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Linear and Non-Linear Stability Analyses of Thermal Convection in a Sparsely Packed Anisotropic Porous Medium with Non-Inertial Acceleration

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Abstract: *Linear and non-linear analyses of thermal convection in a sparsely packed porous medium with the external constraint of rotation are studied. The condition for stationary and oscillatory modes in the case of linear theory is obtained using the normal mode analysis. The non-linear analysis is based on the truncated representation of Fourier series. The influence of various parameters on the onset of convection has been analyzed. Using the non-linear theory, the thermal Nusselt number is calculated for different values of Rayleigh number and other parameters arising in the problem. New results in the realm of nonlinear convection are also discussed. A discussion is also made of low-porosity medium results for constant viscosity liquids.*
Key-words: *convection, porous medium, anisotropy, stability, heat transport.*

I. Introduction

Thermal convection in a rotating porous medium is a phenomenon relevant to many fields. Many authors have investigated the effect of external constraint like rotation and magnetic field on convection in a porous medium. (Rudraiah and Rohini, 1975; Rudraiah and Srimani, 1976; Rudraiah and Vortmeyer, 1978; Patil and Vaidyanathan, 1983; Friedrich, 1983; Palm and Tyvand, 1984; Rudraiah, 1984; Rudraiah and Chandna, 1985; Jou and Liaw, 1987; Vadasz, 1993; 1994; 1997; 1998a; 1998b; 2000; Qin and Kaloni, 1995; Vadasz and Olek, 1998; Straughan, 2000; Govender and Vadasz, 2002; Riahi, 2003; 2006). Most of the above investigators have studied onset of convection in a low-porosity, rotating, isotropic porous medium with constant viscosity. Vadasz (1998b), Qin and Kaloni (1995) and Straughan (2000) have performed a non-linear stability analysis of convection in a low-porosity, rotating, isotropic porous medium with constant viscosity. The object of this paper is to study the effect of non-inertial acceleration on heat transport in a high-porosity, anisotropic porous medium occupied by a Boussinesq fluid with constant viscosity. This is a first step to the more general non-linear problem involving variable viscosity.

II. Mathematical formulation

Consider a horizontal anisotropic porous layer of infinite extent occupied by a Boussinesquian fluid, confined between stress free isothermal boundaries at *z*= 0 and z= d, at which the temperatures are T_0 and T_1 respectively, which is kept rotating at constant rate (see fig.1). Let Ω denote the angular velocity of rotation. The porous medium is assumed to have high porosity and hence the fluid flow is governed by Brinkmann model with effect of Coriolis force and centrifugal acceleration. An appropriate single-phase heat transport equation is chosen with effective heat capacity ratio and effective thermal diffusivity. Thus the governing equations for the Rayleigh-Bénard situation in a Boussinesquian fluid with a rotating anisotropic porous layer are:

Conservation of mass

$$\nabla.\vec{q} = 0 \tag{1}$$

Conservation of linear momentum

$$\rho_R\left[\frac{1}{\Phi}\frac{\partial \vec{q}}{\partial t} + \frac{1}{\Phi^2}\left(\vec{q}.\nabla\right)\vec{q}\right] = -\nabla p + \rho\vec{g} - \mu_f \boldsymbol{k}.\vec{q} + \mu_e \nabla^2 \vec{q} + 2\frac{\rho_R}{\Phi}\left(\vec{q}\times\vec{\Omega}\right), \tag{2}$$

Conservation of energy

$$\gamma\frac{\partial T}{\partial t} + \vec{q}.\nabla T = \chi_{Tv}\left[\eta_1\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\partial^2 T}{\partial z^2}\right] \tag{3}$$

Equation of state

$$\rho = \rho_R\left[1-\alpha\left(T-T_0\right)\right]. \tag{4}$$

2.1 Basic state

The aim of this paper is to investigate the stability of a quiescent state to infinitesimal perturbations superposed on the basic state. The basic state of the liquid being quiescent is described by

$$\frac{\partial(\,)}{\partial t}=0,\ \vec{q}_b=(0,0,0),\ T=T_b(z),\ \rho=\rho_b(z). \tag{5}$$

The temperature T_b, pressure p_b and density ρ_b satisfy

$$\frac{dp_b}{dz}=-\rho_b g,$$

$$\frac{d^2T_b}{dz^2}=0,$$

$$\rho_b=\rho_R\left[1-\alpha(T_b-T_0)\right], \tag{6a, b, c}$$

We now superpose infinitesimal perturbations on the quiescent basic state and study the stability of the system.

2.2. Perturbed state

Let the basic state be disturbed by an infinitesimal thermal perturbation. We now have

$$\vec{q}=\vec{q}_b+\vec{q}\,',\ T_b=T_b(z)+T',\ \ p_b=p_b(z)+p',\ \rho_b=\rho_b(z)+\rho'. \tag{7}$$

The prime indicates that the quantities are infinitesimal perturbations.
Substituting equation (7) into equations (1) to (4) and using the equations (6a, b, c), we get

$$\nabla.\vec{q}\,'=0, \tag{8}$$

$$\rho_R\left[\frac{1}{\Phi}\frac{\partial\vec{q}\,'}{\partial t}+\frac{1}{\Phi^2}(\vec{q}\,'.\nabla)\vec{q}\,'\right]=-\nabla p' + \rho'\vec{g}-\mu_f\,\boldsymbol{k}.\vec{q}\,'+\mu_e\nabla^2\vec{q}\,'+2\frac{\rho_R}{\Phi}\left(\vec{q}\,'\times\vec{\Omega}\right), \tag{9}$$

$$\gamma\frac{\partial T'}{\partial t}+\vec{q}\,'.\nabla T'=-\beta_1 w'+\chi_{Tv}\left[\eta\frac{\partial^2T'}{\partial x^2}+\frac{\partial^2T'}{\partial z^2}\right], \tag{10}$$

$$\rho'=-\alpha\rho_R T'. \tag{11}$$

We consider only two-dimensional disturbances and thus restrict ourselves to the *xz*- plane. We can now introduce a stream function

$$u'=\frac{\partial\psi'}{\partial z},\quad w'=-\frac{\partial\psi'}{\partial x} \tag{12}$$

which satisfies the continuity equation (8).

Operating curl twice on equation (9), to eliminate pressure and introducing the stream function ψ and non-dimensionalizing the resulting equation as well as equation (10) and making use of equation (11), using the following definition:

$$(x^*,z^*)=\left(\frac{x}{d},\frac{y}{d}\right),v^*=\frac{\chi_{Tv}}{d^2}v',T^*=\frac{T'}{\beta_1 d},t^*=\frac{\chi_{Tv}}{d^2}t,\Phi^*=\frac{\Phi}{\chi_{Tv}},\psi^*=\frac{\psi'}{\chi_{Tv}}. \tag{13}$$

we get the dimensionless equations in the form

$$\frac{\partial}{\partial t}(\nabla^2\psi)=\Lambda Pr\nabla^4\psi-PrDa^{-1}\left[\frac{\partial^2\psi}{\partial x^2}+\frac{1}{\varepsilon}\frac{\partial^2\psi}{\partial z^2}\right]+Pr\sqrt{Ta}\frac{\partial v}{\partial z}-RPr\frac{\partial T}{\partial x}+\frac{\partial(\psi,\nabla^2\psi)}{\partial(x,z)}, \tag{14}$$

$$\frac{\partial v}{\partial t}=\Lambda Pr\nabla^2 v-Pr\sqrt{Ta}\frac{\partial\psi}{\partial z}+\frac{\partial(\psi,v)}{\partial(x,z)}, \tag{15}$$

$$\gamma\frac{\partial T}{\partial t}=-\frac{\partial\psi}{\partial x}+\eta\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\partial(\psi,T)}{\partial(x,z)}, \tag{16}$$

where the asterisks have been dropped for simplicity, the non-dimensional parameters $R, Pr, Ta, Da^{-1}, \Lambda, \varepsilon$ and η are as defined below:

$R=\alpha g\Delta T d^3/\chi$ (Rayleigh number), $\Pr=\nu/\chi$ (Prandtl number), $Ta=-$ (Taylor number), $Da^{-1}=d^2/k_v$ (Inverse Darcy number), $\Lambda=\mu_p/\mu_f$ (Brinkmann number), $\varepsilon=k_h/k_v$ (Mechanical anisotropy parameter) and

$\eta=\chi_h/\chi_v$ (Thermal anisotropy parameter).

The equations (14) to (16) are solved for free-free isothermal boundary conditions

$$\psi=\nabla^2\psi=\frac{\partial v}{\partial z}=T=0 \text{ at } z=0,1. \tag{17}$$

III. Linear stability analysis

In this section, we discuss the linear stability analysis considering both stationary and overstable states. The solution of this analysis is of great utility in the local non-linear stability analysis discussed in the next section. To make this study we neglect the Jacobians in equations (14) to (16) and assume the solutions to be periodic waves (see Chandrasekhar, 1961) of the form

$$-\pi Pr\sqrt{Ta}\,A+\pi^2 aAE-\Lambda PrK^2 D=0, \tag{18}$$

Substituting equation (18) in the linearized version of equations (14) to (16), we get

$$\left[\left(\sigma+\Lambda Pr\delta^2\right)\delta^2+PrDa^{-1}\delta_1^2\right]\psi_0+\left[\pi aPrR\right]T_0-\left[\pi a\Pr Ta\right]v_0=0, \tag{19}$$

$$\left[Pr\sqrt{Ta}\pi\right]\psi_0+\left[\sigma+\Lambda Pr\delta^2\right]v_0=0, \tag{20}$$

$$\left[\pi a\right]\psi_0+\left[\sigma+(1+\eta a^2)\pi^2\right]T_0=0, \tag{21}$$

where $\delta^2=\pi^2(1+a^2),\ \delta_1^2=\pi^2\left(\frac{1}{\varepsilon}+a^2\right).$

For a non-trivial solution of the homogeneous system of equations (19) to (21) for ψ_0, v_0 and T_0, we require

$$R=\frac{\left[\left\{\left(\sigma+\Lambda Pr\delta^2\right)K^2+Da^{-1}\delta_1^2\right\}\left(\sigma+\Lambda Pr\delta^2\right)+TaPr^2\pi^2\right]\left\{\sigma+(1+\eta a^2)\pi^2\right\}}{Pr\pi^2a^2\left(\sigma+\Lambda Pr\delta^2\right)}. \tag{22}$$

3.1. Stationary state

If σ is real, then stationary stability occurs when $\sigma=0$. This gives the stationary Rayleigh number R in the form

$$R^s=\frac{\left\{\left(\Lambda\delta^4+Da^{-1}\delta_1^2\right)\Lambda\delta^2+Ta\pi^2\right\}\left(1+\eta a^2\right)}{\Lambda\delta^2a^2}. \tag{23}$$

The critical wave number a_c satisfies the equation

$$G_1a_c^{10}+G_2a_c^8+G_3a_c^6+G_4a_c^4+G_5a_c^2+G_6=0, \tag{24}$$

where

$$G_1 = 2\eta\Lambda^2\pi^4,\ G_2 = \Lambda^2\pi^4(1+6\eta) + \Lambda\eta Da^{-1}\pi^2,$$
$$G_3 = \Lambda^2\pi^4(2+6\eta) + 2\Lambda\eta Da^{-1}\pi^2,$$
$$G_4 = 2\eta\Lambda^2\pi^4 + \Lambda Da^{-1}\pi^2\left(\eta - \frac{1}{\varepsilon}\right),$$
$$G_5 = -2\left(\Lambda^2\pi^4 + \frac{\Lambda Da^{-1}\pi^2}{\varepsilon} + Ta\right), G_6 = \frac{G_5}{2}.$$

The critical wave number a_c depends on $Da^{-1}, \Lambda, \varepsilon, \eta$ and Ta. When $\gamma = 1$, $Da^{-1} = 0, Ta = 0, \Lambda = \varepsilon = \eta = 1$, we get the classical result $a_c^2 = 0.5$ and $R_c^s = 657.5$ for clean fluids and for very small values of Da^{-1} we get $a_c = \left(\sqrt{\eta\varepsilon}\right)^{-1}$, $R_c^s = \pi^2\left(1+\sqrt{\eta\varepsilon}\right)^2$, the classical result of Epherre (1977), from which the classical Lapwood(1948) result follows for isotropic porous media. In the presence of rotation a_c and R_c^s are given by $a_c = \left(\sqrt{\eta\varepsilon}\right)^{-1}\left[1+Ta\varepsilon^{-2}\right]$, $R_c^s = \pi^2\left(1+\sqrt{\eta\varepsilon\left(\varepsilon^{-2}+Ta\right)}\right)^2$, and the corresponding result for isotropic porous media (the classical result of Vadasz, 1998) is obtained by taking $\varepsilon = \eta = 1$.

3.2. Oscillatory motions

We put $\sigma = i\omega$ (ω : real) in equation (22) and rearranging we get the Rayleigh number R of marginal stability in the form

$$R = \frac{\left[\begin{array}{l}\left\{\left(\Lambda\delta^4 + Da^{-1}\delta_1^2\right)\Lambda\delta^2 + \pi^2 Ta\right\}\left\{\Lambda Pr^3\pi^2\delta^2\left(1+\eta a^2\right)\right\} \\ \qquad + \omega^2\left\{Pr\pi^2\left(1+\eta a^2\right)\left(\Lambda\delta^4 + Da^{-1}\delta_1^2\right) + Pr^2\left(\pi^2 Ta - \Lambda^2\delta^6\right) - \omega^2\delta^2\right\}\end{array}\right] + i\omega N}{Pr\pi^2 a^2\left(\Lambda^2 Pr^2\delta^4 + \omega^2\right)}, \quad (25)$$

where

$$N = \left[\left(Pr^2\left\{\left(\Lambda\delta^4 + Da^{-1}\delta_1^2\right)\Lambda\delta^2 + \pi^2 Ta\right\} - \omega^2\delta^2\right)\left\{\Lambda Pr\delta^2 - \pi^2\left(1+\eta a^2\right)\right\}\right]$$
$$+\left[\left\{Pr\left(\Lambda\delta^4 + Da^{-1}\delta_1^2\right) + \Lambda\delta^4\right\}\left\{\omega^2 + \Lambda Pr\pi^2\delta^2\left(1+\eta a^2\right)\right\}\right]. \quad (26)$$

Since R is a real quantity, either $\omega = 0$ (stationary) or $N = 0$ ($\omega \neq 0$, oscillatory). The latter condition yields

$$\omega^2 = -\frac{Pr^2\left[\Lambda^2\delta^4 Pr\left(\Lambda\delta^4 + Da^{-1}\delta_1^2\right) + \pi^2\left\{\Lambda PrTa\delta^2 + \left(1+\eta a^2\right)\left(\Lambda^2\delta^6 - Ta\pi^2\right)\right\}\right]}{Pr\left(\Lambda\delta^4 + Da^{-1}\delta_1^2\right) + \delta^2\pi^2\left(1+\eta a^2\right)}. \quad (27)$$

Oscillatory Rayleigh number R^o is now given by

$$R^o = \frac{\left[\begin{array}{l}\left\{\left(\Lambda\delta^4 + Da^{-1}\delta_1^2\right)\Lambda\delta^2 + \pi^2 Ta\right\}\left\{\Lambda Pr^3\pi^2\delta^2\left(1+\eta a^2\right)\right\} \\ \qquad + \omega^2\left\{Pr\pi^2\left(1+\eta a^2\right)\left(\Lambda\delta^4 + Da^{-1}\delta_1^2\right) + Pr^2\left(\pi^2 Ta - \Lambda^2\delta^6\right) - \omega^2\delta^2\right\}\end{array}\right]}{Pr\pi^2 a^2\left(\Lambda^2 Pr^2\delta^4 + \omega^2\right)}, \quad (28)$$

and is minimized with respect to the wave number to obtain the critical value of R^o.

In the next section we perform a non-linear stability analysis and obtain the finite amplitude Rayleigh number, and also quantify the heat transfer by conduction and convection and see the effect of anisotropic parameters.

IV. Non-linear theory

The finite-amplitude analysis is carried out here via a double Fourier series representation for the stream function ψ, v and temperature T in the form

$$\psi = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}(t)\, sin(m\pi ax)\, sin(n\pi z), \tag{29}$$

$$v = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}(t)\, sin(m\pi ax)\, cos(n\pi z). \tag{30}$$

$$T = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} B_{mn}(t)\, cos(m\pi ax)\, sin(n\pi z), \tag{31}$$

Substituting equations (29) to (31) into the set of three coupled non-linear partial differential equations (14) to (16) we get a system of coupled, non-linear ordinary differential equations. Once a set of partial differential equations has been converted to a system of ordinary differential equations via a Fourier series it is logical to use the observed fact that laboratory and physical system often exhibit flows dominated by a few spatial harmonics to truncate the system as for as possible. The primary advantage of creating these truncated spectral models is that their steady states and temporally periodic solutions can be obtained analytically in many cases. Although the relationship between the solutions of the governing partial differential equations and the corresponding severely truncated ordinary differential system has not been established, these low-order spectral models may reproduce qualitatively, the convection phenomena observed in the full system. This allows one to choose a minimal representation from the above Fourier series. The results from such a simple analysis also serve as a starting value in solving a general non-linear convection problem.

The first effect of non-linearity is to disort the temperature field through the interaction of ψ and T and also v and T. The distortion of the temperature field will correspond to a change in the horizontal mean, i.e. a component of the form $sin(2\pi z)$ will be generated. Thus a truncated system which describes the finite amplitude free convection is given by (Veronis, 1959).

$$\psi = A(t)\, sin(\pi ax)\, sin(\pi z), \tag{32}$$

$$v = D(t)\, sin(\pi ax)\, cos(\pi z) + E(t)\, sin(2\pi ax). \tag{33}$$

$$T = B(t)\, cos(\pi ax)\, sin(\pi z) + C(t)\, sin(2\pi z), \tag{34}$$

where the amplitudes A, B, C, D, and E are to be determined from the dynamics of the system. The function ψ does not contain an x-independent term because the spontaneous generation of large scale flow has been discounted.

Substituting equations (32) to (34) into equations (14) to (16) and equating the coefficients of like terms we obtain the following non-linear autonomous system (generalized Lorenz model) of differential equations:

$$\dot{A} = \frac{-\left(\Lambda\delta^4 + Da^{-1}\delta_1^2\right)Pr}{\delta^2}A - \frac{RPr\pi a}{\delta^2}B + \frac{Pr\pi\sqrt{Ta}}{\delta^2}D, \tag{35}$$

$$\dot{B} = -\pi aA - \pi^2\left(1+\eta a^2\right)B - \pi^2 aAC, \tag{36}$$

$$\dot{C} = \frac{\pi^2 a}{2}AB - 4\pi^2 C, \tag{37}$$

$$\dot{D} = -Pr\pi\sqrt{Ta}\,A + \pi^2 aAE - \Lambda Pr\pi^2 D, \tag{38}$$

$$\dot{E} = -\frac{\pi^2 a}{2}AD - 4\Lambda Pr\pi^2 a^2 E, \tag{39}$$

where the over dot denotes the time derivative.

The non-linear system of autonomous differential equations is not amenable to analytical treatment for the general time-dependent variable and we have to solve using a numerical method. However, one can make qualitative predictions as discussed below. The generalized Lorenz model is uniformly bounded in time and possesses many properties of the full problem. Also the phase-space volume contracts at a uniform rate given by

$$\frac{\partial\dot{A}}{\partial A}+\frac{\partial\dot{B}}{\partial B}+\frac{\partial\dot{C}}{\partial C}+\frac{\partial\dot{D}}{\partial D}+\frac{\partial\dot{E}}{\partial E}=\frac{-\left(\Lambda\delta^4+Da^{-1}\delta_1^2\right)Pr}{\delta^2}-\pi^2\left(1+\eta\alpha^2\right)-4\pi^2-\Lambda Pr\delta^2-4\Lambda Pr\pi^2a^2, \tag{40}$$

which is always negative and therefore the system is bounded and dissipative. As a result, the trajectories are attracted to a set of measure zero in the phase-space; in particular they may be attracted to a fixed point, a limit cycle or perhaps, a strange attractor.

From qualitative predictions we now look into the possibility of an analytical solution. In the case of steady motions, equations (35) to (39) can be solved in closed form. Setting the left hand sides of equations (35) to (39) equal to zero, we get

$$\left(\Lambda\delta^4+Da^{-1}\delta_1^2\right)A+R\pi aB-\pi\sqrt{Ta}D=0, \tag{41}$$

$$\pi aA+\pi^2aAC+\pi^2\left(1+\eta a^2\right)B=0, \tag{42}$$

$$aAB-8C=0, \tag{43}$$

$$-\pi Pr\sqrt{Ta}A+\pi^2aAE-\Lambda PrK^2D=0, \tag{44}$$

$$AD+8\Lambda PraE=0. \tag{45}$$

Solving for *B*, *C*, *D* and *E* in terms of *A*, we get

$$B=-\frac{8aA}{\pi\left[8\left(1+\eta a^2\right)+a^2A^2\right]}, \tag{46}$$

$$C=-\frac{a^2A^2}{\pi\left[8\left(1+\eta a^2\right)+a^2A^2\right]}, \tag{47}$$

$$D=-\frac{8\Lambda\pi Pr^2\sqrt{Ta}A}{8\Lambda^2Pr^2K^2+\pi^2A^2}, \tag{48}$$

and

$$E=-\frac{\pi Pr\sqrt{Ta}A^2}{a\left[8\Lambda^2Pr^2\delta^2+\pi^2A^2\right]}, \tag{49}$$

Substituting *B* and *D* from equations (46) and (48) in (41) and writing in terms of *A*, we get

$$\left[\begin{array}{l}\left(\Lambda\delta^4+Da^{-1}\delta_1^2\right)\left(8\Lambda^2Pr^2\delta^2+\pi^2A^2\right)\left\{8\left(1+\eta a^2\right)+\pi^2a^2A^2\right\}\\ \quad -8R\pi^2a^2\left(8\Lambda^2Pr^2\delta^2+\pi^2A^2\right)+8\Lambda Ta\pi^2Pr^2\left\{8\left(1+\eta a^2\right)+\pi^2a^2A^2\right\}\end{array}\right]A=0 \tag{50}$$

The solution *A* = 0 corresponds to pure conduction which we know to be a possible solution though it is unstable when *R* is sufficiently large. The remaining solutions are given by

$$F_1\left(\frac{A^2}{8}\right)^2+F_2\left(\frac{A^2}{8}\right)+F_3=0, \tag{51}$$

Where

$$F_1=-\left(\Lambda\delta^4+Da^{-1}\delta_1^2\right)\pi^4a^2,$$

$$F_2=a^2\left[\left(\Lambda\delta^4+Da^{-1}\delta_1^2\right)\left\{\Lambda^2a^2Pr^2\delta^2+\pi^2\left(1+\eta_1a^2\right)\right\}-\pi^2a^2R+\Lambda Ta\pi^2a^2Pr^2\right],$$

$$F_3=\Lambda\pi^2Pr^2\left[\Lambda\delta^2\left\{\left(\Lambda\delta^4+Da^{-1}\delta_1^2\right)\left(1+\eta_1a^2\right)-a^2R\right\}+Ta\pi^2\left(1+\eta_1a^2\right)\right].$$

The finite amplitude Rayleigh number is obtained by solving the discriminant of equation (51) and is obtained in the form:

$$R^{f}=\frac{1}{\pi^{2}a^{2}}\left[\begin{array}{c}\pi^{2}\left(1+\eta a^{2}\right)\left(Da^{-1}\delta_{1}^{2}+\delta^{4}\Lambda\right)+a^{2}Pr^{2}\Lambda\left\{\pi^{2}Ta-\delta^{2}\Lambda\left(Da^{-1}\delta_{1}^{2}+\delta^{4}\Lambda\right)\right\}\\ -2\pi aPr^{2}\Lambda\sqrt{\Lambda Ta\left(Da^{-1}\delta_{1}^{2}+\delta^{4}\Lambda\right)\left\{\pi^{2}\left(1+\eta a^{2}\right)-K^{2}Pr^{2}a^{2}\Lambda^{2}\right\}}\end{array}\right] \quad (52)$$

V. Heat transport

In the study of convection problems the determination of heat transport across the fluid layer is important. This is because; the onset of convection as Rayleigh number is increased is more readily detected by its effect on the heat transfer. In the basic state, the heat transfer is by conduction alone.
If H_T is the rate of heat transfer / unit area, then

$$H_T=-\chi\left\langle\frac{\partial T_{total}}{\partial z}\right\rangle_{z=0}, \quad (53)$$

where the angular bracket corresponds to a horizontal average and

$$T_{total}=\left[T_0-\frac{\Delta T}{d}z\right]+T(x,z,t). \quad (54$$

The first term of the RHS of equation (54) is the temperature distribution of conduction state prevalent before convection sets in. The second term on the RHS of equation (54) represents the convective contribution to heat transport.
Substituting equation (34) in equation (6b) and using the resultant equation in the equation (53), we get

$$H_T=\frac{\chi\Delta T}{d}-\frac{\chi\Delta T}{d}2\pi C. \quad (55)$$

The Nusselt number *Nu* is defined by

$$Nu=\frac{H_T}{\left(\chi\Delta T/d\right)}=1-2\pi C. \quad (56)$$

Alternately, *Nu* may be directly defined in terms of the non-dimensional quantities as follows:

$$Nu_f=\frac{\left[\frac{a_c}{2\pi}\int_0^{2\pi/k}(1-z+T)_z\,dx\right]_{z=0}}{\left[\frac{a_c}{2\pi}\int_0^{2\pi/k}(1-z)_z\,dx\right]_{z=0}}=1-2\pi C.$$

Substituting for *C* from equation (47) and then using the solution of equation (51) we can calculate the Nusselt number *Nu* for different values of *R* and other parameters of the problem.

VI. Results and discussions

In the chapter a study is made of the effect of rigid-body rotation on linear and nonlinear convection in a fluid saturated anisotropic porous medium at the onset of convection.
With the motivation of control of convection, the following effects on the classical Rayleigh- Bénard problem are considered:

(i) porous medium inhibition of convection,
(ii) anisotropy of the medium and
(iii) Coriolis force.

These three effects are, respectively, represented by the inverse Darcy number Da^{-1}, anisotropy parameters (ε,η), and the Taylor number *Ta*. The present formulation of the porous media problem for an infinite porous layer with rotation parallel to gravity is based on the Chandrasekhar (1961) formulation of the problem in a clear fluid layer. This formulation involves several assumptions (see Knobloch, 1998)- the lateral boundaries are far enough not to influence rotating convection and that the Froude number is quite small. The latter assumption facilitates the restoration of the conduction state as an equilibrium solution. Experimentally, the lateral boundary effect and the centrifugal effect have been shown by Ecke *et al*. (1992) to be quite

important but in a theoretical study to keep the problem manageable and focus on Benard-like situations, it is common to exclude these effects. The other types of instabilities possible in the case of systems as this is the Kuppers-Lortz instability (Küppers and Lortz, 1969) and Cox-Matthews instability (Cox and Matthews, 2000). In the former instability, convection rolls become unstable to rolls with a different orientation and in the case of the latter it arises at rapid rotation and at infinitesimal angles between two roll orientations. This is beyond the scope of the present chapter. The main emphasis of the present study is to consider the effect of non-inertial acceleration on the onset of convection via the stationary/oscillatory modes of linear instability or the finite amplitude steady mode of instability. Before embarking on a discussion of the results depicted by the figs. 2 to 18, we note that as in the case of clear fluid critical convection is always stationary as overstable motion is restricted to very low values of the Prandtl number, when Da^{-1} is quite large. For low values of Da^{-1}, the critical value is stationary or overstable. Another point to be noted is that all the observations made in the previous chapter on high rotation rates holds good in the case of the present problem as well. We now discuss the results depicted by the figs. 2 to 18. Fig.2, reveals that, the effect of increasing Da^{-1} is to stabilize the system. We further find from the figure subcritical instability exists in the case of this problem. We have observed in the computations that as the rotation rate is increased then its effect on onset of convection is not altered by varying Da^{-1}. This is due to the fact that large rotations do not allow the internal structure of the porous medium to affect convection. Fig.3, reveals that, the effect of increasing Da^{-1} is to dampen the oscillations, at the onset of convection, for all rotation rates. The effect of increasing *Ta* is similar to that of increasing Da^{-1} on the Rayleigh number and opposite on the frequency of oscillations (see figs. 4 and 5). Further, we find that finite amplitude steady convection precedes marginal convection for all values of *Ta*.
The effect of varying ε is more pronounced in the case of finite amplitude steady convection compared to marginal convection and this is seen in fig. 6. Further, for all value of ε, subcritical instability exists. The effect of increasing ε is to destabilize the system. Fig.7, shows that, the effect of increasing ε is to enhance the oscillations. The effect of increasing η on *R* is opposite to that of increasing ε and the same can be seen in fig.8, but the effect of increasing η on the frequency of oscillations is similar to the effect of increasing ε (see fig. 6.9). The effect of increasing Brinkman number Λ is to increase the Rayleigh number and decrease the frequency of oscillations (see figs. 10 and 11 respectively). The effect of increasing *Pr* is, shown in fig.12, to destabilize the system and this is true because of the fact that the rotation rate has been chosen to be high (same as in Chandrasekhar, 1961). The effect of increasing *Pr* on the frequency of oscillations (see fig.13) is to amplify the oscillations. In the plots of the Rayleigh numbers it is obvious that subcritical instability is preferred to marginal stability.

We note at this point that the oscillatory convection curves are below those of the stationary ones because of the choice of small Prandtl numbers in the case of high-porosity media. While calculating the Nusselt number *Nu* a large value of *Pr* is chosen which essentially implies stationary curve is below the oscillatory one and calculations have been done using the stationary Rayleigh number. The results of this calculation are shown in figs. 14 to 18. The reduced-heat-transfer effect of increasing Da^{-1} and *Ta* on *Nu* is shown respectively in figs.14 and 15. The effect of increasing ε and η on *Nu* is shown to be opposite to each other in figs.16 and 17.

The effect of increasing ε is to clearly enhance the heat transfer. The effect of increasing Λ on *Nu* is to enhance the heat transfer (see fig.18).
We conclude the results and discussion with the remark that subcritical instability is not possible in the case of very small Da^{-1}.

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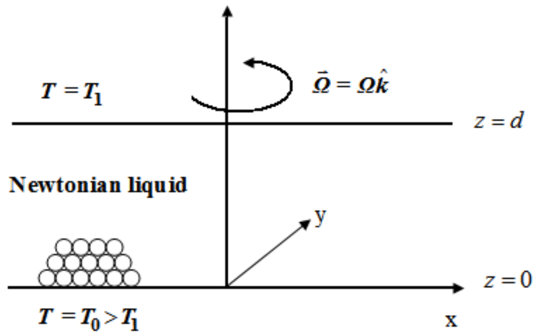

Fig.1 : Physical configuration.

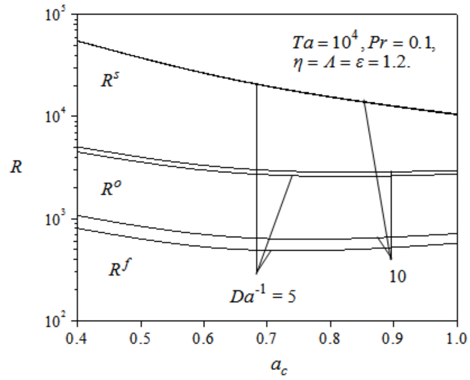

Fig. 2: Rayleigh number R Vs. the critical wave number a_c for different values of porous parameter Da^{-1}.

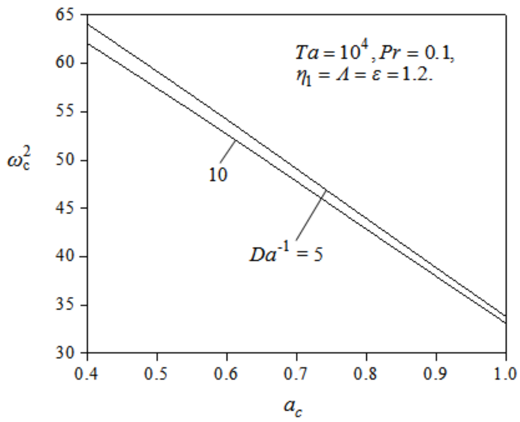

Fig. 3: ω_c^2 Vs. a_c for different values of Da^{-1}.

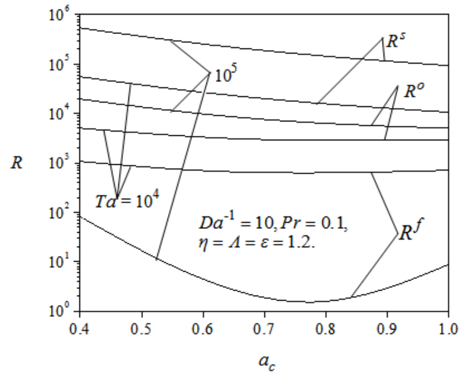

Fig. 4: R Vs. a_c for different values of Taylor number Ta.

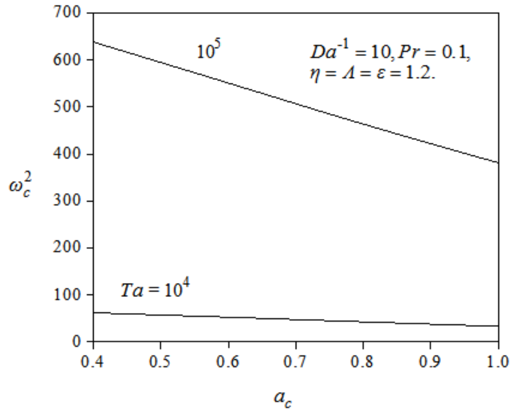

Fig. 5: ω_c^2 Vs. a_c for different values of Ta.

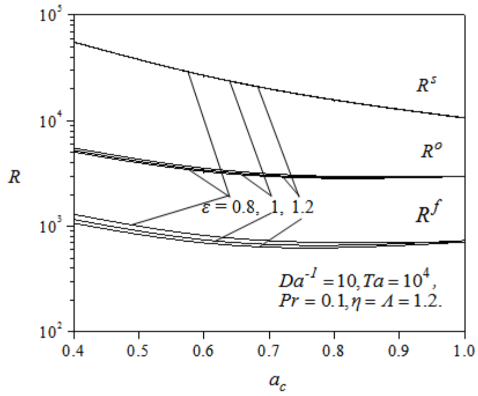

Fig. 6: R Vs. a_c for different values of mechanical anisotropy parameter ε.

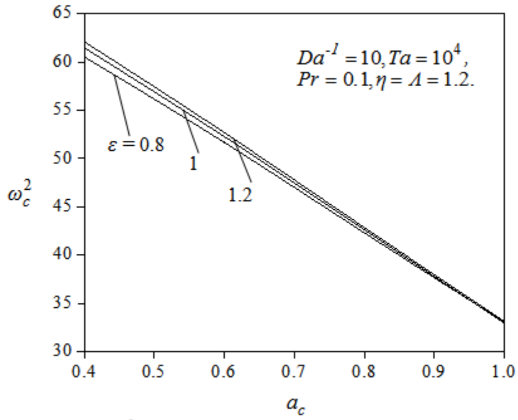

Fig. 7: ω_c^2 Vs. a_c for different values of ε.

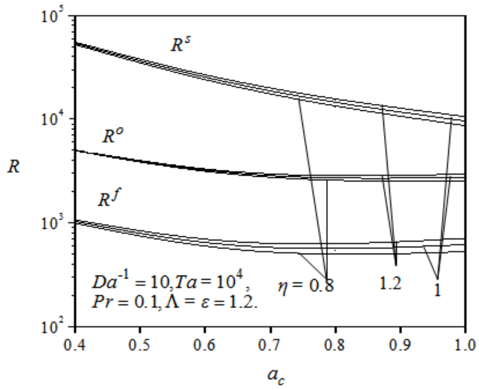

Fig. 8: R Vs. a_c for different values of thermal anisotropy parameter η.

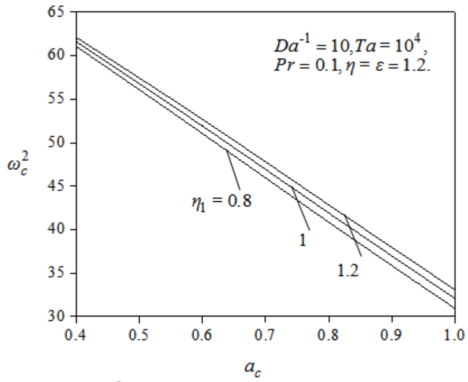

Fig. 9: ω_c^2 Vs. a_c for different values of η.

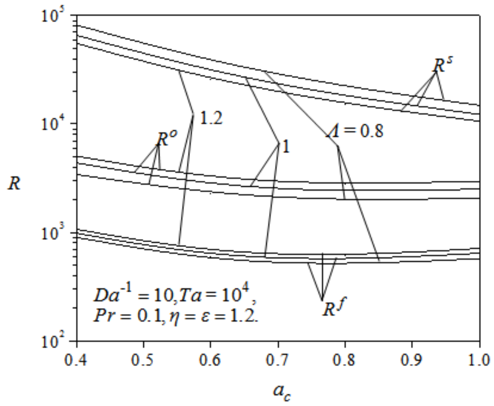

Fig. 10: R Vs. a_c for different values of Brinkmann number Λ.

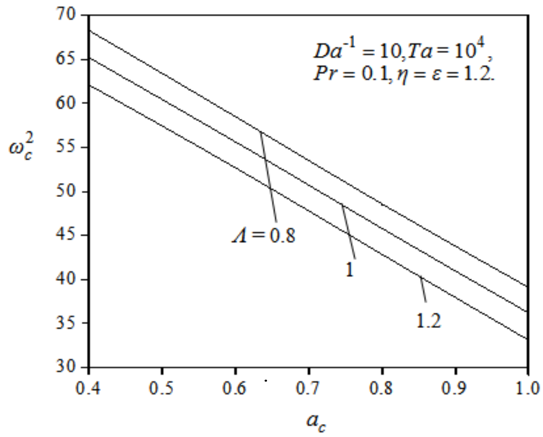

Fig. 11: ω_c^2 **Vs.** a_c **for different values of** Λ.

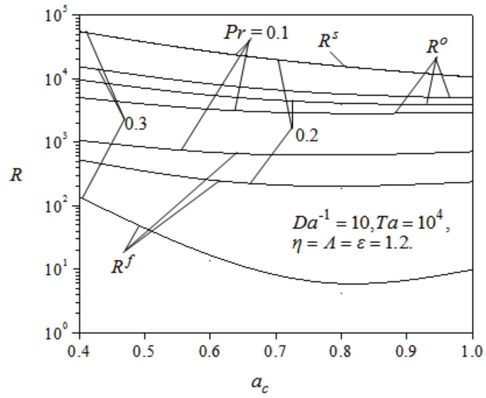

Fig. 12: R **Vs.** a_c **for different values of Prandtl number** Pr.

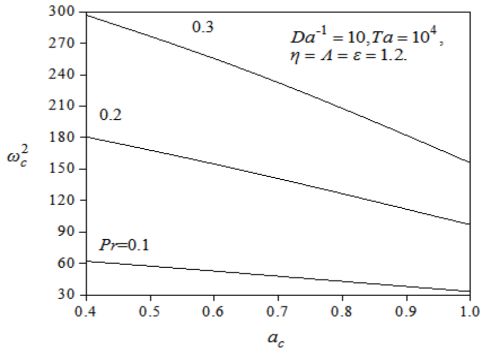

Fig. 13: ω_c^2 **Vs.** a_c **for different values of** Pr.

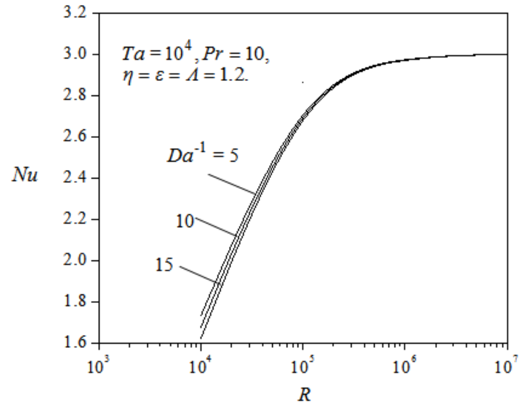

Fig. 14: Nusslet number Nu vs. R for different values of Da^{-1}.

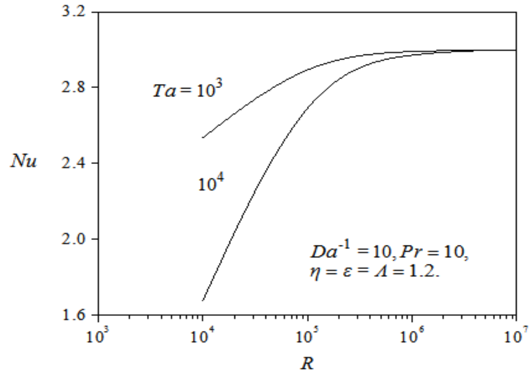

Fig. 15: Nu Vs. R For different values Ta.

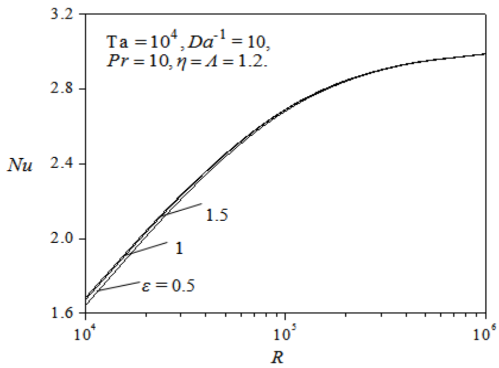

Fig. 16: Nu Vs. R For different values of ε.

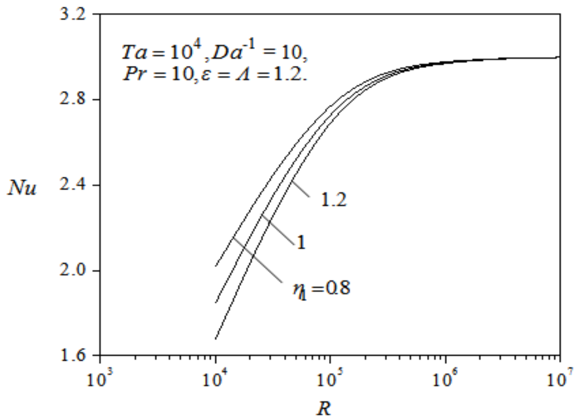

Fig. 17: *Nu* Vs. *R* For different values of η.

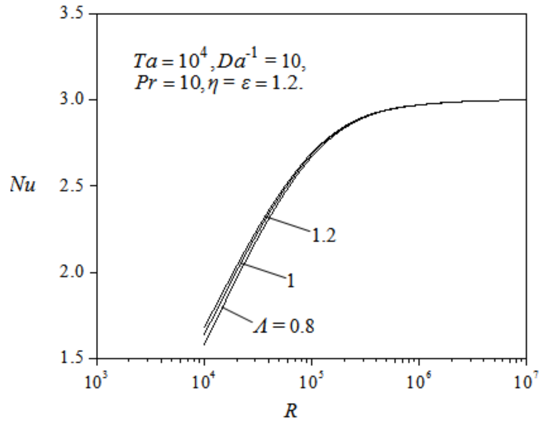

Fig. 18. *Nu* Vs. *R* For different values of Λ.

Bibliography

[1]. Rudraiah, N. and Rohini, G. (1975). Cellular convection in a rotating fluid through porous medium, Vignana Bharathi, India, 4, 23.

[2]. Rudraiah, N. and Srimani, P. K. (1976). Thermal convection in a rotating fluid through a porous medium, Vignana Bharati, India, 2, 11.

[3]. Rudraiah, N. and Vortmeyer, D. (1978). Stability of finite amplitude and overstable convection of conducting fluid through fixed porous bed, Warme-and Stoffibertragung therme and Fluid Dynamics, 11, 241.

[4]. Patil, P. R. and Vaidyanathan, G. (1983). On setting up of convection currents in a rotating porous medium under the influence of variable viscosity, Int. J. Engg. Sci., 21, 123.

[5]. Palm, E and Tyvand, P. A. (1984). Thermal convection in a rotating porous layer, ZAMP, 35, 122.

[6]. Rudraiah, N. (1984). Linear and non-linear magneto-convection in a porous medium. Proc. Indian Academy of Sciences (Mathematical Sciences), 3, Nos. 2 and 3

[7]. Rudraiah, N., and Chandna, O. P. (1985). Effects of Coriolis force and non-uniform temperature gradient on the Rayleigh-Benard convection, Can. J. Phys., 64, 90.

[8]. Straughan, B. (2000). A sharp nonlinear stability threshold in rotating porous convection, Proc. R. Soc. Lond., 457, 87.

[9]. Vadasz, P. (1993). Three dimensional free convection in a long rotating porous box, J. Heat Transfer, 115,639.

[10]. Vadasz, P. (1994): Stability of free convection in a narrow porous layer subject to rotation, Int. Comm. Heat Transfer, 21, 881.

[11]. Vadasz, P. (1996). Stability of free convection in a rotating porous layer distant from the axis of rotation, Transport in porous media, 23, 155.

[12]. Vadasz, P. (1997): Flow in rotating porous media, (Ed. P.Du Plessis, Ch. 4, Southampton), Computational Mechanics Publications.

[13]. Vadasz, P. (1998a). Coriolis effect on gravity-driven convection in a rotating porous layer heated from below, J. Fluid Mech., 376, 351.

[14]. Vadasz, P. (1998b). Free convection in a porous media, Transport Phenomena in Porous Media, Elsevier(Ed. By Ingham, D. B. and Pop, I.).

[15]. Riahi, D. N. (2003). On stationary and oscillatory modes of flow instability in a rotating porous layer during alloy solidification, J. Porous Media, 6,

[16]. Riahi, D. N. (2006). Non linear convection in a rotating mushy layer, J. Fluid Mech., 553, 389.

[17]. Qin, Y. and Kaloni, P. N. (1992). Thermal instability problem in a rotating micropolar fluid, Int. J. Eng. Sci., 30, 1117.

[18]. Vadasz, P. and Olek, S. (1998). Transition and chaos for free convection in a rotating porous layer, Int. J. Heat Mass Transfer, 41, 1417.

[19]. Govender, S. and Vadasz, P. (2002). Moderate time linear study of moderate Stephan number convection in rotating mushy layers, J. Porous Media, 5, 113.

[20]. Jou, J. J. and Liaw, J. S. (1987). Thermal convection in a porous medium subject to transient heating and rotation. Int. J. Heat Mass Transfer, 30, 208.

[21]. Chandrasekhar, S. (1961). Hydrodynamic and Hydromagnetic stability, Oxford University Press, London.

[22]. Veronis, G. (1959). Cellular convection with finite amplitude in a rotating fluid. J. Fluid Mech.,5, 401.
[23]. Ecke, R. E., Zhong, F. and Knobloch, E. S. (1992). Hopf bifurcation with broken reflection symmetry in rotating Rayleigh-Bénard convection, Europhy.Lett., 19, 177.
[24]. Küppers, G. and Lortz, D. 1969. Transition from laminar convection to thermal turbulence in a rotating fluid layer, J. Fluid Mech., 35, 609.
[25]. Cox, S. M. and Matthews, P. C. 2000. Instability of rotating convection, J. Fluid Mech., 403, 153.