IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 4 Ver. I (Jul. - Aug.2016), PP 42-46
www.iosrjournals.org

Generalized Analytic Difference Sequence Spaces Defined By Musielak-Orlicz Function

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Abstract: *In this paper, the generalized analytic difference sequence space defined by Musielak-Orlicz function, are introduced, and some of their algebraic and topological properties are explored. Few inclusion relations involving the introduced spaces are also discussed.*
Keywords: *Analytic Sequences, Difference Sequence Space, Entire Sequences, Musielak- Orlicz Function,*

I. Introduction

A complex sequence, whose k^{th} term is denoted by (x_k).A sequence x= (x_k) is said to be analytic, if $sup_k |x_k|^{\frac{1}{k}} < \infty$ .The vector space of all analytic sequences will be denoted by Λ. A sequence is entire sequence, if $\lim_k |x_k|^{\frac{1}{k}} = 0$. The vector space of all entire sequences will be denoted by Γ.

The notion of difference sequence space was introduced by Kizmaz [1], who studied the difference sequence spaces $\ell_\infty(\Delta)$, c(Δ) and $c_0(\Delta)$. Kizmaz defined the following Difference Sequence Spaces:

$$Z(\Delta) = \{x = (x_k) : \Delta_x \in Z\} \text{ where } \Delta_x = (\Delta_x)_{k=1}^{\infty} = (x_k - x_{k+1})_{k=1}^{\infty}.$$

Here Z stands for one of the spaces c_0, c and ℓ_∞.
The notion was further generalized by Et and Colak [2] by introducing the spaces $c(\Delta^m)$, $c_0(\Delta^m)$ and $\ell_\infty(\Delta^m)$

Let m, v be non-negative integers, then for $z = \{\ell_\infty, c, c_0\}$, we have sequence spaces

$$Z(\Delta_v^m) = \{x = (x_k) \in \omega : (\Delta_v^m x_k) \in Z\} \qquad \text{(see Raji et al [3])}$$

where $\Delta_v^m x = (\Delta_v^m x_k) = (\Delta_v^{m-1} x_k - \Delta_v^{m-1} x_{k+1})$ and $\Delta_v^0 x_k = x_k, \forall\, k \in \mathbb{N}$, which is equivalent to the following binomial expansion

$$\Delta_v^m x_k = \sum_{i=0}^{m} (-1)^i \binom{m}{i} x_{k+iv}$$

Taking $v = 1$, we have the spaces which were studied by Et and Colak [2].
Taking $m = v = 1$, we get the spaces which were introduced and studied by Kizmaz [1].

An Orlicz function M : $[0, \infty) \to [0, \infty)$ is a continuous, non decreasing and convex function such that M (0) = 0, M (x) > 0, for all $x > 0$ and $M(x) \to 0$ $as\ x \to \infty$.

Lindenstrauss and Tzafriri [4] used the idea of Orlicz function to define the following sequence space:

$$\ell_M = \left\{ x \in \omega: \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty \right\},$$

where $\omega = \{all\ complex\ sequences\}$, which is called an Orlicz sequence space. Also ℓ_M is a Banach space with norm

$$\| x \| = \inf\{\rho > 0: \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \le 1\}$$

It was proved that every Orlicz sequence space ℓ_M contains a subspace isomorphic to $\ell_p (1 \le p < \infty)$

II. Definitions And Preliminaries

Definition 2.1: Let M_k be an Orlicz function. The space consisting of all these sequences x in ω, such that $sup_k \left(M_k\left(\frac{|x_k|^{\frac{1}{k}}}{\rho}\right)\right) < \infty$ for some arbitrary fixed $\rho > 0$ is denoted by Λ_M and is known as the space of analytic sequence defined by a sequence of Orlicz function.

Definition 2.2 (see (Musielak [5])**:** A sequence space E is said to be solid or normal if $(\alpha_k x_k) \in E$, where $(x_k) \in E$ and for all sequence of scalars (α_k) with $|\alpha_k| \le 1$.

Definition 2.3: Let V be a vector space over scalar field K. A seminorm v on V is a real valued function on V so that

1. v (x) ≥ 0, for all $x \in V$
2. v$(\alpha x) = |\alpha| v(x)$, for all $\alpha \in k, x \in V$
3. v(x+ y) $\le v(x) + v(y)$, for all $x, y \in V$

Definition 2.4 (Maddox [6])**:** Let X be a linear metric space. A function $p : X \to R$ is called paranorm, if

1. $p(x) \ge 0, \forall\, x \in X$,
2. $p(-x) = p(x), \forall\, x \in X$,
3. $p(x + y) \le p(x) + p(y), \forall\, x, y \in X$.

4. If (λ_n) is a sequence of scalars with $\lambda_n \to \lambda \; as \; n \to \infty$ and (x_n) is a sequence of vectors with $p(x_n - x \to 0$ as $n \to \infty$, then $p\lambda nxn - \lambda x \to 0 \; as \; n \to \infty$.

Remark 2.1: The following inequality will be used throughout the paper.

Let $p = (p_k)$ be a sequence of positive real numbers with $0 \le p_k \le sup(p_k) = G, K = \max(1, 2^{G-1})$, then

$$|a_k + b_k|^{p_k} \le K\{|a_k|^{p_k} + |b_k|^{p_k}\}, \quad (2.1)$$

where $a_k, b_k \in \mathbb{C}$. Also $|a|^{p_k} \le \max(1, |a|^G)$ for all $a \in \mathbb{C}$.

Definition 2.5 (Musielak [5])**:** Musielak-Orlicz function is defined to be a sequence of Orlicz functions.

Definition 2.6: Let $\mathcal{M} = (M_k)$ be a sequence of Orlicz functions, X be locally convex Hausdorff topological linear space whose topology is determined by a set of continuous seminorms q. The symbols $\Lambda(X) and \, \Gamma(X)$ denote the space of all analytic and entire sequences, respectively defined over X.

Now we define a new sequence space:

$$\Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s) = \{x \in \Lambda(X): sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k} < \infty, \text{for some } \rho > 0\}$$

Remarks 2.2: We get the following analytic sequence spaces from the above space by giving particular values to p and s.

Taking $p_k = 1$, for all n ∈ ℕ,we have

$$\Lambda_{\mathcal{M}}(\Delta_v^m, q, s) = \{x \in \Lambda(X): sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right] < \infty, \text{for some } \rho > 0\}$$ (Abbas and Kamel [7])

If we take s = 0, we have

$$\Lambda_{\mathcal{M}}(\Delta_v^m, p, q) = \{x \in \Lambda(X): sup_n \frac{1}{n}\sum_{k=1}^{n}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k} < \infty, \text{for some } \rho > 0\}$$ (Raj et al [3])

If we take s = 0, m = v = 1, we get

$$\Lambda_{\mathcal{M}}(\Delta, p, q,) = \{x \in \Lambda(X): sup_n \frac{1}{n}\sum_{k=1}^{n}\left[M_k\left(q\left(\frac{|\Delta x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k} < \infty, \text{for some } \rho > 0\}$$

(Lindenstrauss and Tzafriri [4])

III. Main Results

The following results are obtained in this work.

Theorem 3.1: Let M=(M_k) be any Musielak Orlicz function, and $p = (p_k)$ a sequence of strictly positive real numbers,then $\Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$ is a linear space over the set of complex numbers $\mathbb{C}$.

Proof. Let $x = (x_k)$, $y = (y_k) \in \Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$ and $\alpha, \beta \in \mathbb{C}$, then we have

$$sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho_1}\right)\right)\right]^{p_k} < \infty, \text{for some } \rho_1 > 0 \quad (3.11)$$

$$sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m y_k|^{\frac{1}{k}}}{\rho_2}\right)\right)\right]^{p_k} < \infty, \text{for some } \rho_2 > 0 \quad (3.12)$$

Since $M = (M_k)$ is a non decreasing modulus function, q seminorm and Δ_v^m is linear, then

$$sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m (\alpha x_k + \beta y_k)|^{\frac{1}{k}}}{\rho_3}\right)\right)\right]^{p_k} < \infty, \text{for some } \rho_3 > 0 \quad (3.13)$$

where $\rho_3 = \max\{|\alpha|^{\frac{1}{k}}\rho_1, |\beta|^{\frac{1}{k}}\rho_2\}$.

Now,

$$sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m (\alpha x_k + \beta y_k)|^{\frac{1}{k}}}{\rho_3}\right)\right)\right]^{p_k}$$

$$\le sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\alpha|^{\frac{1}{k}}|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho_3} + \frac{|\beta|^{\frac{1}{k}}|\Delta_v^m y_k|^{\frac{1}{k}}}{\rho_3}\right)\right)\right]^{p_k}$$

$$\le K sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho_1} + \frac{|\Delta_v^m y_k|^{\frac{1}{k}}}{\rho_2}\right)\right)\right]^{p_k}$$

$$\leq K sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho_1}\right)\right) + M_k\left(q\left(\frac{|\Delta_v^m y_k|^{\frac{1}{k}}}{\rho_2}\right)\right)\right]^{P_K}$$

$$\leq K sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho_1}\right)\right) + K sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m y_k|^{\frac{1}{k}}}{\rho_2}\right)\right)\right]^{P_K} < \infty$$

This proves that $\Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$ is a linear space.

Theorem 3.2: Let $\mathcal{M}' = (M_k')$ and $\mathcal{M}'' = (M_k'')$ be Musielak- Orlicz functions. Then

$$\Lambda_{\mathcal{M}'}(\Delta_v^m, p, q, s) \cap \Lambda_{\mathcal{M}''}(\Delta_v^m, p, q, s) \subseteq \Lambda_{\mathcal{M}'+\mathcal{M}''}(\Delta_v^m, p, q, s). \quad (3.21)$$

Proof. Let $x \in \Lambda_{\mathcal{M}'}(\Delta_v^m, p, q, s) \cap \Lambda_{\mathcal{M}''}(\Delta_v^m, p, q, s)$.Then there exist ρ_1 and ρ_2 such that

$$sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k'\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho_1}\right)\right)\right]^{p_k} < \infty, \text{for some } \rho_1 > 0 \quad (3.22)$$

$$sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k''\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho_2}\right)\right)\right]^{p_k} < \infty, \text{for some } \rho_2 > 0 \quad (3.23)$$

Let $\rho = \min\left(\frac{1}{\rho_1}, \frac{1}{\rho_2}\right)$. Then we have

$$sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[(M_k' + M_k'')\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k} \leq K sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k'\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho_1}\right)\right)\right]^{p_k}$$

$$+ K sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k''\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho_2}\right)\right)\right]^{p_k} < \infty$$

by (3.21) and (3.21). Then

$$sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[(M_k' + M_k'')\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k} < \infty \text{ for some } \rho > 0$$

Therefore, $x \in \Lambda_{\mathcal{M}'+\mathcal{M}''}(\Delta_v^m, p, q, s)$.

Theorem 3.3: The sequence space $\Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$ is solid.

Proof. Let $x = (x_k) \in \Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$, then

$$sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k} < \infty.$$

Let (α_k) be a sequence of scalars such that $|\alpha_k| \leq 1, \ \forall\, k \in \mathbb{N}$, then we have

$$sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\alpha_k \Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k} \leq sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\alpha_k|^{\frac{1}{k}}|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k}$$

$$\leq sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k} < \infty$$

Hence, $(\alpha_k x_k) \in \Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$.

Theorem 3.4: Suppose $sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k} \leq |x_k|^{\frac{1}{k}}, then\ \Lambda \subset \Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$.

Proof. Let $x \in \Lambda$. Then we have,

$$sup|x_k|^{\frac{1}{k}} < \infty. \quad (3.41)$$

But

$$sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k} \leq sup|x_k|^{\frac{1}{k}}$$

by our assumption. It implies that

$$sup_n \frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k} \leq \infty$$

by (3.41). Then $x \in \Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$ and $\Lambda \subset \Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$.

Theorem 3.5: Let $0 \leq p_k \leq r_k$ $and \{\frac{r_k}{p_k}\}$ be bounded. Then $\Lambda_{\mathcal{M}}(\Delta_v^m, r, q, s) \subset \Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$.

Proof. Let $x \in \Lambda_{\mathcal{M}}(\Delta_v^m, r, q, s)$, then

$$sup_n \frac{1}{n} \sum_{k=1}^{n} k^{-s} \left[M_k \left(q \left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho} \right) \right) \right]^{r_k} < \infty. \quad (3.51)$$

Let

$$t_k = sup_n \frac{1}{n} \sum_{k=1}^{n} k^{-s} \left[M_k \left(q \left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho} \right) \right) \right]^{q_k}$$

and $\lambda_k = \frac{p_k}{r_k}$. $Since\ p_k \leq r_k$, we have $0 \leq \lambda_k \leq 1$.

Take $0 < \lambda < \lambda_k$. Define

$$u_k = \begin{cases} t_k, & if\ t_k \geq 1 \\ 0, & if\ t_k < 1 \end{cases}$$

and

$$v_k = \begin{cases} 0, & if\ t_k \geq 1 \\ t_k, & if\ t_k < 1 \end{cases}$$

$t_k = u_k + v_k$, $\quad {t_k}^{\lambda_k} = {u_k}^{\lambda_k} + {v_k}^{\lambda_k}$. It follows that ${u_k}^{\lambda_k} \leq u_k \leq t_k$, ${v_k}^{\lambda_k} \leq {v_k}^{\lambda}$. Since ${t_k}^{\lambda_k} = {u_k}^{\lambda_k} + {v_k}^{\lambda_k}$, then ${t_k}^{\lambda_k} \leq t_k + {v_k}^{\lambda}$. Thus

$$sup_n \frac{1}{n} \sum_{k=1}^{n} k^{-s} \left[M_k \left(q \left(\frac{|\Delta_k^m x_k|^{\frac{1}{k}}}{\rho} \right) \right)^{r_k} \right]^{\lambda_k} \leq sup_n \frac{1}{n} \sum_{k=1}^{n} k^{-s} \left[M_k \left(q \left(\frac{|\Delta_k^m x_k|^{\frac{1}{k}}}{\rho} \right) \right) \right]^{r_k}$$

$$\Rightarrow sup_n \frac{1}{n} \sum_{k=1}^{n} k^{-s} \left[M_k \left(q \frac{|\Delta_k^m x_k|^{\frac{1}{k}}}{\rho} \right)^{r_k} \right]^{\frac{p_k}{r_k}} \leq sup_n \frac{1}{n} \sum_{k=1}^{n} k^{-s} \left[M_k \left(q \left(\frac{|\Delta_k^m x_k|^{\frac{1}{k}}}{\rho} \right) \right) \right]^{r_k}$$

$$\Rightarrow sup_n \frac{1}{n} \sum_{k=1}^{n} k^{-s} \left[M_k \left(q \left(\frac{|\Delta_k^m x_k|^{\frac{1}{k}}}{\rho} \right) \right)^{r_k} \right]^{p_k} \leq sup_n \frac{1}{n} \sum_{k=1}^{n} k^{-s} \left[M_k \left(q \left(\frac{|\Delta_k^m x_k|^{\frac{1}{k}}}{\rho} \right) \right) \right]^{r_k}$$

But, $sup_n \frac{1}{n} \sum_{k=1}^{n} k^{-s} \left[M_k \left(q \left(\frac{|\Delta_k^m x_k|^{\frac{1}{k}}}{\rho} \right) \right) \right]^{r_k} < \infty$.

Therefore, $sup_n \frac{1}{n} \sum_{k=1}^{n} k^{-s} \left[M_k \left(q \left(\frac{|\Delta_k^m x_k|^{\frac{1}{k}}}{\rho} \right) \right)^{r_k} \right]^{p_k} < \infty$.

Hence $x \in \Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$. From (3.51), we get

$$\Lambda_{\mathcal{M}}(\Delta_v^m, r, q, s) \subset \Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s).$$

Theorem 3.6 (i) Let $0 < inf p_k \leq p_k \leq 1$. Then, $\Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s) \subset \Lambda_{\mathcal{M}}(\Delta_v^m, q, s)$

(ii) Let $1 \leq p_k \leq sup\ p_k < \infty$. Then $\Lambda_{\mathcal{M}}(\Delta_v^m, q, s) \subset \Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$

Proof. (i) Let $x \in \Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$. Then

$$sup_n \frac{1}{n} \sum_{k=1}^{n} k^{-s} \left[M_k \left(q \left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho} \right) \right) \right]^{p_k} < \infty \quad (3.61)$$

Since $0 < inf p_k \leq p_k \leq 1$,

$$sup_n \frac{1}{n} \sum_{k=1}^{n} k^{-s} \left[M_k \left(q \left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho} \right) \right) \right] \leq sup_n \frac{1}{n} \sum_{k=1}^{n} k^{-s} \left[M_k \left(q \left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho} \right) \right) \right]^{p_k} < \infty \quad (3.62)$$

From (3.61) and (3.62), it follows that $x \in \Lambda_{\mathcal{M}}(\Delta_v^m, q, s)$.Thus $\Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s) \subset \Lambda_{\mathcal{M}}(\Delta_v^m, q, s)$.

(ii) Let $p_k \geq 1$ for each k and $\sup p_k < \infty$, and let $x \in \Lambda_{\mathcal{M}}(\Delta_v^m, q, s)$.Then

$$sup_n \frac{1}{n} \sum_{k=1}^{n} k^{-s} \left[M_k \left(q \left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho} \right) \right) \right] < \infty \quad (3.63)$$

Since $1 \leq p_k \leq supp_k < \infty$, we have

$$sup_n\frac{1}{n}\sum_{k=1}^{n}k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k} \leq sup_n\frac{1}{n}\sum_{k=1}^{n}k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]$$

$$\Longrightarrow sup_n\frac{1}{n}\sum_{k=1}^{n}k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k} \to 0 \ \ as \ \ n \to \infty$$

This implies that $x \in \Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$. Therefore, $\Lambda_{\mathcal{M}}(\Delta_v^m, q, s) \subset \Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s)$.

IV. Conclusion

We conclude that the sequence space that we have introduced, namely

$$\Lambda_{\mathcal{M}}(\Delta_v^m, p, q, s) = \{x \in \ \Lambda(X): sup_{n_n}\frac{1}{n}\sum_{k=1}^{n} k^{-s}\left[M_k\left(q\left(\frac{|\Delta_v^m x_k|^{\frac{1}{k}}}{\rho}\right)\right)\right]^{p_k} < \infty, \text{for some } \rho > 0\}$$

is not only a linear space but is also solid. The space extends the results of Raj et al [3] and Abbas and Kamel [7]. It further open doors for the extension of similar types of result for other spaces defined by Musielak-Orlicz functions.

Acknowledgement

The authors are thankful to Prof. M. A. Babagana and Dr. H. R. Bakari for their supports during the preparation of this paper.

References

[1]. H. Kizmaz, On certain sequence spaces, *Canadian Math. Bull.*, *24 (2)*, 1981, 169-176.

[2]. M. Et and R. Colak, On generalized difference sequence spaces, *Soochow J. Math*., *21 (4)*, 1995, 377-386.

[3]. K. S. Raj, S. K. Sharma and A. Gupta, Entire sequence spaces defined by Musielak-Orlicz function, *Int. J. Math. Sci. and Appl. 1 (2)*, 2011, 953-960.

[4]. J. Lindenstrauss and L. Tzafriri, On Orlicz sequence spaces, *Israel J. Math. I0*, 1971, 379-390.

[5]. J. Musielak, Orlicz spaces and modular spaces, *Lecture Notes in Mathematics, 1034*. 1983

[6]. I. J. Maddox, Elements of functional analysis, Marcel Dekker Inc., New York and Basel, 1981.

[7]. N. M. Abbas, and R. A. Kamel, Orlicz space of difference analytic sequences, *Mathematical Theory and Modeling*, *4(3)*, 2014, 51-57.