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Recognizable Infinite Triangular Array Languages

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Abstract: *In this paper we extend to infinite triangular arrays the concept of triangular domino systems and triangular tiling systems which recognize infinite triangular arrays and show that the class of ωω-triangular array languages recognized by triangular domino systems is same as the class of ωω-triangular array languages recognized by triangular tiling systems. Also we introduce triangular Wang systems and prove that the class of ωω-triangular languages obtained by triangular Wang systems is the same as the class of recognizable ωω-triangular languages*

Keywords: *Infinite triangular domino systems,Wang recognizable infinite triangular array languages, labelled triangular Wang tile.*

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I. Introduction

Infinite triangular pictures are the digitized images which occur in the two dimensional plane. Infinite triangular picture p is a triangular array of elements of alphabets. It is useful to introduce the notation $\Sigma_T^{\omega\omega}$ for the set of all infinite triangular pictures over the same alphabet Σ. Infinite triangular picture has infinite number of rows and infinite number of right slanting lines. The size |p| of a picture p is specified by the pair ($|p|_{row}$, $|p|_{rsline}$) of its number of rows and number of right slanting lines. A pixel p(i, j), $1 \le i \le |p|_{row}$, $1 \le j \le |p|_{rsline}$ is the element at position (i, j) in the triangular array P. Conventionally the indices grow from bottom to top for the rows and from left to right for right slanting lines.

For convenience we usually consider the bordered version of picture p obtained by surrounding the picture with the special boundary symbol # which is assumed not to be in the alphabet. In [4] domino recognizability of triangular picture languages and hrl-domino systems are defined. Also in [4] we define the overlapping of iso-triangular pictures. In this paper we extend to infinite triangular arrays the concept of triangular domino systems and triangular tiling systems which recognize infinite triangular arrays and show that the class of ωω-triangular array languages recognized by triangular domino systems is same as the class of ωω-triangular array languages recognized by triangular tiling systems. In [1,2] we study about infinite arrays.In [5] we study the Wang systems for rectangular pictures. Here we introduce triangular Wang systems and prove that the class of ωω-triangular languages obtained by triangular Wang systems is the same as the class of recognizable ωω-triangular languages.

II. Preliminaries

We will see some definitions of infinite rectangular arrays [20].

Definition 2.1

A tiling system T = (Σ, Γ, θ, π) where Σ and Γ are two finite alphabets, θ is a finite set of triangular tiles over $\Gamma \cup \{\#\}$ and $\pi : \Gamma \to \Sigma$ is a projection from Γ to Σ. The tiling system T recognizes an ωω- language L over Σ if L = π(L′) where L′ = $L^{\omega\omega}(\theta)$ is the local ωω- language over Γ corresponding to the set of tiles. We denote by $\mathcal{L}^{\omega\omega}(TS)$, the family of ωω - languages recognized by tiling systems. This family is also denoted by ωω-REC.

Definition 2.2:

A labelled Wang tile is a 5-tuple, consisting of 4 colours,choosen in a finite set Q of colours and a label chosen in a finite alphabet Σ.

Definition 2.3:

A Wang system is a triplet W=(Σ,Q,T) where Σ is a finite alphabet,Q is a finite set of colours and T is a finite set of Wang tiles ,$T \subseteq Q^4 \times \Sigma$.

Definition 2.4:
Let W=(Σ,Q,T) be a Wang system. An infinite array M over T is a tiling of W,if it satisfies the following conditions.

1. $M(1,1) = B\ \underset{B}{\overset{q}{\boxed{a}}}\ p$, $M(1,n) = r\ \underset{B}{\overset{q}{\boxed{b}}}\ p$, n = 2, 3, ...

   $M(m,1) = B\ \underset{p}{\overset{r}{\boxed{c}}}\ q$, m = 2, 3, ...

2. $M(m,n) = s\ \underset{p}{\overset{r}{\boxed{a}}}\ q$, m, n = 2, 3, ...

   Here p, q, r, s ≠ B.

If M is a tiling of W, the label of M, denoted by |M|, is an infinite array over Σ,defined by

$$|M|(m,n)=a \Leftrightarrow M(m,n) = s\ \underset{p}{\overset{r}{\boxed{a}}}\ q \text{ , for some p,q,r,s.}$$

Definition 2.5:
Let W is a Wang system. An infinite array w is generated by W if there exists a tiling M such that |M| = w. We denote by $L^{\omega\omega}(W)$,the language of infinite arrays generated by the Wang system W.

Definition 2.6:
An infinite array language L is Wang recognizable if there exists a Wang system W such that L = $L_T^{\omega\omega}(W)$. The family of all Wang recognizable infinite array languages is denoted by $\mathcal{L}^{\omega\omega}(WS)$.

Definition 2.7:
Let M = (Q, Σ, δ, I, F) be a two direction online tessellation automata (2DOTA). We say that $L \subseteq \Sigma^{\omega\omega}$ is recognized by M in (inf, ∩)-mode if L = {p ∈ $\Sigma^{\omega\omega}$: inf(r(p)) ∩ F ≠ ϕ for some run r(p)} and we write L = $L^{\omega\omega}(M)$.

III. Recognizability Of ωω-Triangular Languages

Definition 3.1:
A triangular tiling system T = (Σ, Γ, θ, π) where Σ and Γ are two finite alphabets, θ is a finite set of triangular tiles over Γ ∪ {#} and π : Γ → Σ is a projection from Γ to Σ. The triangular tiling system T recognizes an ωω -triangular array language L over Σ if L = π(L′) where L′ = $L^{\omega\omega}(\theta)$ is the local ωω-triangular array language over Γ corresponding to the set of tiles. We denote by $\mathcal{L}_T^{\omega\omega}(ITTS)$, the family of ωω-triangular array languages recognized by triangular tiling systems.This family is also denoted by ωω-TREC.

Theorem 3.1
$$\mathcal{L}_T^{\omega\omega}(ITDS) = \mathcal{L}_T^{\omega\omega}(ITTS)$$

For proving this theorem we need to prove the following propositions.

Proposition 3.1.

Clearly $\mathcal{L}_T^{\omega\omega}(\mathrm{ITDS}) \subseteq \mathcal{L}_T^{\omega\omega}(\mathrm{ITTS})$.

To prove the converse part we need the following proposition.

Proposition 3.2.

The family $\mathcal{L}_T^{\omega\omega}(\mathrm{ITTS})$ is closed under projection.

Proof.

Let Σ_1, Σ_2 be two alphabets. Let $p : \Sigma_1 \to \Sigma_2$ be a projection.

Let $L \subseteq \Sigma_T^{\omega\omega}$ be recognized by the triangular tiling system.

$T_1 = (\Sigma_1, \Gamma, \theta, \pi_1)$ where $L = \pi_1(L')$ and $L' = L_T^{\omega\omega}(\theta)$

Consider the triangular tiling system $T_2 = (\Sigma_2, \Gamma, \theta, \pi_2)$ where

$\pi_2 = p \circ \pi_1 : \Gamma \to \Sigma_2$.

Now $p(L) = p(\pi_1(L')) = (p \circ \pi_1)(L') = \pi_2(L')$.

Therefore $p(L)$ is recognized by the triangular tiling system T_2.

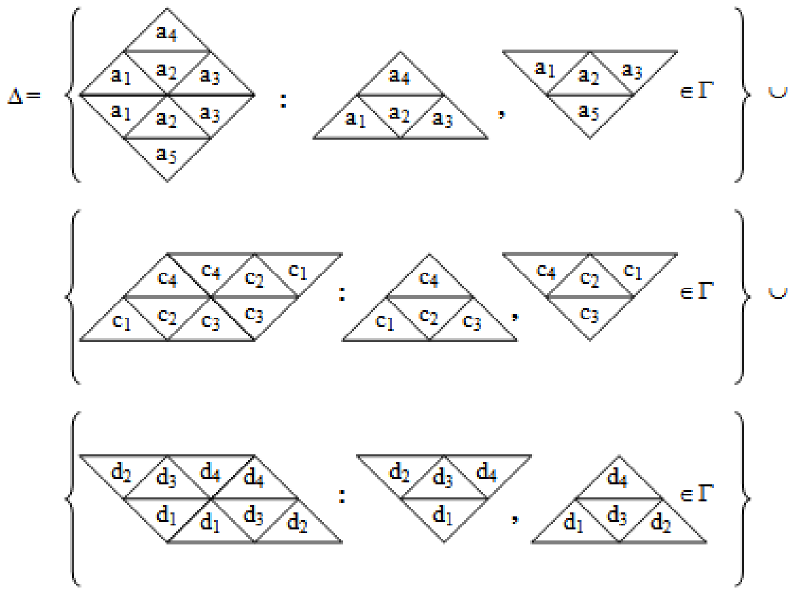
Hence $\mathcal{L}_T^{\omega\omega}(\mathrm{ITTS})$ is closed under projection.

Proposition 3.3.

If L is a local $\omega\omega$-triangular language over Σ, then there is an hrl-local $\omega\omega$-triangular language L' over an alphabet Γ and a mapping $\pi : \Gamma \to \Sigma$ such that $L = \pi(L')$.

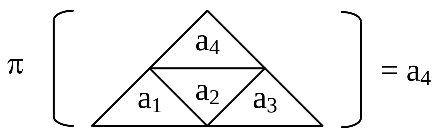
Proof.

Let $L = L_T^{\omega\omega}(\theta)$ where θ is a finite set of triangular tiles over $\Sigma \cup \{\#\}$. Let $\Gamma = \theta$ and



Let $L' = L_T^{\omega\omega}(\Delta)$. Then L' is an hrl-local $\omega\omega$-triangular language over Γ.

Define π: $\Gamma \to \Sigma$ by



Then $L = \pi(L')$.

Combining propositions we have the following result.

IV. Labeled Wang Tiles

In this section we use a different formalism to recognize ωω-triangular picture languages. Wang tiles are introduced for the tiling of Euclidean plane. De Prophetics and Varricchio [5] have introduced the notion of Wang tile by adding a label taken as a finite alphabet, to a Wang tile.They have also defined Wang systems and proved that the family of array languages recognized by Wang systems coincides with the family of recognizable array languages.

We extend the concept of ωω-languages to ωω-triangular array languages and prove that the family of ωω-triangular array languages recognized by triangular Wang systems is the same as the family of recognizable ωω-triangular array languages.

Definition 4.1:

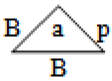
A labelled triangular Wang tile is a 4-tuple consisting of three colours chosen in a finite set Q of colours and a label chosen in a finite alphabet Σ.

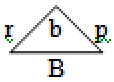
Definition 4.2:

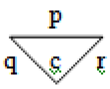
A triangular Wang system is a triple W = <Σ, Q, T> where Σ is a finite alphabet, Q is a finite set of colours, T is a finite set of triangular Wang tiles, $T \subseteq Q^3 \times \Sigma$.

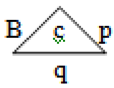
Definition 4.3:

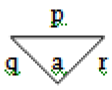
Let W = <Σ, Q, T> be a triangular Wang system. An infinite triangular array M over T is a tiling of W, if it satisfies the following conditions:

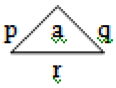
1. M(1, 1) = 

M(1, n) =  if n = 3, 5, 7, …

M(1, n) =  if n = 2, 4, 6, …

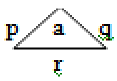
M(m, 1) =  if m = 2, 3, …

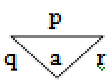
2. M(m,n) =  if (m, n) = (2, 2), (2, 4), ...
(3, 2), (3, 4), ...
(4, 2), (4, 4), ...

M(m,n) =  if (m, n) = (2, 3), (2, 5), ...
(3, 3), (3, 5), ...
(4, 3), (4, 5), ...

Here p, q, r ≠ B.

If M is a tiling of W, the label of M, denoted by |M|, is an infinite over Σ, defined by

|M|(m, n) = a ⇔ M(m, n) =  for some p, q, r.

M(m, n) =  for some p, q, r.

Definition 4.5:

Let W be a triangular Wang system. An infinite triangular array w is generated by W if there exists a tiling M such that |M| = w. We denote by $L_T^{\omega\omega}(W)$, the language of infinite triangular arrays generated by the triangular Wang system W.

Definition 4.6:

An infinite triangular array language L is Wang recognizable if there exists a triangular Wang system W such that L = $L_T^{\omega\omega}(W)$. The family of all Wang recognizable infinite triangular array languages is denoted by $\mathcal{L}_T^{\omega\omega}(\mathrm{ITWS})$.

Theorem 4.1

$\omega\omega\text{-TREC} = \mathcal{L}_T^{\omega\omega}(\mathrm{ITWS})$

For proving this theorem we need to prove the following propositions.

Proposition 4.1.

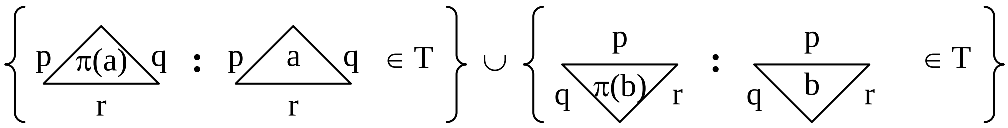
$\mathcal{L}_T^{\omega\omega}(\mathrm{ITWS})$ is closed under projection.

Proof.

Let W = (Γ,Q,T) be a triangular Wang system and $\pi : \Gamma \to \Sigma$ be a projection. We have to show that if L = $L_T^{\omega\omega}(W)$, then L′ = π(L) = $L_T^{\omega\omega}(W')$ for some W′.

Let W′ = (Σ, Q, T ′) where

T′=



Then L′ = $L_T^{\omega\omega}(W')$.

Proposition 4.2.

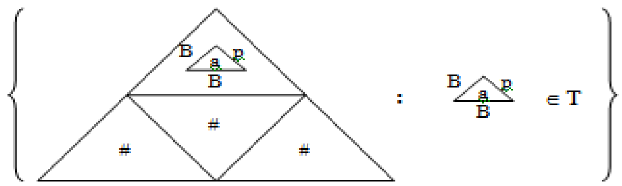
$\mathcal{L}_T^{\omega\omega}(\mathrm{ITWS}) \subseteq \omega\omega\text{-TREC}$.

Proof.

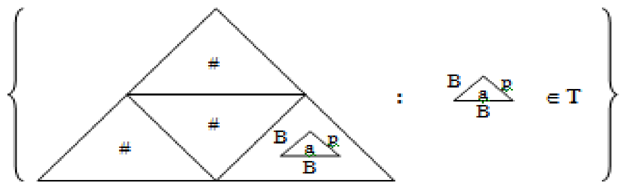
Let L ∈ $\mathcal{L}_T^{\omega\omega}(\mathrm{ITWS})$ and L = $L_T^{\omega\omega}(W)$ where W = (Σ, Q, T).

Let Γ = T and $\theta = \theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_4 \cup \theta_5$ where

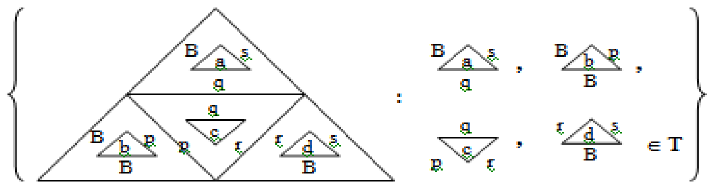
$\theta_1 =$

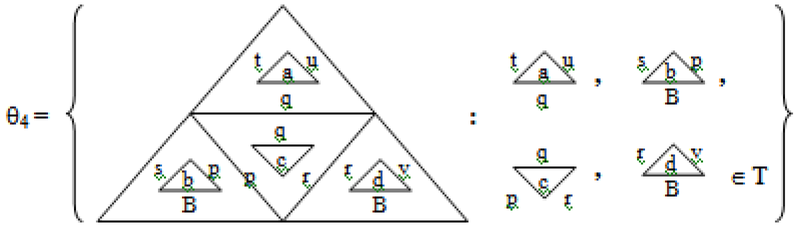


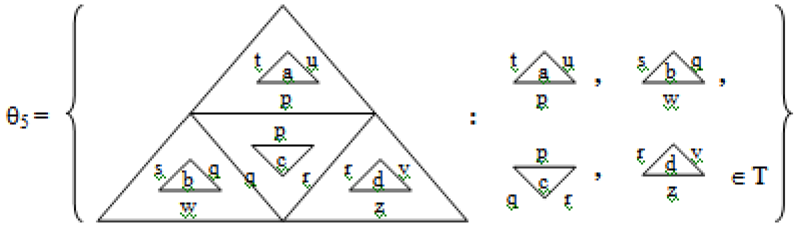
$\theta_2 =$



$\theta_3 =$







Let $L_1 = L_T^{\omega\omega}(\theta)$. Define $\pi : \Gamma \rightarrow \Sigma$ by = a.
Then $L = \pi(L_1)$ and therefore $L \in \omega\omega$-TREC.

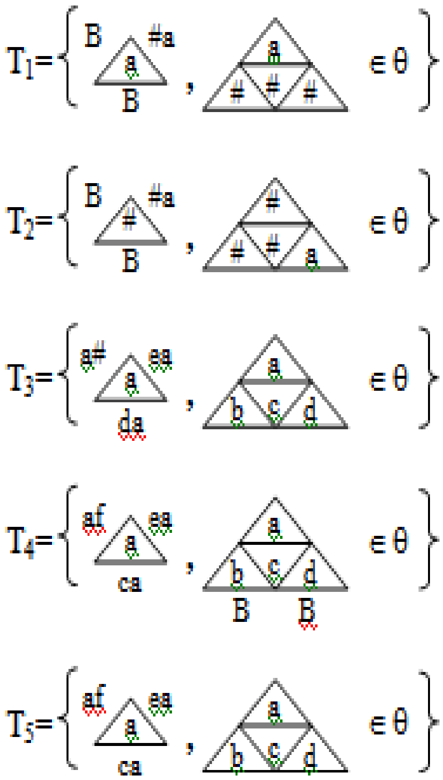
Proposition 4.3.

$$\omega\omega\text{-TREC} \subseteq \mathcal{L}_T^{\omega\omega}(\text{TWS}).$$

Proof.

Let $L \in \omega\omega$-TREC. Then there exists a local $\omega\omega$-triangular array language L_1 over Γ and a projection $\pi : \Gamma \rightarrow \Sigma$ such that $L = \pi(L_1)$. Let $L_1 = L_T^{\omega\omega}(\theta)$ where θ is a finite set of triangular tiles over $\Gamma \cup \{\#\}$.

Consider the triangular Wang system $W = <\Sigma, Q, T>$ where $Q = (\Gamma \cup \{\#\}^2) \cup \{B\}$ where $B \notin \Gamma \cup \{\#\}$
$T = T_1 \cup T_2 \cup T_3 \cup T_4 \cup T_5$ where



Then L = $L_T^{\omega\omega}(W)$.

Thus L $\in$ $\mathcal{L}_T^{\omega\omega}(\mathrm{ITWS})$ and therefore $\omega\omega\text{-TREC} \subseteq \mathcal{L}_T^{\omega\omega}(\mathrm{ITWS})$.

Combining propositions 5.3.1,5.3.2 and 5.3.3 we have the result.

V. Automata Characterization Of Recognizable Infinite Triangular Array Languages

In this section we give automata characterization of recognizable infinite triangular array languages.

Definition 5.1:

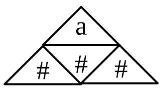
Let M = (Q, Σ, δ, I, F) be a two direction online tessellation automata (2DOTA). We say that L $\subseteq$ $\Sigma_T^{\omega\omega}$ is recognized by M in (inf, $\cap$)-mode if L = {p $\in$ $\Sigma_T^{\omega\omega}$: inf(r(p)) $\cap$ F $\neq$ ϕ for some run r(p)} and we write L = $L_T^{\omega\omega}(M)$.

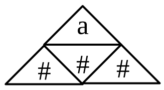
Theorem 5.1.

L $\in$ ωω-TREC if and only if L is recognized by a 2DOTA in (inf, $\cap$)-mode in which every state is a final state.

Proof.

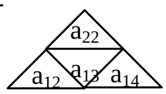
Let L $\in$ ωω-TLOC. Then there exists a finite set θ of triangular tiles such that L = $L_T^{\omega\omega}(\theta)$. Consider the 2DOTA, M = (Q, Σ, δ, q_0, F) where Q = θ

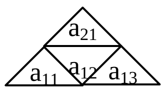
q_0 = {


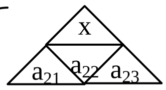
:


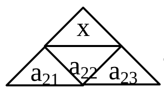
$\in \theta$ }

and δ : Q × Q × Σ → 2^Q such that

δ [


,


, x] = [


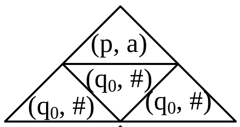
:


$\in \theta$]

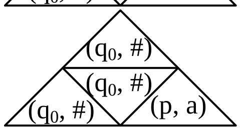
Then L = $L_T^{\omega\omega}(M)$. Since the languages recognized by 2DOTA is closed under morphism, every L$\in$ωω-TREC is recognized by a 2DOTA with F = Q.

Conversely let L be recognized by a 2DOTA M = (Q, Σ, δ, q_0, F).

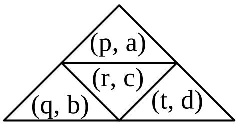
Let Γ = Q × (Σ ∪ {#}),

θ_1 = {


/ p $\in$ δ(q_0, q_0, a) }

θ_2 = {


/ q_0 $\in$ δ(q_0, q_0, #) }

θ_3 = {


/ p $\in$ δ(r, q, a) }

and θ = $\theta_1 \cup \theta_2 \cup \theta_3$.

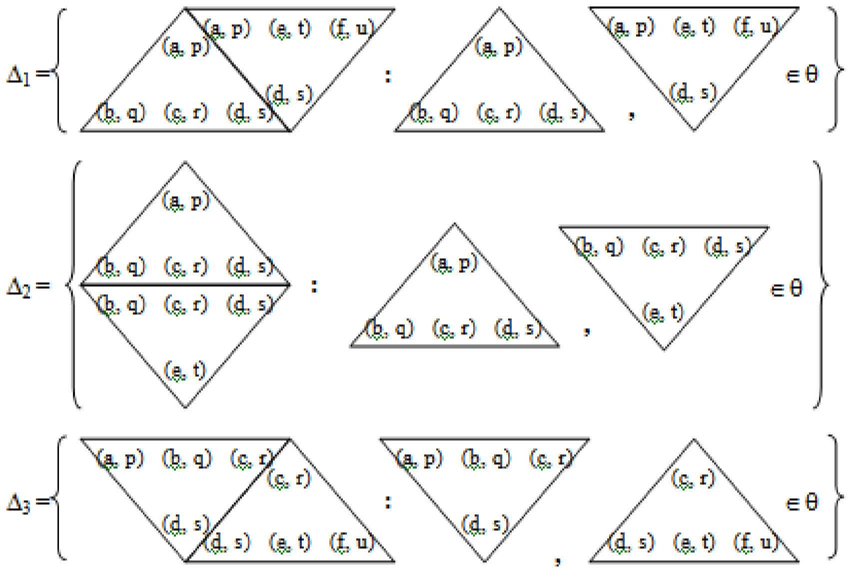
Let L_1 = $L_T^{\omega\omega}(\theta)$. Define π : Γ → Σ by π(a, p) = a. Then L = π(L_1). Therefore L $\in$ ωω-TREC.

Remark 5.1.

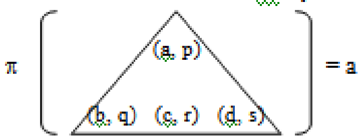
If L is recognized by a 2DOTA M = (Q, Σ, δ, q_0, F), then we can show that L is morphic image of an hrl-local ωω-language. Using the notation of theorem let

Γ_1 = θ

$\Gamma_1 = \theta$



$\Delta = \Delta_1 \cup \Delta_2 \cup \Delta_3$. Let $L_2 = L_T^{\omega\omega}(\Delta)$. Define $\pi_1 : \Gamma_1 \to \Sigma$ by



Then L_2 is hrl-local and $L = \pi_1(L_2)$.

VI. Learning Of Recognizable Infinite Triangular Array Languages

In this section we give learning algorithm for recognizable infinite triangular array languages. In [3], we have given an algorithm to learn hrl-local ωω-triangular array languages in the limit from positive data that are ultimately periodic arrays. We have proved that an ωω-triangular array language is recognizable if and only if it is a projection of a hrl-local ωω-triangular array language.

We now show how to derive a learning algorithm, for recognizable ωω-triangular array languages, from one that learn hrl-local ωω-triangular array languages.

Let $L \in$ ωω-TREC. Then L is recognized by some 2DOTA,
$M = (Q, \Sigma, \delta, q_0, F)$.
Let $\Gamma = Q \times (\Sigma \cup \{\#\})$ and let π_1 and π_2 be projections on Γ defined by $\pi_1(q, a) = q$ and $\pi_2(q, a) = a$. A triangular array p over Γ is called a computation description array if $\pi_1(p)$ is an accepting run of M on $\pi_2(p)$.

We note that

1. The alphabet Γ contains $n(m+1)$ elements, where n is the number of states of minimum 2DOTA for L and $m = |\Sigma|$.
2. For any positive example p of L, let C(p) the set of all computation description triangular arrays for p. Then C(p) has atmost $n^{A(p)}$ triangular arrays.
3. If U is an hrl-local ωω-triangular language over Γ such that $\pi(U)=L$ and E is a characteristic sample for U, then there is a finite set S_L of positive data of L such that $E \subseteq \pi^{-1}(S_L)$.

   We obtain a learning algorithm for the class ωω-TREC.

Algorithm TREC
Input: A positive presentation of an unknown recognizable ωω-triangular array language L,
$n = |Q|$ for the minimal 2DOTA which accepts L.
Output: A finite set Δ for dominoes such that $L=\pi(L_T^{\omega\omega}(\Delta))$

Procedure:

Procedure:

Initialize all parameters:

$E_0 = \Delta_0 = \phi$, answer = "no"

repeat

while answer = "no" do

i=i+1;

read the positive example p;

let C(p)={$w_1, w_2, \ldots\ldots. w_k$} be the set of all computation

descriptions for p;

let C($p_{(2)}$) = { $w_1', w_2', \ldots\ldots. w_k'$} where w_i' is a prefix of w_i and size of

w_i' = size of $p_{(2)}$;

let j = 0;

while (j<k) and answer = "no" do

j=j+1;

$E_i = E_i \cup$ { w_j};

scan w_j' to compute $B_2(w_j')$;

let $\Delta_i = \Delta_i \cup$ { $\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}$: α , $\beta \in B_2(w_j')$ and first row of α = second row

To prove the learning algorithm TREC terminates we need to prove the following result.

Proposition 6.1

Let n be the number of states for the 2DOTA recognizing the unknown recognizable ωω-triangular array language L. After atmost t(n) number of queries, Algorithm TREC produces a conjecture Δ_i such that E_i includes a characteristic sample for an hrl-local ωω-triangular language U with the property that L = π(U) where t(n) is a polynomial in n, depending on L.

Proof.

Let L be an unknown recognizable ωω-triangular array language. Let M = (Q,Σ,δ,q_0,F) be a 2DOTA which accepts L.Let |Q| = n and |Σ| = m.Then there is an hrl-local ωω-triangular array language U over $\Gamma = Q\times(\Sigma\cup\{\#\})$ and a projection π such that π(L′) = L. Let E be a characteristic sample for U. We have already proved that $U=U_1\oplus U_2$ where U_1 and U_2 be two local hrl ωω-triangular languages and $E = E_1 \oplus E_2$ are characteristic samples for U_1 and U_2 respectively,Saoudi and Yokomoni [1] have mentioned that the lengths of all strings in E_1 and E_2 are not more than $3m_1^2$ and $3m_2^2$ respectively,where $m_1 = |\Sigma||Q_1|^2$ and $m_2=|\Sigma||Q_2|^2$ (refer lemma 15 in [1]).Therefore the sizes of the arrays in $E = E_1\oplus E_2$ are (i,j) where $1 \le i \le 3m_2^2$ and $1 \le j \le 3m_1^2$.Now we can find a finite set of positive data S_L of L such that $E \subseteq \pi^{-1}(S_L)$.Since π is area preserving,the areas of all the arrays in S_L are not more that $9m_1^2m_2^2$.Let $S_L = \{w_1,w_2,\ldots\ldots. w_p\}$ and $l=\max\{A(w_1),\ldots\ldots. A(w_p)\}$.Then the number of computation descriptions in $\pi^{-1}(S_L)$ is atmost $n^{A(w_1)} + \ldots\ldots\ldots + n^{A(w_2)} \le pm^l = t(n)$.Note with at most t(n) number of queries,Algorithm TREC finds a finite set E_i of positive data of U with the property that E_i of positive data of U with the property that E_i includes a characteristic sample for U and π(U)=L.

Summarizing we obtain the following result.

Theorem 6.1.

Given an unknown recognizable ωω-triangular array language L, Algorithm TREC learns from positive data and superset queries, a finite set of dominoes Δ such that $L=\pi(L_T^{\omega\omega}(\Delta))$

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