Let $y_1(t)$ 、 $y_2(t)$ are as the density of fish A, B in turn. Let r_1 、 r_2 are as their growth rates. Let y_1^m 、 y_2^m are as the maximum environmental capacities of the ocean for their individual growth.

$$\begin{cases} \dfrac{dy_1}{dt} = r_1 y_1\left(1 - \dfrac{y_1}{y_1^m} + \dfrac{b_1 y_2}{y_2^m}\right) \\ \dfrac{dy_2}{dt} = r_2 y_2\left(-1 - \dfrac{y_2}{y_2^m} + \dfrac{b_2 y_1}{y_1^m}\right) \end{cases} \quad (1)$$

Among them: b_1 refers that the amount of food which unit quantity of the School of fish B provide to the fish A is b_1 times than the amount of food which unit quantity of the School of fish A provide to the fish B . Similarly, among them : b_2 refers that the amount of food which unit quantity of the School of fish A provide to the fish B is b_2 times than the amount of food which unit quantity of the School of fish B provide to the fish A .

Equation of the model (1)

$$f_1 = 0;\ f_2 = 0 \quad (2)$$

System (1) admits three nonnegative equilibrium point:

$S_1(0,0)$， $S_2(y_1^m,0)$， $S_3(y_1^m(1+b_1)/(1-b_1b_2), y_2^m(1+b_2)/(1-b_1b_2))$

In order to facilitate, let $x_0 = y_1^m$， $x_1 = y_1^m(1+b_1)/(1-b_1b_2)$， $x_2 = y_2^m(1+b_2)/(1-b_1b_2)$.

For the model (1) , we obtain the derivative (3) .[2][5]

$$\begin{cases} \dfrac{\partial f_1}{\partial y_1} = a_{11} = r_1 - 2r_1y_1/y_1^m + r_1b_1y_2/y_2^m, \\ \dfrac{\partial f_1}{\partial y_2} = a_{12} = r_1b_1y_1/y_2^m, \\ \dfrac{\partial f_2}{\partial y_1} = a_{21} = r_2b_2y_2/y_1^m, \\ \dfrac{\partial f_2}{\partial y_2} = a_{22} = -r_2 - 2r_2y_2/y_2^m + r_2b_2y_1/y_1^m. \end{cases} \quad (3)$$

For equilibrium point $S_1(0,0)$， From (3) we know that

$$a_{11} = r_1; a_{12} = 0; a_{21} = 0; a_{22} = -r_2$$

Characteristic root equation

$$\begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{vmatrix} = 0, \text{ we have } \begin{vmatrix} r_1-\lambda & a_{12} \\ a_{21} & r_2-\lambda \end{vmatrix} = 0,$$

The solution of the equation is $\lambda_1 = r_1 > 0; \lambda_2 = -r_2 < 0$ .We not only get equilibrium point S_1 is unstable, but also get equilibrium point S_2 is locally stable if $b_1 < 1, b_2 > 1, b_1b_2 < 1$ .Equilibrium point S_3 is locally stable if $b_1 < 1, b_2 > 1, b_1b_2 < 1$.

We discussed global asymptotical stability of the corresponding nonnegative equilibrium. For equilibrium point S_2， we discuss equilibrium point S_2 whether global asymptotical stability.

We construct the following Lyapunov function[6]

$$V_1 = \alpha(y_1 - x_0 - x_0(\ln y_1 - \ln x_0)) + \beta y_2 \quad (4)$$

where α, β are positive constants determined below.

For (1) , we calculating the derivative, we have

$$\frac{dV_1}{dt} = -(y_1 - x_0, y_2)C_1\begin{pmatrix} y_1 - x_0 \\ y_2 \end{pmatrix} + \beta r_2 y_2\left(\frac{b_2x_0}{y_1^m} - 1\right) \quad (5)$$

where
$$C_1 = \begin{pmatrix} \dfrac{r_1\alpha}{y_1^m} & -\dfrac{1}{2}\left(\dfrac{\alpha r_1b_1}{y_2^m} + \dfrac{\beta r_2b_2}{y_1^m}\right) \\ -\dfrac{1}{2}\left(\dfrac{\alpha r_1b_1}{y_2^m} + \dfrac{\beta r_2b_2}{y_1^m}\right) & \dfrac{r_2\beta}{y_2^m} \end{pmatrix}$$

When $b_2 < 1$， we get $\beta r_2x_2\left(\dfrac{b_2x_0}{N_1} - 1\right) < 0$.

We show that there exist suitable constants α, β such that the matrix

$$|C_1| = \frac{\alpha\beta r_1r_2}{y_1^m y_2^m}(1 - b_1b_2) - \frac{1}{4}\left(\frac{\alpha r_1b_1}{y_2^m} - \frac{\beta r_2b_2}{y_1^m}\right)^2 \quad (6)$$

Also,

$$\frac{r_1\alpha}{y_1^m} > 0 \quad (7)$$

One could choose $\alpha = \dfrac{y_2^m}{r_1b_1}, \beta = \dfrac{y_1^m}{r_2b_2}$， when $b_1b_2 < 1$， for any $y_1, y_2 > 0$,we have $\dfrac{dV_1}{dt} < 0$,except the boundary equilibrium point S_2 ,where $\dfrac{dV_1}{dt} = 0$. So, equilibrium point S_2 is global asymptotical stability if $b_1 < 1, b_2 > 1$， $b_1b_2 < 1$.

For equilibrium point S_3 , we discuss whether global asymptotical stability.

We construct the following Lyapunov function

$$V_1 = \alpha(y_1 - x_1 - x_1(\ln y_1 - \ln x_1)) + \beta(y_2 - x_2 - x_2(\ln y_2 - \ln x_2)) \qquad (8)$$

where $\alpha = \frac{y_2^m}{r_1 b_1}, \beta = \frac{y_1^m}{r_2 b_2}$.

For (1) , we calculating the derivative, we have

$$\frac{dV_2}{dt} = -(y_1 - x_1, y_2 - x_2)C_2\begin{pmatrix} y_1 - x_1 \\ y_2 - x_2 \end{pmatrix} \qquad (9)$$

where $$C_2 = \begin{pmatrix} \frac{r_1\alpha}{y_1^m} & -\frac{1}{2}(\frac{\alpha r_1 b_1}{y_2^m} + \frac{\beta r_2 b_2}{y_1^m}) \\ -\frac{1}{2}(\frac{\alpha r_1 b_1}{y_2^m} + \frac{\beta r_2 b_2}{y_1^m}) & \frac{r_2\beta}{y_2^m} \end{pmatrix}$$

From (6) and (7) we know that C_2 is positive definite. When $b_1 b_2 < 1$, for any $y_1, y_2 > 0$, we have $\frac{dV_1}{dt} < 0$, except the boundary equilibrium point S_3, where $\frac{dV_1}{dt} = 0$. So, equilibrium point S_3 is global asymptotical stability if $b_1 < 1$, $b_2 > 1$, $b_1 b_2 < 1$.

II. Conclusion

1. Equilibrium point S_1 is unstable. Two species of fish will eventually die.

2. Equilibrium point S_2 is global asymptotical stability if $b_1 < 1, b_2 > 1, b_1 b_2 < 1$. The fish A will eventually stable, the fish B will eventually stable.

3. Equilibrium point S_3 is global asymptotical stability if $b_1 < 1, b_2 > 1, b_1 b_2 < 1$. Two species of fish tend to be stable.

Acknowledgements

This research was financially supported by the National Science Foundation of Zhejiang Province (LY12A01010) and by the College Students' Scientific and Technological Innovation of Zhejiang Province (2015R411035).

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