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Properties of α-Interior and α-Closure in Intuitionistic Topological Spaces

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Abstract: *The purpose of this research article is to study about intuitionistic α-open sets and discuss interior and closure properties of intuitionistic sets.*

I. Introduction

After the introduction of fuzzy sets by Zadeh[11], there have been a number of generalizations of this fundamental concept. Using the notion of intuitionistic fuzzy sets, Coker[4] introduced the notion of intuitionistic fuzzy topological spaces. The concept of intuitionistic set in topological space was first introduced by Coker[3].He has studied some fundamental topological properties on intuitionistic sets. Open sets play a vital role in general topology and they are now the research topics of many researchers worldwide. Njastad[9] studied semi open sets, pre open sets, α-open sets and semipro open sets in general topological spaces. Maheswari[8] has studied the properties of α-interior and α-closure in topological spaces. In this paper, the properties of intuitionistic α-open sets are introduced and characterized.

II. Preliminaries

Definition 2.1 [5] : Let X be a non empty set. An intuitionistic set (IS for short) A is an object having the form A = < X, A_1, A_2 >, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of members of A, while A_2 is called the set of non-members of A.

Definition 2.2 [5] : Let X be a non empty set and let A, B are intuitionistic sets in the form A = < X, A_1, A_2 >, B = < X, B_1, B_2 > respectively. Then
(a) $A \subseteq B$ iff $A_1 \subseteq B_1$ and $B_2 \subseteq A_2$
(b) A = B iff $A \subseteq B$ and $B \subseteq A$
(c) A^c = < X, A_2, A_1 >
(d) [ ] A = < X, A_1, $(A_1)^c$ >
(e) $A - B = A \cap B^C$.
(f) $\phi_\sim$ = < X, ϕ, X >, $X_\sim$ = < X, X, ϕ >
(g) $A \bigcup B$ = < X, $A_1 \bigcup B_1$, $A_2 \cap B_2$ >.
(h) $A \cap B$ = < X, $A_1 \cap B_1$, $A_2 \cup B_2$ >.

Definition 2.3 [5] : An intuitionistic topology (for short IT) on a non empty set X is a family of IS's in X satisfying the following axioms.
(i) $\phi_\sim, X_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.
(iii) $\bigcup G_\alpha \subseteq \tau$ for any arbitrary family $\{G_i: G_\alpha / \alpha \in J\} \subseteq \tau$ where (X,τ) is called an intuitionistic topological space (for short ITS(X)) and any intuitionistic set in is called an intuitionistic open set (for short IOS) in X. The complement A^c of an IOS A is called an intuitionistic closed set (for short ICS) in X.

Definition 2.4[5] : Let (X,τ) be an intuitionistic topological space (for short ITS(X)) and A = < X, A_1, A_2 > be an IS in X. Then the interior and closure of A are defined by
Icl (A) = $\cap$ {K: K is an ICS in X and $A \subseteq K$},
Iint(A) = $\bigcup$ {G : G is an IOS in X and $G \subseteq A$}.
It can be shown that Icl(A) is an ICS and Iint(A) is an IOS in X and A is an ICS in X iff Icl(A) = A and is an IOS in X iff Iint(A) = A.

Definition 2.5[5]: Let X be a non empty set and $p \in X$. Then the IS p defined by p = < X, {p}, $\{p\}^c$ > is called an intuitionistic point (IP for short) in X. The intuitionistic point p is said to be contained in A = < X, A_1, A_2 > (i.e $p \in A$) if and only if $p \in A_1$.

Definition 2.6[10]
Let (X,τ) be an ITS(X). An intuitionistic set A of X is said to be
(i) Intuitionistic semiopen if A$\subseteq$ Icl(Iint(A)).
(ii) Intuitionistic preopen if A$\subseteq$ Iint(Icl(A)).
(iii) Intuitionistic regular open if A = Iint(Icl(A)) .
(iv) Intuitionistic α-open if A$\subseteq$ Iint(Icl(Iint(A))).
The family of all intuitionistic pre open, intuitionistic regular open and intuitionistic α-open sets of (X,τ) are denoted by IPOS, IROS and IαOS respectively.

Definition 2.7 [5]
(a) If B = < Y, B_1, B_2 > is an IS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the IS in X defined by $f^{-1}(B)$ = < X, $f^{-1}(B_1)$, $f^{-1}(B_2)$ >.

If A = < X, f (A_1), $f(A_2)$ > is an IS in X, then the image of A under f, denoted by f(A) is the IS in Y defined by f(A) = < Y, $f(A_1)$, $f_{-}(A_2)$ >, where $f_{-}(A_2)$ = Y– (f(X-A_2)).

III. Properties of Intuitionistic α -Open Set

Definition 3.1: An intuitionistic point x in an intuitionistic topological space (X, τ) is said to be an intuitionistic α -interior point of A if and only if there exists an intuitionistic α-open set U in X such that U$\subseteq$A.
Definition 3.2: The set of all intuitionistic α-interior points of A$\subseteq$X is said to be the intuitionistic α-interior of A or equivalently the union of all intuitionistic α-open sets which are contained in A is called the intuitionistic α -interior of A and is denoted by Iαint(A).
Note 3.3: Since every open set is intuitionistic α-open, it follows that every intuitionistic interior point of A$\subseteq$X is an intuitionistic α-interior point of A. Hence Iint(A) $\subseteq$I αint(A).But converse is not true.

Theorem 3.4: If S is a nonempty intuitionistic α-open set in an intuitionistic topological space (X, τ) then Iint(S)$\neq$ phi.
Proof: Since S is a intuitionistic α-open set, S$\subseteq$Iint(Icl(Iint(S))). Let us suppose that Iint(S) is empty. Then we have S$\subseteq$ ϕ and hence S=ϕ . It is contrary to the hypothesis that S is nonempty. Therefore, Iint(S) is not empty.

Theorem 3.5: If A$\subseteq$B$\subseteq$Iint(Icl(IintA)), then B is an intuitionistic α-open set.
Proof: Since A$\subseteq$B, Iint(Icl(IintA)) $\subseteq$Iint(Icl(IintB)). This inclusion along with the hypothesis implies B $\subseteq$Iint(Icl(IintB)). That is B is an intuitionistic α -open set.

Theorem 3.6: A set B is intuitionistic α -open iff there exists an intuitionistic open set D such that D$\subseteq$B$\subseteq$Iint(Icl(D))
Proof: Let us suppose that there exists an intuitionistic open set D such that D$\subseteq$B$\subseteq$Iint(Icl(D)). Since B$\subseteq$ Iint(Icl(D)) = Iint(Icl(Iint(D)))$\subseteq$Iint(Icl(Iint(B))). This implies B is intuitionistic α-open. Conversely, B is intuitionistic α-open, then B$\subseteq$Iint(Icl(Iint(B))). Let IintB = D. Since IintB$\subseteq$B. D$\subseteq$B and also B$\subseteq$ Iint(Icl(D)). Hence D$\subseteq$B$\subseteq$Iint(Icl(D)).

Theorem 3.7: A is intuitionistic α -closed iff there exists an intuitionistic closed set B such that Icl(Iint(B)) $\subseteq$A$\subseteq$B.
Proof: Let A be intuitionistic α -closed.Then Icl(Iint(Icl(A)))$\subseteq$A.Let Icl(A) = B.Since A$\subseteq$Icl(A),A$\subseteq$B and by hypothesis, Icl(Iint(B))$\subseteq$A.Thus there exists B such that Icl(Iint(B))$\subseteq$A$\subseteq$B. Conversely, suppose that there exists B such that Icl(Iint(B))$\subseteq$A$\subseteq$B. Since B is intuitionistic closed Icl(B) =B. By hypothesis, Icl(int(B))$\subseteq$A this implies Icl(Iint(Icl(B)))$\subseteq$A. As A$\subseteq$B, Icl(Iint(Icl(A)))$\subseteq$Icl(Iint(Icl(B)))$\subseteq$A.Thus A is intuitionistic α -closed.

Lemma 3.8: Let A be an intuitionistic subset of X. Then x$\in$Iα cl(A) iff for any intuitionistic α -open set U containing x, A$\cap$U$\neq$ ϕ .
Proof: Necessity: Let x$\in$Iα cl(A) and U be an intuitionistic α -open set containing x such that A$\cap$U = ϕ . This implies, A$\subseteq$X-U. But, X-U is intuitionistic α -closed set. Since Iα cl(A) is the smallest intuitionistic

α-closed set containing A, Iα cl(A) $\subseteq$ X-U. Since x $\notin$ X-U $\Rightarrow$ x $\notin$ I α cl(A) which is a contradiction. Hence for any intuitionistic α-open set U containing x, A $\bigcap$ U $\neq \phi$

Sufficiency: Let us suppose that every intuitionistic α-open set of X containing x meets A. If x $\notin$ Iα cl(A), there exists intuitionistic α-closed set F of X such that A $\subseteq$ F and x $\notin$ F. Therefore x $\in$ X-F $\in$ Iα OS(X). Hence X-F is an intuitionistic α-open set in X containing x but (X-F) $\bigcap$ A = ϕ which is a contradiction to the hypothesis. Consequently x $\in$ Iα cl(A).

Theorem 3.9: Let (X, τ) be an intuitionistic topological space and A = < X, A_1, A_2 > and B = < Y, B_1, B_2 > be two intuitionistic sets over X. Then,

1. A is an intuitionistic α-closed set iff A = Iα cl(A).
2. A is an intuitionistic α-open set iff A = Iα int(A).
3. (Iα cl(A))^c = Iα int(A^c)
4. (Iα int(A))^c = Iα cl(A^c)
5. A $\subseteq$ B $\Rightarrow$ Iα int(A) $\subseteq$ Iα int(B)
6. A $\subseteq$ B $\Rightarrow$ Iα cl(A) $\subseteq$ Iα cl(B)
7. Iα cl(ϕ) = ϕ and Iα cl(X) = X
8. Iα int(ϕ) = ϕ and Iα int(X) = X
9. Iα cl (A $\bigcup$ B) = Iα cl(A) $\bigcup$ Iα cl(B)
10. Iα int(A $\bigcap$ B) = Iα int(A) $\bigcap$ Iα int(B)
11. Iα cl (A $\bigcap$ B) $\subset$ Iα cl(A) $\bigcap$ Iα cl(B)
12. Iα int (A $\bigcup$ B) $\supset$ Iα int(A) $\bigcup$ Iα int(B)
13. I α cl(Iα cl(A)) = Iα cl(A)
14. I α int(Iα int(A)) = Iα int(A)

Proof:

1. Let A be an intuitionistic α-closed set.Then it is the smallest intuitionistic α-closed set containing itself (Since arbitrary intersection of intuitionistic α-closed set is intuitionistic α-closed). Hence A= Iα cl(A). Conversely, let A = Iα cl(A). Since Iα cl(A) being the intersection of intuitionistic α-closed sets is intuitionistic α-closed, so Iα cl(A) is an intuitionistic α-closed set. This implies A is an intuitionistic α-closed set of an intuitionistic topological space.
2. Let A be intuitionistic α-open. Since arbitrary union of intuitionistic α-open sets is intuitionistic α-open, A is the largest intuitionistic α-open set contained in A. Hence A = Iα int(A). Conversely, let A = I α int(A). Since Iα int(A) being the union of intuitionistic α-open sets is intuitionistic α-open, this implies A is intuitionistic α-open in an intuitionistic topological space.

3. (Iα cl(A))^c = ($\bigcap$ { K:K is an Iα CS in X and A $\subseteq$ K })^c
 = ($\bigcup$ {K^c}:{K^c} is an Iα OS in X and {K^c} $\subseteq$ {A^c} })
 = Iα int {A^c}
4. (Iα int(A))^c = ($\bigcup$ {G:G is an Iα OS in X and G $\subseteq$ A})
 = ($\bigcap$ { G^c : G^c is an Iα CS in X and G^c $\supseteq$ A^c })
 = ($\bigcap$ {G^c : G^c is an Iα CS in X and A^c $\subseteq$ G^c})
 = Iα cl{A^c}
5. Iα int(A) = ($\bigcup$ { G:G is an Iα OS in X and G $\subseteq$ A })
 Iα int(B) = ($\bigcup$ { G:G is an Iα OS in X and G $\subseteq$ B })
 Now Iα int(A) $\subseteq$ A $\subseteq$ B. This implies Iα int(A) $\subseteq$ B. Since Iα int(B) is the largest intuitionistic α-open set contained in B. Hence Iα int(A) $\subseteq$ Iα int(B).
6. Iα cl(A) = ($\bigcap$ {K:K is an I α CS in X and A $\subseteq$ K })
 Iα cl(B) = ($\bigcap$ {K:K is an I α CS in X and B $\subseteq$ K })
 Since A $\subseteq$ Iα cl(A) and B $\subseteq$ Iα cl(B) $\Rightarrow$ A $\subseteq$ B $\subseteq$ Iα cl(B) $\Rightarrow$ A $\subseteq$ Iα cl(B).
 But Iα cl(A) is smallest intuitionistic α-closed containing A.
 Therefore, Iα cl(A) $\subseteq$ Iα cl(B).

7. Since ϕ and X are intuitionistic α-closed sets, then by (1) Iα cl(ϕ) = ϕ and Iα cl(X) = X.
8. Since ϕ and X are intuitionistic α- open sets, then by (2) Iα int(ϕ) = ϕ and α int(X) = X.
9. Since $A \subset A \bigcup B$, $B \subseteq A \bigcup B$ and $A \subseteq B \Rightarrow$ Iα cl(A) $\subseteq$ Iα cl(B). Iα cl(A) $\subseteq$ Iα cl($A \bigcup B$), Iα cl(B) $\subseteq$ Iα cl($A \bigcup B$) $\Rightarrow$ Iα cl(A)$\bigcup$Iα cl(B)$\subseteq$Iα cl($A \bigcup B$).Now Iα cl(A), Iα cl(B) is intuitionistic α-closed. This implies Iα cl(A) $\bigcup$Iα cl(B) is intuitionistic α-closed. Then $A \subseteq$ Iα cl(A) and $B \subseteq$Iα cl(B) $\Rightarrow$ $A \bigcup B \subseteq$ Iα cl(A) $\bigcup$ Iα cl(B) that is Iα cl(A)$\bigcup$Iα cl(B) is intuitionistic α-closed containing $A \bigcup B$. But Iα cl($A \bigcup B$) is smallest intuitionistic α-closed containing $A \bigcup B$. Hence Iα cl($A \bigcup B$)$\subseteq$Iα cl(A)$\bigcup$ Iα cl(B).Therefore Iα cl($A \bigcup B$) =Iα cl(A) $\bigcup$ Iα cl(B).
10. Since $A \bigcap B \subset A$, $A \bigcap B \subset B$ and $A \subseteq B \Rightarrow$I$\alpha$ int(A) $\subseteq$ Iα int(B). Then, Iα int($A \bigcap B$) $\subseteq$ Iα int(A) and Iα int($A \bigcap B$)$\subseteq$Iα int(B).This implies Iα int($A \bigcap B$)$\subset$Iα int(A)$\bigcap$Iα int(B). Now Iα int(A), Iα int(B) is intuitionistic α-open in X. This implies Iα int($A \bigcap$) Iα int(B) is intuitionistic α-open in X. Then Iα int(A) $\subset$ A and Iα int(B)$\subset$ B $\Rightarrow$Iα int(A) $\bigcap$Iα int(B) $\subset A \bigcap B$ that is Iα int(A) $\bigcap$ Iα int(B) is an intuitionistic α-open set contained in $A \bigcap B$. Therefore Iα int(A) $\bigcap$Iα int(B) $\subset$Iα int($A \bigcap B$). Iα int($A \bigcap B$) = Iα int(A) $\bigcap$Iα int(B).
11. $A \bigcap B \subset A$ and $A \bigcap B \subset B$ $\Rightarrow$Iα cl($A \bigcap B$) $\subset$Iα cl(A) and Iα cl($A \bigcap B$) $\subset$ Iα cl(B) $\Rightarrow$Iα cl($A \bigcap B$)$\subset$ Iα cl(A) $\bigcap$ Iα cl(B).
12. $A \subset A \bigcup B$ and $B \subset A \bigcup B$ $\Rightarrow$Iα int(A) $\subset$ Iα int($A \bigcup B$) and Iα int(B) $\subset$ Iα int($A \bigcup B$) $\Rightarrow$ Iα int($A \bigcup B$) $\Rightarrow$ Iα int(A)$\bigcup$Iα int(B) $\subset$ Iα int($A \bigcup B$).
13. A is intuitionistic α-closed iff A = Iα cl(A). Since Iα cl(A) is intuitionistic α-closed, Iα cl(Iα cl(A)) = Iα cl(A).
14. Since Iα int(A) is intuitionistic α-open and A is intuitionistic α-open iff A=Iα int(A), therefore Iα int(Iα int(A)) = Iα int(A).

Example 3.10: Let X={a,b,c} and τ = {ϕ, X, A, B} where A= < X,{a},{b,c} >, B = < X,{a,b},{c} >} C = <X,{c},{a}> then Iα cl(B) = < X,{a,b,c},ϕ > ,Iα cl(C) = <X,{c},{a}> , Iα cl($B \bigcap C$) = <X, ϕ ,{a,c} >, Iα cl(B)$\bigcap$ Iα cl (C) = <X,{c},{a}>. From above we get that Iα cl($B \bigcap C$) $\subset$ Iα cl(B)$\bigcap$ Iα cl (C) but the converse is not true.

Example 3.11: Let X={a,b,c} and τ = { ϕ, X, A, B} where A= < X,{a},{b,c} >, B = < X,{a,b},{c} >} C = <X,{c},{a}> then Iα int(B) = <X,{a,b},{c}> ,Iα int(C) = <X,ϕ,{a,b,c}> ,Iα int($B \bigcup C$)= <X,{a,b,c},ϕ >, Iα int(B)$\bigcup$Iα int (C)= <X,{a,b},{c}>. From above we get that Iα int(B) $\bigcup$ Iα int (C) $\subset$ Iα int($B \bigcup C$) but the converse is not true.

Definition 3.12: In an intuitionistic topological space (X,τ) a point $p \in X$ is called intuitionistic α-limit point of A if any intuitionistic α-open set containing p contains a point of A disjoint from p. The set of all intuitionistic α-limit points is denoted as Iα d(A).

Theorem 3.13: A is intuitionistic α-closed iff Iα d(A) $\subseteq$A

Proof: Necessity:

Let A be an intuitionistic α-closed set and $p \in$ Iα d(A). Assume $p \notin A$ then $p \notin X-A$. As X-A is intuitionistic α-open and disjoint from A, $p \notin$ Iα d(A), which is a contradiction. Hence $p \in A$. Thus Iα d(A) $\subseteq$A.

Sufficiency:

Suppose Iα d(A) $\subseteq$A. Let $p \in X-A$, then $p \notin A$ and so $p \notin$ Iα d(A). Hence there is an intuitionistic α-open set B which contains p but contains no point of A. Since $p \notin A \Rightarrow p \in B \subseteq X-A$. As p is an arbitrary point of X-A and arbitrary union of intuitionistic α-open sets is intuitionistic α-open, X-A is the union of intuitionistic α-open sets and hence intuitionistic α-open. Hence A is intuitionistic α-closed.

Theorem 3.14: Iα d(Iα d(A)) - A$\subseteq$Iα d(A).

Proof: Let $p_{\sim} \in$ Iα d(Iα d(A))-A and B be any intuitionistic α-open set containing $p_{\sim}$.Then $B \bigcap$ (Iα d(A)- $p_{\sim}$) $\neq \phi$. Let $q_{\sim} \in B \bigcap$ (Iα d(A)- $p_{\sim}$). Since $q_{\sim} \in$ Iα d(A) and $q_{\sim} \in B$, so $B \bigcap (A- q_{\sim}) \neq \phi$. Let $r_{\sim} \in B \bigcap (A- q_{\sim})$. Then $r_{\sim} \neq p_{\sim}$ for $r_{\sim} \in A$ but $p_{\sim} \notin A$. Therefore $B \bigcap (A- p_{\sim}) \neq \phi$. Hence $p_{\sim} \in$ Iα d(A).

Theorem 3.15: $I\alpha d(A \bigcup I\alpha d(A)) \subseteq A \bigcup I\alpha d(A)$.

Proof: Let $p_\sim \in I\alpha d(A \bigcup I\alpha d(A))$ and $p_\sim \notin A$. If B is an intuitionistic α -open set containing $p_\sim$ then $B \bigcap [(A \bigcup I\alpha d(A)) - p_\sim] \neq \phi$. This implies $B \bigcap (A - p_\sim) \neq \phi$. For, if $B \bigcap [I\alpha d(A) - p_\sim] \neq \phi$ then by proof in previous theorem, $B \bigcap [A - p_\sim] \neq \phi$. Hence $p_\sim \in I\alpha d(A)$.

References

[1]. Andrijevic D, Some Properties of the Topology of α Sets, Mat. Vesnik 36(1984),1-10.
[2]. Andrijevic D, Semi Preopen Sets, Mat. Vesnik 38(1986), 24-32.
[3]. Coker D, A note on intuitionistic sets and intuitionistic points. Turkish J. Math. 1996; 20(3):343-351.
[4]. Coker D, An introduction to intuitionistic fuzzy topological spaces. Fuzzy Sets and Systems.1997; 88(1):81-89.
[5]. Coker D, An introduction to intuitionistic topological spaces. Busefal. 2000; 81:51-56.
[6]. Gnanambal Ilango, S. Selvanayaki. Generalized preregular closed sets in intuitionistic topological spaces. Internat. J. Math. Archive. 2014; 5(4):1-7.
[7]. Gnanambal Ilango, S.Girija. Some More Results on Intuitionistic Semi Open Sets. Int. Journal of Engineering Research and Applications.Vol 4,Issue 11, Nov 2014,pp.70-74
[8]. Maheswari and Thakur,{Some Problems In Topology}, Ph.D Thesis.
[9]. Olav Njastad, On Some Classes of Nearly Open Sets, Pacific Journal of Mathematics 15, No.3,(1965) 961 -970.
[10]. Younis J. Yaseen, Asmaa G. Raouf. On generalization closed set and generalized continuity on intuitionistic topological spaces. J.of Al-Anbar University for Pure Science, 2009; 3(1).
[11]. Zadeh L.A, Fuzzy Sets, Inform and Control,8,(1965),338-353.