*IOSR Journal of Mathematics (IOSR-JM)*
*e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 3 Ver. VI (May. - Jun. 2016), PP 14-20*
*www.iosrjournals.org*

# Strongly Soft G-Closed Sets and Strongly Soft $\partial$-Closed Sets in Soft $\check{C}ech$ Closure Space

R.Gowri[1], G.Jegadeesan[2]
[1]*Department of Mathematics, Govt.College for Women's (A), Kumbakonam- India.*
[2] *Department of Mathematics, Anjalai Ammal Mahalingam College, Kovilvenni-India.*

***Abstract:*** *In this paper, we introduce strongly soft generalized closed sets and strongly soft $\partial$-closed sets in soft $\check{C}ech$ closure spaces, which are defined over an initial universe with a fixed set of parameters. Also, we show that every strongly soft $\partial$-closed set is strongly soft g-closed and studied some of their basic properties.*
***Keywords:*** *Soft set, Strongly soft g-closed set, Strongly soft $\partial$-closed set*

## I. Introduction

Fuzzy sets [1], theory of rough sets [2], theory of vague sets [3], theory of intuitionistic fuzzy sets [4], and theory of interval mathematics [5,6] are the tools, which are dealing with uncertainties. But all these theories have their own difficulties, namely inadequacy of parameterization. In 1999, D. Molodtsov [7] introduced the notion of soft set to deals with inadequacy of parameterization. Later, he applied this theory to several directions [8,9].

Levine [10] introduced generalized closed sets in topological space in order to extend some important properties of closed sets to a large family of sets. For instance, it was shown that compactness, normality and completeness in a uniform space are inherited by g-closed subsets.

E. $\check{C}ech$ [12] introduced the concept of closure spaces. In $\check{C}ech's$ approach the operator satisfies idempotent condition among Kuratowski axioms. This condition need not hold for every set A of X. When this condition is also true, the operator becomes topological closure operator. Thus the concept of closure space is the generalization of a topological space. In 2010, Chawalit Boonpok [11] introduced generalized closed sets in $\check{C}ech$ closure spaces.

R. Gowri and G. Jegadeesan [13,14,15,16] introduced and studied the concept of lower separation axioms, higher separation axioms, soft generalized closed sets and soft $\partial$-closed sets in soft $\check{C}ech$ closure spaces.

In this paper, we introduce the stronger form of soft generalized closed sets and soft $\partial$-closed sets in soft $\check{C}ech$ closure spaces. Also, we investigate some of their basic properties.

## II. Preliminaries

In this section, we recall the basic definitions of soft Čech closure space.

Definition2.1 [12]. Let X be an initial universe set, A be a set of parameters. Then the function $k: P(X_{F_A}) \to P(X_{F_A})$ defined from a soft power set $P(X_{F_A})$ to itself over X is called Č$ech$ Closure operator if it satisfies the following axioms:

(C1) $k(\emptyset_A) = \emptyset_A$ .
(C2) $F_A \subseteq k(F_A)$
(C3) $k(F_A \cup G_A) = k(F_A) \cup k(G_A)$

Then (X, $k$, A) or $(F_A, k)$ is called a soft Č$ech$ closure space.

Definition2.2 [12]. A soft subset $U_A$ of a soft Č$ech$ closure space $(F_A, k)$ is said to be soft $k$-closed (soft closed) if $k(U_A) = U_A$.

Definition 2.3 [12]. A soft subset $U_A$ of a soft Č$ech$ closure space $(F_A, k)$ is said to be soft $k$-open (soft open) if $k({U_A}^C) = {U_A}^C$.

Definition 2.4 [12]. A soft set $Int(U_A)$ with respect to the closure operator $k$ is defined as $Int(U_A) = F_A - k(F_A - U_A) = [k({U_A}^C)]^C$. Here $U_A^C = F_A - U_A$.

Definition 2.5 [12]. A soft subset $U_A$ in a soft Č$ech$ closure space $(F_A, k)$ is called Soft neighbourhood of $e_F$ $if$ $e_F \in Int(U_A)$.

Definition 2.6 [12]. If $(F_A, k)$ be a soft Č$ech$ closure space, then the associate soft topology on $F_A$ is $\tau = \{{U_A}^C : k(U_A) = U_A\}$.

Definition 2.7 [12]. Let $(F_A, k)$ be a soft Č$ech$ closure space. A soft Č$ech$ closure space $(G_A, k^*)$ is called a soft subspace of $(F_A, k)$ if $G_A \subseteq F_A$ and $k^*(U_A) = k(U_A) \cap G_A$, for each soft subset $U_A \subseteq G_A$.

Definition 2.8 [14]. A soft subset $U_A$ of a soft Č$ech$ closure space $(F_A, k)$ is said to be soft generalized closed (briefly soft g-closed) set if $k[U_A] \subseteq G_A$, whenever $U_A \subseteq G_A$ and $G_A$ is soft open subset of $(F_A, k)$.

Definition 2.9 [15]. A soft subset $U_A$ of a soft Č$ech$ closure space $(F_A, k)$ is said to be soft $\partial$-closed set if $k[U_A] \subseteq G_A$, whenever $U_A \subseteq G_A$ and $G_A$ is soft g-open subset of $(F_A, k)$.

## III. Strongly Soft Generalized Closed Set

In this section, we introduce strongly soft generalized closed sets in soft Č$ech$ closure space and investigate some basic properties.

Definition 3.1. Let $U_A$ be a soft subset of a soft $\check{C}ech$ closure space $(F_A, k)$ is said to be

1. Soft semi-open set if $U_A \subseteq k[int(U_A)]$ and a soft semi-closed set if $int(k[U_A]) \subseteq U_A$.
2. Soft regular-open set if $int(k[U_A]) = U_A$ and a soft regular-closed set if $U_A = k[int(U_A)]$.
3. Soft pre-open set if $U_A \subseteq int(k[U_A])$ and a soft pre-closed set if $k[int(U_A)] \subseteq U_A$.
4. Soft $\alpha$-open set if $U_A \subseteq int(k[int(U_A)])$ and soft $\alpha$-closed set if $k[int(k[U_A])] \subseteq U_A$.
5. Soft semi pre-open (soft $\beta$-open) set if $U_A \subseteq k[int(k[U_A])]$ and soft semi pre-closed set if $int(k[int(U_A)]) \subseteq U_A$.

The smallest soft Č$ech$ semi-closed set containing $U_A$ is called soft Č$ech$ semi-closure of $U_A$ with respect to $k$ and it is denoted by $k_s(U_A)$.

The largest soft Č$ech$ semi-open set contained in $U_A$ is called soft Č$ech$ semi-interior of $U_A$ with respect to $k$ and it is denoted by $int_s(U_A)$.

The smallest soft Č$ech$ pre-closed set containing $U_A$ is called soft Č$ech$ pre-closure of $U_A$ with respect to $k$ and it is denoted by $k_p(U_A)$.

The largest soft Č$ech$ pre-open set contained in $U_A$ is called soft Č$ech$ pre-interior of $U_A$ with respect to $k$ and it is denoted by $int_p(U_A)$.

The smallest soft Č$ech$ $\alpha$-closed set containing $U_A$ is called soft Č$ech$ $\alpha$-closure of $U_A$ with respect to $k$ and it is denoted by $k_\alpha(U_A)$.

The largest soft Č$ech$ $\alpha$-open set contained in $U_A$ is called soft Č$ech$ $\alpha$-interior of $U_A$ with respect to $k$ and it is denoted by $int_\alpha(U_A)$.

The smallest soft Č$ech$ semi pre-closed set containing $U_A$ is called soft Č$ech$ semi pre-closure of $U_A$ with respect to $k$ and it is denoted by $k_{sp}(U_A)$.

The largest soft Č$ech$ semi pre-open set contained in $U_A$ is called soft Č$ech$ semi pre-interior of $U_A$ with respect to $k$ and it is denoted by $int_{sp}(U_A)$.

Definition 3.2. A soft subset $U_A$ of a soft $\check{C}ech$ closure space $(F_A, k)$ is said to be soft semi-generalized closed set (briefly soft sg-closed) if $k_s(U_A) \subseteq G_A$, whenever $U_A \subseteq F_A$, $G_A$ is soft semi open in $F_A$.

Definition 3.3. A soft subset $U_A$ of a soft $\check{C}ech$ closure space $(F_A, k)$ is said to be soft generalized semi-closed set (briefly soft gs-closed) if $k_s(U_A) \subseteq G_A$, whenever $U_A \subseteq F_A$, $G_A$ is soft open in $F_A$.

Definition 3.4. A soft subset $U_A$ of a soft $\check{C}ech$ closure space $(F_A, k)$ is said to be soft generalized semi pre-closed set (briefly soft gsp-closed) if $k_{sp}(U_A) \subseteq G_A$, whenever $U_A \subseteq F_A$, $G_A$ is soft open in $F_A$.

Definition 3.5. Let $(F_A, k)$ be a soft $\check{C}ech$ closure space. A soft subset $U_A \subseteq F_A$ is called soft regular generalized closed (briefly soft rg-closed) set if $k[U_A] \subseteq G_A$, whenever $U_A \subseteq G_A$ and $G_A$ is soft regular open subset of $(F_A, k)$.

Definition 3.6. A soft subset $U_A$ of a soft $\check{C}ech$ closure space $(F_A, k)$ is said to be soft generalized pre-closed set (briefly soft gp-closed) if $k_p(U_A) \subseteq G_A$, whenever $U_A \subseteq F_A$, $G_A$ is soft open in $F_A$.

Definition 3.7. Let $(F_A, k)$ be a soft $\check{C}ech$ closure space. A soft subset $U_A \subseteq F_A$ is called soft generalized pre-regular closed (briefly soft gpr-closed) set if $k_p(U_A) \subseteq G_A$, whenever $U_A \subseteq G_A$ and $G_A$ is soft regular open subset of $(F_A, k)$.

Definition 3.8. Let $(F_A, k)$ be a soft $\check{C}ech$ closure space. A soft subset $U_A \subseteq F_A$ is called a strongly soft generalized closed (briefly strongly soft g-closed) set if $k[int(U_A)] \subseteq G_A$, whenever $U_A \subseteq G_A$ and $G_A$ is soft open subset of $(F_A, k)$.

Example 3.9. Let the initial universe set $X = \{u_1, u_2\}$ and $E = \{x_1, x_2, x_3\}$ be the parameters.

Let $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$.Then $P(X_{F_A})$ are

$F_{1A} = \{(x_1, \{u_1\})\}, F_{2A} = \{(x_1, \{u_2\})\}, F_{3A} = \{(x_1, \{u_1, u_2\})\}, F_{4A} = \{(x_2, \{u_1\})\}, F_{5A} = \{(x_2, \{u_2\})\},$

$F_{6A} = \{(x_2, \{u_1, u_2\})\}, F_{7A} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}, F_{8A} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\},$

$F_{9A} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\}, F_{10A} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}, F_{11A} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\},$

$F_{12A} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}, F_{13A} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\},$

$F_{14A} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}, F_{15A} = F_A, F_{16A} = \emptyset_A.$

An operator $k: P(X_{F_A}) \to P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = k(F_{7A}) = k(F_{8A}) = k(F_{11A}) = F_{11A}, k(F_{4A}) = k(F_{5A}) = k(F_{6A}) = F_{6A}, k(F_{2A}) = F_{10A},$
$k(F_{9A}) = k(F_{10A}) = k(F_{12A}) = F_{12A}, k(F_{3A}) = k(F_{13A}) = k(F_{14A}) = k(F_A) = F_A, k(\emptyset_A) = \emptyset_A.$
Here, $\emptyset_A, F_{4A}, F_{5A}, F_{6A}, F_{7A}, F_{8A}, F_{9A}, F_{10A}, F_{11A}, F_{12A}, F_{13A}, F_{14A}, F_A$ are strongly soft g-closed sets in $F_A$.

Theorem 3.10. In a soft $\check{C}ech$ closure space $(F_A, k)$, every soft closed set is strongly soft g-closed.

Proof. The proof is obvious from the definition (2.2) of soft closed set.

Result 3.11. The converse of the above theorem 3.10, is need not be true from the following example.

Example 3.12. In example 3.9, $F_{4A} = \{(x_2, \{u_1\})\}$ is strongly soft g-closed but not soft closed.

Theorem 3.13. In a soft $\check{C}ech$ closure space $(F_A, k)$, every soft g-closed set is strongly soft g-closed.

Proof. The proof is obvious.

Result 3.14. The converse of the above theorem 3.13, is need not be true as shown from the following example.

Example 3.15. Let us consider the soft subsets of $F_A$ that are given in example 3.9. An operator $k: P(X_{F_A}) \to P(X_{F_A})$is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = F_{1A}, k(F_{2A}) = k(F_{9A}) = F_{12A}, k(F_{4A}) = F_{4A}, k(F_{5A}) = k(F_{8A}) = F_{14A}, k(F_{7A}) = F_{7A},$
$k(F_{3A}) = k(F_{6A}) = k(F_{10A}) = k(F_{11A}) = k(F_{12A}) = k(F_{13A}) = k(F_{14A}) = k(F_A) = F_A, k(\emptyset_A) = \emptyset_A.$
Here, $F_{2A} = \{(x_1, \{u_2\})\}$ is strongly soft g-closed set but not soft g-closed.

Theorem 3.16. Let $(F_A, k)$ be a soft $\check{C}ech$ closure space. If $U_A$ be a soft subset of $F_A$ is both soft open and strongly soft g-closed, then $U_A$ is soft closed.

Proof. Suppose $U_A$ is both soft open and strongly soft g-closed. Since, $U_A$ is strongly soft g-closed. Then $k[int(U_A)] \subseteq U_A$. That is, $k[U_A] = k[int(U_A)] \subseteq U_A$. Since, $U_A \subseteq k[U_A]$. Hence, $U_A$ is soft closed.

Theorem 3.17. Let $(F_A, k)$ be a soft $\check{C}ech$ closure space and let $U_A \subseteq F_A$. If $U_A$ is strongly soft g-closed, then $k[int(U_A)] - U_A$ has no non-empty soft closed subset.

Proof. Suppose that $U_A$ is strongly soft g-closed. Let $V_A$ be a soft closed subset of $k[int(U_A)] - U_A$. Then $V_A \subseteq k[int(U_A)] \cap (F_A - U_A)$ and so $U_A \subseteq (F_A - V_A)$. Consequently, $V_A \subseteq F_A - k[int(U_A)]$. Since,

$V_A \subseteq k[int(U_A)]$, $V_A \subseteq k[int(U_A)] \cap (F_A - k[int(U_A)]) = \emptyset_A$. Thus, $V_A = \emptyset_A$. Therefore, $k[int(U_A)] - U_A$ contains no non-empty soft closed set.

Result 3.18. The converse of the above theorem 3.17, is not true as shown in the following example.

Example 3.19. In example 3.9, take $U_A = F_{2A}$. $k[int(U_A)] - U_A = \{(x_1, \{u_2\}), (x_2, \{u_2\})\} - \{(x_1, \{u_2\})\} = \{(x_2, \{u_2\})\}$, which does not contain non-empty soft closed subset of $F_A$. But $F_{2A} = \{(x_1, \{u_2\})\}$ is not strongly soft g-closed.

Theorem 3.20. Let $(F_A, k)$ be a soft $\check{C}ech$ closure space. If $U_A$ be a soft subset of $F_A$ is both strongly soft g-closed and soft semi open, then $U_A$ is soft g-closed.

Proof. Suppose $U_A$ is both strongly soft g-closed and soft semi open. Then, $k[int(U_A)] \subseteq G_A$, whenever $U_A \subseteq G_A$ and $G_A$ is soft open subset of $F_A$. $U_A \subseteq k[int(U_A)]$, since $U_A$ is soft semi-open. Then, $k[U_A] \subseteq k[int(U_A)] \subseteq G_A$. Hence, $U_A$ is soft g-closed.

Corollary 3.21. If $U_A$ is soft subset of a soft $\check{C}ech$ closure space $(F_A, k)$ is both strongly soft g-closed and soft open, then $U_A$ is soft g-closed in $F_A$.

Proof. Since, every soft open set is soft semi-open. Then, by the above theorem 3.20 result follows.

Result 3.22. Let $U_A$ and $V_A$ are two non-empty strongly soft g-closed subsets of a soft $\check{C}ech$ closure space $(F_A, k)$. Then the following example shows that, $U_A \cap V_A$ and $U_A \cup V_A$ need not be strongly soft g-closed.

Example 3.23. In example 3.9, take $U_A = F_{7A}$ and $V_A = F_{8A}$. Then, $U_A \cap V_A = F_{7A} \cap F_{8A} = F_{1A}$ , which is not a strongly soft g-closed subset in $F_A$. Also in example 3.15, take $U_A = F_{2A}$ and $V_A = F_{5A}$. Then, $U_A \cup V_A = F_{2A} \cup F_{5A} = F_{10A}$ , which is not a strongly soft g-closed subset in $F_A$.

Theorem 3.24. In a soft $\check{C}ech$ closure space $(F_A, k)$, every soft $\partial$-closed set is strongly soft g-closed.

Proof. The proof is obvious.

Result 3.25. The converse of the above theorem 3.24, is not be true as shown in the following example.

Example 3.26. In example 3.9, $F_{8A} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$ is strongly soft g-closed but not soft $\partial$-closed.

Theorem 3.27. Let $U_A \subseteq H_A \subseteq F_A$ and if $U_A$ is strongly soft g-closed in $F_A$, then $U_A$ is strongly soft g-closed relative to $H_A$.

Proof. Let $U_A \subseteq H_A \subseteq F_A$, and suppose that $U_A$ is strongly soft g-closed in $F_A$. Let $U_A \subseteq H_A \cap G_A$, where $G_A$ is soft open in $F_A$. Since, $U_A$ is strongly soft g-closed in $F_A$, $U_A \subseteq G_A$ implies $k[int(U_A)] \subseteq G_A$. That is, $H_A \cap k[int(U_A)] \subseteq H_A \cap G_A$, where $H_A \cap k[int(U_A)]$ is closure of interior of $U_A$ with respect to $k$ in $H_A$. Thus, $U_A$ is strongly soft g-closed relative to $H_A$.

## IV. Strongly Soft $\partial$-Closed Set

In this section, we introduce strongly soft $\partial$-closed sets in soft Č$ech$ closure space and investigate some of their basic properties.

Definition 4.1. Let $(F_A, k)$ be a soft $\check{C}ech$ closure space. A soft subset $U_A \subseteq F_A$ is called a strongly soft $\partial$-closed set if $k[int(U_A)] \subseteq G_A$, whenever $U_A \subseteq G_A$ and $G_A$ is soft g-open subset of $(F_A, k)$.

Example 4.2. In example 3.9, the strongly soft $\partial$-closed subsets are $\emptyset_A, F_{4A}, F_{5A}, F_{6A}, F_{10A}, F_{11A}, F_{12A}, F_A$.

Theorem 4.3. In a soft $\check{C}ech$ closure space $(F_A, k)$, every strongly soft $\partial$-closed set is strongly soft g-closed.

Proof. The proof is obvious.

Result 4.4. The converse of the above theorem 4.3, is not be true as shown in the following example.

Example 4.5. In example 3.9, $F_{7A} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}$ is strongly soft g-closed but not strongly soft $\partial$-closed.

Theorem 4.6. In a soft $\check{C}ech$ closure space $(F_A, k)$, every soft closed set is strongly soft $\partial$-closed.

Proof. The proof is obvious.

Result 4.7. The following example shows that the converse of above theorem 4.6, need not to be true.

Example 4.8. In example 3.9, $F_{10A} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$ is strongly soft $\partial$-closed but not soft closed.

Theorem 4.9. In a soft $\check{C}ech$ closure space $(F_A, k)$, every soft $\partial$-closed set is strongly soft $\partial$-closed in $F_A$.

Proof. Suppose $U_A$ is soft $\partial$-closed subset of $F_A$. That is, $k[U_A] \subseteq G_A$, whenever $G_A$ is a soft g-open subset of $F_A$ with $U_A \subseteq G_A$. Since, $k[int(U_A)] \subseteq k[U_A]$. This implies, $k[int(U_A)] \subseteq G_A$, whenever $G_A$ is a soft g-open subset of $F_A$ with $U_A \subseteq G_A$. Therefore, $U_A$ is strongly soft $\partial$-closed in $F_A$.

Result 4.10. The converse of the above theorem 4.9, is not true as shown in the following example.

Example 4.11. In example 3.9, $F_{10A} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$ is strongly soft $\partial$-closed but not soft $\partial$-closed.

Theorem 4.12. Let $(F_A, k)$ be a soft $\check{C}ech$ closure space. If $U_A$ be a soft subset of $F_A$ is both soft open and strongly soft $\partial$-closed then $U_A$ is soft closed.

Proof. It is similar to the proof of the theorem 3.16.

Corollary 4.13. If $U_A$ is both soft open and strongly soft $\partial$-closed in $F_A$, then $U_A$ is both soft regular open and soft regular closed in $F_A$.

Proof. Since, $U_A$ is soft open, then $U_A = int(U_A)$. Since, $U_A$ is soft closed, then $U_A = int(U_A) = int(k[U_A])$. Hence, $U_A$ is soft regular open. Also, $k[int(U_A)] = k[U_A]$. Since, $U_A$ is soft closed, then $k[int(U_A)] = U_A$. Hence, $U_A$ is soft regular closed.

Corollary 4.14. If $U_A$ is both soft open and strongly soft $\partial$-closed, then $U_A$ is soft rg-closed.

Theorem 4.15. Let $U_A$ be a soft subset of soft $\check{C}ech$ closure space $(F_A, k)$ is both strongly soft $\partial$-closed and soft semi-open, then $U_A$ is soft $\partial$-closed.

Proof. Suppose $U_A$ is both strongly soft $\partial$-closed and soft semi-open. Then, $k[int(U_A)] \subseteq G_A$, whenever $U_A \subseteq G_A$ and $G_A$ is soft g-open subset of $F_A$. Since, $U_A$ is soft semi open, then $U_A \subseteq k[int(U_A)]$. Then, $k[U_A] \subseteq k[int(U_A)] \subseteq G_A$. Then, $U_A$ is soft $\partial$-closed set in $F_A$.

Corollary 4.16. If $U_A$ is soft subset of a soft $\check{C}ech$ closure space $(F_A, k)$ is both strongly soft $\partial$-closed and soft open, then $U_A$ is soft $\partial$-closed set in $F_A$.

Proof. Since, every soft open set is soft semi-open. Then, by the above theorem 4.15, result follows.

Theorem 4.17. Let $(F_A, k)$ be a soft $\check{C}ech$ closure space and let $U_A \subseteq G_A$. If $U_A$ is strongly soft $\partial$-closed then $k[int(U_A)] - U_A$ has no non-empty soft closed subset.

Proof. It is analogous to the proof of the theorem 3.17.

Result 4.18. The converse of the above theorem 4.17, is not true as shown in the following example.

Example 4.19. Let us consider the soft subsets of $F_A$ that are given in example 3.9. An operator $k: P(X_{F_A}) \to P(X_{F_A})$is defined from soft power set $P(X_{F_A})$ to itself over X as follows.
$k(F_{1A}) = k(F_{5A}) = F_{8A}, k(F_{2A}) = F_{3A}, k(F_{3A}) = k(F_{9A}) = k(F_{13A}) = F_{13A}, k(F_{4A}) = F_{4A}, k(F_{7A}) = F_{7A},$
$k(F_{6A}) = k(F_{8A}) = k(F_{11A}) = F_{11A}, k(F_{10A}) = F_{14A}, k(F_{12A}) = k(F_{14A}) = k(F_A) = F_A,\ k(\emptyset_A) = \emptyset_A.$

Take, $U_A = F_{2A}$. Then, $k[int(U_A)] - U_A = \{(x_1, \{u_1, u_2\})\} - \{(x_1, \{u_2\})\} = \{(x_1, \{u_1\})\}$, which does not contain non-empty soft closed subset of $F_A$. But, $F_{2A} = \{(x_1, \{u_2\})\}$ is not strongly soft $\partial$-closed.

Corollary 4.20. A strongly soft $\partial$-closed set $U_A$ is soft regular closed if and only if $k[int(U_A)] - U_A$ is soft closed.

Proof. Suppose that $U_A$ is soft regular closed. Since, $U_A = k[int(U_A)]$, then $k[int(U_A)] - U_A = \emptyset_A$ is soft regular closed. Thus, $k[int(U_A)] - U_A$ is soft closed. Conversely, assume that $k[int(U_A)] - U_A$ is soft closed. By the theorem 4.17, $k[int(U_A)] - U_A$ contains no non-empty soft closed set. This implies, $k[int(U_A)] - U_A = \emptyset_A$. Thus, $U_A$ is soft regular closed.

Theorem 4.21. In a soft $\check{C}ech$ closure space $(F_A, k)$, every strongly soft $\partial$-closed set is soft gsp-closed.

Proof. The proof of the theorem is immediate from the definition of strongly soft $\partial$-closed and soft gsp-closed.

Result 4.22. The converse of the above theorem 4.21, is not true as shown in the following example.

Example 4.23. In example 3.9, take $F_{2A} = \{(x_1, \{u_2\})\}$ is soft gsp-closed set but not strongly soft $\partial$-closed.

Result 4.24. Let $U_A$ and $V_A$ are two non-empty strongly soft $\partial$-closed subsets of a soft $\check{C}ech$ closure space $(F_A, k)$. Then the following example shows that, $U_A \cap V_A$ and $U_A \cup V_A$ need not be strongly soft $\partial$-closed.

Example 4.25. In example 4.19, take $U_A = F_{6A}$ and $V_A = F_{8A}$. Then, $U_A \cap V_A = F_{6A} \cap F_{8A} = \{(x_2, \{u_1, u_2\})\} \cap \{(x_1, \{u_1\}), (x_2, \{u_2\})\} = F_{5A}$, which is not a strongly soft $\partial$-closed subset in $F_A$. Also in example 3.15, take $U_A = F_{2A}$ and $V_A = F_{5A}$. Then, $U_A \cup V_A = F_{2A} \cup F_{5A} = F_{10A}$ , which is not a strongly soft $\partial$-closed subset in $F_A$.

Result 4.26. The following example shows that, strongly soft $\partial$-closed and soft gs-closed sets are independent.

Example 4.27. In example 4.19, $F_{1A} = \{(x_1, \{u_1\})\}$ is strongly soft $\partial$-closed set but not soft gs-closed. Also $F_{2A} = \{(x_1, \{u_2\})\}$ is soft gs-closed but not strongly soft $\partial$-closed.

Theorem 4.28. In a soft $\check{C}ech$ closure space $(F_A, k)$, every strongly soft $\partial$-closed set is soft gp-closed.

Proof. The proof is obvious.

Result 4.29. The converse of the above theorem 4.28, is not true as shown in the following example.

Example 4.30. In example 4.19, $F_{5A} = \{(x_2, \{u_2\})\}$ is soft gp-closed but not strongly soft $\partial$-closed.

Result 4.31. The following example shows that, strongly soft $\partial$-closed and soft rg-closed sets are independent.

Example 4.32. In example 4.19, $F_{6A} = \{(x_2, \{u_1, u_2\})\}$ is strongly soft $\partial$-closed set but not soft rg-closed. Also $F_{10A} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$ is soft rg-closed but not strongly soft $\partial$-closed.

Result 4.33. In a soft $\check{C}ech$ closure space $(F_A, k)$, soft g-closed and strongly soft $\partial$-closed sets are independent as shown in the following examples.

Example 4.34. In example 3.9, $F_{7A} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}$ is soft g-closed set but not strongly soft $\partial$-closed.

Example 4.35. In example 3.15, $F_{2A} = \{(x_1, \{u_2\})\}$ is strongly soft $\partial$-closed set but not soft g-closed.

Theorem 4.36. Let $U_A \subseteq H_A \subseteq F_A$ and if $U_A$ is strongly soft $\partial$-closed in $F_A$, then $U_A$ is strongly soft $\partial$-closed relative to $H_A$.

Proof. Let $U_A \subseteq H_A \subseteq F_A$, and suppose that $U_A$ is strongly soft $\partial$-closed in $F_A$. Let $U_A \subseteq H_A \cap G_A$, where $G_A$ is soft g-open in $F_A$. Since, $U_A$ is strongly soft $\partial$-closed in $F_A$, $U_A \subseteq G_A$ implies $k[int(U_A)] \subseteq G_A$. That is, $H_A \cap k[int(U_A)] \subseteq H_A \cap G_A$, where $H_A \cap k[int(U_A)]$ is closure of interior of $U_A$ with respect to $k$ in $H_A$. Thus, $U_A$ is strongly soft $\partial$-closed relative to $H_A$.

## V. Conclusion

In the present work, we have introduced strongly soft g-closed sets and strongly soft $\partial$-closed sets in soft $\check{C}ech$ closure spaces, which are defined over an initial universe with a fixed set of parameters. We studied the behavior relative to union, intersection of strongly soft g-closed sets and strongly soft $\partial$-closed sets. Also, we proved that every soft closed set is strongly soft g-closed as well as strongly soft $\partial$-closed. In future, findings of this paper will contribute to a new types of soft generalized closed sets in soft $\check{C}ech$ closure spaces and it may leads to find a new types of separation axioms. Also, the findings in this paper will help to carry out a general framework for their applications in practical life.

## References

[1]. L.A Zadeh, " Fuzzy Sets", *Inf.Control, Vol.8*, 1965, pp. 338-353.
[2]. Z.Pawlak, "Rough Sets", *Int.J.Comput.Sci, Vol.11*, 1982, pp.341-356.
[3]. K.Atanassov, "Intuitionistic Fuzzy Sets", *Fuzzy Sets and Systems, Vol.20*, 1986, pp.87-96.
[4]. K.Atanassov, " Operators Over Interval Valued Intuitionistic Fuzzy Sets", *Fuzzy Sets and Systems, Vol.64*, 1994, pp.159-174.
[5]. M.B. Gorzalzany, " A Method of Inference in Approximate Reasoning Based on Interval-Valued Fuzzy Sets", *Fuzzy Sets and Systems, Vol.21*, 1987, pp1-17.
[6]. D.A. Molodtsov, " Soft Set Theory First Results", *Comput.Math.Appl, Vol.37*, 1999, pp.19-31.
[7]. D.A. Molodtsov, "The Description of a Dependence with the Help of Soft Sets", *J.Comput.Sys.Sc.Int, 40*, 2001, 977-984.
[8]. D.A. Molodtsov, "The Theory of Soft Sets (In Russian)", *URSS publishers, Moscow*.
[9]. N. Levine, "Generalized Closed Sets in Topology", *Rend.Circ.Mat.Palermo, Vol.19, No.2*, 1970, pp.89-96.
[10]. Chawalit Boonpok, "Generalized Closed Sets in $\check{C}ech$ Closed Spaces", *Acta Universitatis Apulensis, No.22*, 2010, pp.133-140.
[11]. E. $\check{C}ech$, "Topogical Spaces", *Inter Science Publishers, John Wiley and Sons, Newyork*, 1966.
[12]. R.Gowri and G.Jegadeesan, " On Soft $\check{C}ech$ Closure Spaces", *International Journal of Mathematics Trends and Technology,Vol.9*, 2014,32-37.
[13]. R.Gowri and G.Jegadeesan, " Some Higher Separation Axioms in Soft $\check{C}ech$ Closure Spaces", *Int.Mat.Forum,Vol.10, No.11*, 2015,519-525.
[14]. R.Gowri and G.Jegadeesan, " Soft Generalized Closed Sets in Soft $\check{C}ech$ Closure Spaces", *Global Journal of Pure and Applied Mathematics,Vol.2, No.1*, 2016, 909-916.
[15]. R.Gowri and G.Jegadeesan, " Soft $\partial$-Closed Sets in Soft $\check{C}ech$ Closure Spaces", (Accepted).