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Solving Transformed Differential Equation Using Adomian Decomposition Method

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Abstract: *The purpose of this work is to transform the linear initial value problem (IVP) to a voltera integral equation of the second kind and provide a reliable solution for the transformed voltera integral equation using the well-knownAdomian decomposition method (ADM).Some few examples are discussed to show the reliability and the performance of the decomposition method. The results obtained are in better agreement with the exact solutions.*

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I. Introduction

The decomposition method has been shown to solve effectively, easily, and accurately a large class of linear and nonlinear, ordinary, partial, deterministic or stochastic differential equations and integro-differential equations with approximate solutions which converge rapidly to accurate solutions,Cherrualt (1989), Abbaoui and cherruault(1994).The basic motivation of this work is to apply the ADM to the transformed volterra integral equation. This algorithm provides the solution in a rapidly convergent series Wazwaz (2011)

II. Analysis Of The Transformation Method.

Consider the linear initial value problem of the form

$$\frac{d^n y}{dx^n} + a_{1(x)}\frac{d^{n-1}y}{dx^{n-1}} + . \quad . \quad . + a_{n(x)}y = f(x) \qquad \text{... (2.1)}$$

With continuous coefficients and initial conditions as follows

$$y(0) = c_0 , y'(0) = c_1 , y''(0) = c_2 , \quad \ldots \ y^{n-1}(0) = c_{n-1} \qquad \text{... (2.2)}$$

In order to reduce the above IVP to a voltera integral equation.
We make the following transformation.

Let

$$\frac{d^n y}{dx^n} = u(x) \qquad \text{... (2.3)}$$

Integrating both sides of (2.3) with respect to $x\ from\ 0\ to\ x\ gives$

$$\frac{d^{n-1}y}{dx^{n-1}} = \int_0^x u(t)\,dt + c_{n-1} \qquad \text{... (2.4)}$$

Thus, successive integrals yields

$$\frac{d^{n-2}y}{dx^{n-2}} = \int_0^x \int_0^x u(t)\,dtdt + c_{n-1}x + c_{n-2}$$

Proceeding in this manner, and applying the formula of the n-fold integration we obtain

$$y(x) = \int_0^x \int_0^x \int_0^x . . . \int_0^x u(t)dtdtdt \ \ldots dt + c_{n-1}\frac{x^{n-1}}{(n-1)} + c_{n-2}\frac{x^{n-2}}{(n-2)} + \ldots c_1 x + c_0$$

$$= \int_o^x \frac{(x-t)^{n-1}}{(n-1)!}u(t)dt + c_{n-1}\frac{x^{n-1}}{(n-1)!} + c_{n-2}\frac{x^{n-2}}{(n-2)!} + \ldots + c_1 x + c_0 \qquad \text{... (2.5)}$$

Substituting the above into equation 2.1, we get the following results
$u(x) + \int_0^x k\,(x,t)u(t)dt = f(x)$, so that

$$u(x) = f(x) - \int_0^x k(x\,,t)\,u(t)dt \qquad \text{... (2.6)}$$

III. The Adomian Decomposition Method (Adm)

In this section, we use the technique of the Adomian Decomposition Method, Burton (2005). Wazwaz,(2011) and Maturi,(2014)). The ADM consist of decomposing the unknown function $u(x)$ of any equation into a sum of infinite number of components defined by the series

$$u(x) = \sum_{n=0}^{\infty} u_n(x), \qquad \dots (2.7)$$

Or equivalently

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots \qquad \dots (2.8)$$

Where the components $u_n(x)$, $n \geq 0$ will be determined recurrently. The ADM concerns itself with finding the components $u_0(x), u_1(x), u_2(x), u_3(x), \dots$ individually.

To establish the recurrence relation, we substitute (2.7) into the volterra integral equation (2.6) to obtain

$$\sum_{n=0}^{\infty} u_n(x) = f(x) + \int_0^x k(x,t)\left(\sum_0^{\infty} u_n(t)\right)dt, \qquad \dots (2.9)$$

The zeroth component$u_0(x)$ is identified by all the terms that are not included under the integral sign. Therefore, we take

$$u_0(x) = f(x), \qquad u_{n+1}(x) = \int_0^x k(x,t)u_n(t)\,dt, \quad n \geq 0 \qquad \dots (2.10)$$

In view of (2.10), the components $u_0(x), u_1(x), u_2(x), u_3(x), \dots$ are completely determined. As a result, the solution $u(x)$ of the volterra integral equation (2.6) is readily obtained in series form by using the series(2.7)

IV. Some Examples.

Example.1

Consider the IVP $y'' + y = sinx$, $y(0) = 0$, $y'(0) = 0$ $\qquad \dots (2.11)$

The exact solution is$y(x) = \frac{1}{2}sinx - \frac{1}{2}xcosx$

First, we convert the equation to an equivalent volterra integral equation and apply the ADM for analytic solution.

Let $y''(x) = u(x)$ $\qquad \dots (2.12)$

Integrating both sides of the equation (2.8) and applying the initial condition, we get

$$y'(x) = \int_0^x u(t)dt \qquad \dots (2.13)$$

Integrate the equation (2.13) and apply the initial condition, we get

$$y(x) = \int_o^x \int_0^x u(t)dtdt \qquad \dots (2.14)$$

Now, we apply theformula of the $\text{n} - \text{fold}$ integrals given by

$$\int_0^x \int_0^x \int_0^x \dots\dots\dots\dots\dots \int_0^x u(t)dtdtdt\dots dt = \int_0^x \frac{(x-t)^{n-1}}{(n-1)!}u(t)dt$$

The integral equation (2.14) becomes

$$y(x) = \int_0^x (x-t)u(t)\,dt \qquad \dots (2.15)$$

We now substitute (2.12) *and* (2.15) *in equation* (2.11) to get our required volterra integral equation as follows

$$u(x) + \int_0^x (x-t)u(t)dt = sinx$$

$$u(x) = sinx - \int_0^x (x-t)u(t)dt \qquad \dots (2.16)$$

Now, we apply the (ADM) to the volterra integral equation(2.16), we have as follows

$$\sum_{n=0}^{\infty} u_n(x) = sinx - \int_0^x (x-t)\sum_{n=0}^{\infty} u_n(x)\,dt \qquad \dots (2.17)$$

To determine the components of $u(x)$, we use the recurrence relation.

$$u_0(x) = sinx, \quad and \qquad u_{n+1}(x) = -\int_0^x (x-t)u_n(t)dt, \quad n \geq 0 \qquad \dots (2.18).$$

So that

$$u_1(x) = -x + sinx$$

$$u_2(x) = -x + sinx + \frac{1}{6}x^3$$

$$u_3(x) = -x + sinx + \frac{1}{6}x^3 - \frac{1}{120}x^5 \qquad \text{... (2.19)}$$

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Hence,

$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots \qquad \text{... (2.20)}$$

For a four terms solution, we have

$$u(x) = 4sinx - 3x + \frac{1}{3}x^3 - \frac{1}{120}x^5 = x - \frac{1}{3}x^3 + \frac{1}{40}x^5 \qquad \text{... (2.21)}$$

Recall from equation (2.12) *that*

$y''(x) = u(x),$

Therefore

$$y''(x) = x - \frac{1}{3}x^3 + \frac{1}{40}x^5 \qquad \text{... (2.22)}$$

Integrating both sides twice, and applying the initial conditions, we get

$$y(x) = \frac{1}{6}x^3 - \frac{1}{60}x^5 + \frac{1}{1680}x^7 \qquad \text{... (2.23)}$$

Which is the analytic solution.

Example 2.

Consider the IVP

$$y''' + 4y' = x, \qquad y(0) = 0, y' = 0, y''(0) = 1 \qquad \text{... (2.24)}$$

Exact solution is $y(x) = -\frac{3}{16}cos2x + \frac{3}{16} + \frac{1}{8}x^2$

Converting (2.24) to an equivalent Volterra integral equation of second kind,

Let

$$y'''(x) = u(x) \ . \qquad \text{... (2.25)}$$

Integrating both sides and applying the initial condition, we get

$$y''(x) = \int_0^x u(t)dt + 1 \qquad \text{... (2.26)}$$

Integrating again and applying the appropriate initial condition, we get

$$y'(x) = \int_0^x \int_0^x u(t)dtdt + x \qquad \text{... (2.27)}$$

Applying the formula of the n $-fold\ integral, equation\ (2.27)\ becomes$

$$y'(x) = \int_0^x (x - t)u(t)dt + x \qquad \text{... (2.28)}$$

Now, we substitute equation (2.25) and (2.28) into equation (2.24) to get our desired VIE as follow

$$u(x) = -3x - 4\int_0^x (x - t)u(t)dt \qquad \text{... (2.29)}$$

Now, we apply the ADM to the VIE (2.29)

$$\sum_{n=0}^{\infty} u_n(x) = -3x - 4\int_0^x (x - t)\sum_{n=0}^{\infty} u_n(t)dt \qquad \text{... (2.30)}$$

To determine the components of $u(x)$, we use the recurrence relation.

$$u_0(x) = -3x, \qquad u_{n+1}(x) = -4\int_0^x (x - t)u_n(t)dt\ ,\ n \geq 0. \qquad \text{... (2.31)}$$

$$u_0(x) = -3x$$

$$u_1(x) = 2x^3$$

$$u_2(x) = -\frac{2}{5}x^5$$

$$u_3(x) = \frac{4}{105}x^7$$

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.
Hence,
$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) \ldots$
For a four terms solution, we have

$$u(x) = -3x + 2x^3 - \frac{2}{5}x^5 + \frac{4}{105}x^7 \qquad \ldots (2.32)$$

Recall from equation (4.14) that $y'''(x) = u(x)$,
Therefore,

$$y'''(x) = -3x + 2x^3 - \frac{2}{5}x^5 + \frac{4}{105}x^7 \qquad \ldots (2.33)$$

Integrating both sides of equation (2.33) three times and applying the appropriate initial condition at each point gives the solution as follows

$$y(x) = \frac{1}{2}x^2 - \frac{1}{8}x^4 + \frac{1}{60}x^6 - \frac{1}{840}x^8 + \frac{1}{18900}x^{10} \qquad \ldots . (2.34)$$

V. Numerical Result.

X	EXACT	ADM	ABSOLUTE ERROR
0	0	0	0
0.2	0.001328	0.001331	2.65607E-06
0.4	0.010497	0.010518	2.10027E-05
0.6	0.034721	0.03479	6.96519E-05
0.8	0.079995	0.080158	0.000162211
1	0.150584	0.1509	0.000315661
1.2	0.248605	0.249171	0.000566074
1.4	0.373748	0.374756	0.001008337
1.6	0.523146	0.525026	0.001879516

TABLE 1: Result for Example 1

Plots of the Exact and ADM solution for example 1

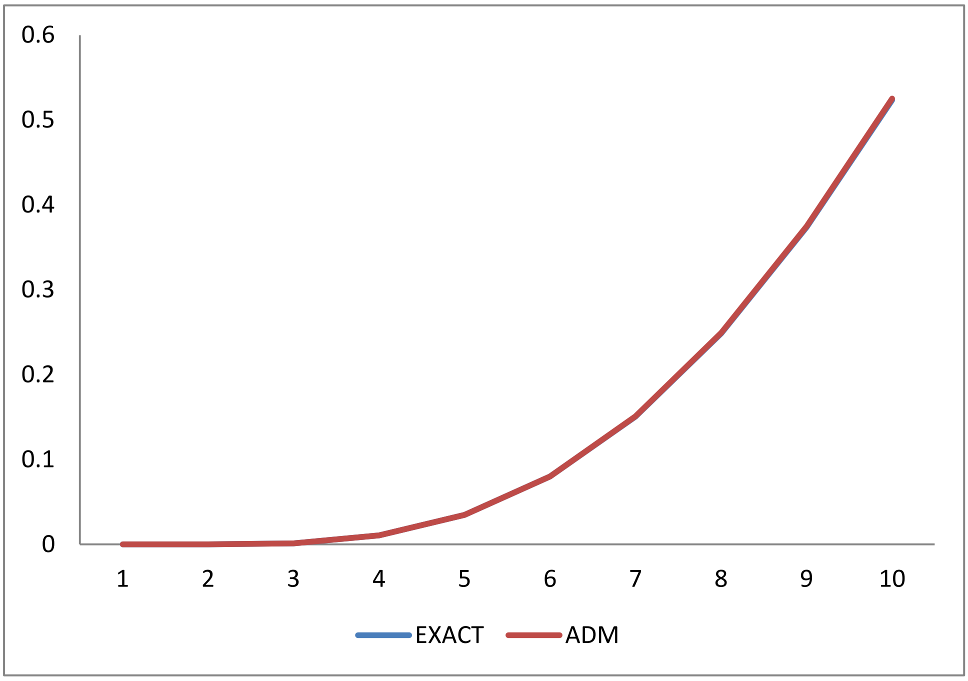
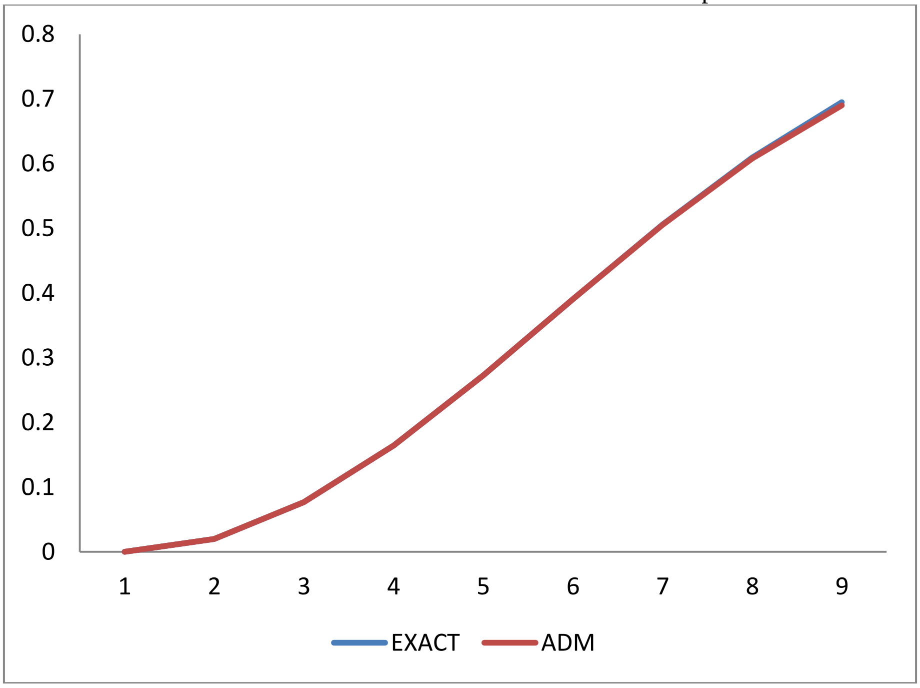


TABLE 2: Result for Example 2

X	EXACT	ADM	ABSOLUTE ERROR
0	0	0	0
0.2	0.019801	0.019801	2.12792E-09
0.4	0.076867	0.076868	1.31012E-07
0.6	0.164558	0.164559	1.23874E-06
0.8	0.272975	0.272978	3.16561E-06
1	0.390528	0.39051	1.80081E-05
1.2	0.505761	0.505547	0.000214219
1.4	0.609167	0.607974	0.001192355
1.6	0.69468	0.689849	0.004831327

Plots of the Exact and the ADM solution for example 2



VI. Conclution

The linear initial value problem(IVP) has been successfully transformed to a volterra integral equation of the second kind, and the solution was readily obtained in series form using the Adomian decomposition method. It is apparently seen from the tables and graphs that ADM is a very powerful and efficient technique in finding analytical solutions for wide classes of integral equations. It is worth pointing out that this method presents a rapid convergence for the solutions. The results show that ADM is powerful mathematical tool for solving linear equations.

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