

Mathematical Assessment of Social Media Impact on Academic Performance of Students in Higher Institution

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Abstract : In this paper, a compartmental model approach was used in formulating a model to investigate the effects of social media on academic performance of students in higher institution. The total population is sub divided in to five compartments which are: social media non users class, less active users class, active users class, high performance class and low performance class. Two equilibria (Media Free Equilibrium MFE and Media Addiction Equilibrium Points AEP) were obtained and analyzed. The basic reproduction number R_0 was obtained and the stability analysis of the MFE reveals that, the MFE is locally asymptotically stable if R_0 is greater than unity and unstable otherwise. The AEP exists whenever $R_0 > 1$. A numerical solution of the model was obtained and it shows that there is a significant effects of social media on academic performance of students in higher institution which proved the local stability analysis of the MFE.

Keywords: Compartmental model, Social media, Equilibrium point, Basic reproduction number, Local Stability.

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I. Introduction

Nowadays, the mode of communication and interaction between people has changed as a result of internet and its platform which composed of several developed technologies. One of the sound technology in communication and sharing information is the social media, such as Facebook, Twitter, Youtube, Whatsapp, Instagram, etc with which its development penetrates through almost all part of our society and contribute positively and negatively in communication, learning, research and education in general. Social media seems very interesting and easy to use as it is being access through smart phones and mobile applications. Academic performance nowadays is influenced by several factors, among which are the social media. Academic performance of students especially in higher institution is the major concern not only for the educational body but for the society in general. Performance of students depends on how they learn, research, etc and depends mostly on the rate of assimilation of the students and how long they devote time to read. Many researchers have been carrying research on how social media influence students' performance in higher institution. According to [1], Facebook users often devoted lesser time to their studies in comparison to non-users did and consequently had lower GPAs. However everyday many student are spending almost all their time using social media, thus, as more students join and continue using social media, one may bear the question in mind whether if social media have impact on academic performance of students or not?

II. Literature Review

In 2014, [2] studied the effects of social networks on students' academic and non-cognitive behavioral outcomes, which the authors estimated the influence of social networks on educational attainment and behavioral outcomes in school. The finding shows that the various types of social networks have negative effect on students' learning outcomes and positive effect on other measures of non-cognitive behavioral outcomes. In a research [3], the impact of social media on higher education in Kosovo was determined. The paper analyzed the impact of social media plat form (email, social networks, blogs, instant messages etc) in higher education. The outcome of the study shows that social media has impact on students and is positive when it comes to communication and interactivity among them. The impact of the use of social media on students learning and performance was investigated by [4], especially the level of engagement and collaboration between them while using Facebook. The results obtained showed that Facebook use has a significant increase impact on students' collaboration and engagement. A paper [5] on the impact of social media on academic performance was published, in which a survey of undergraduate students of a private university in south west Nigeria was

conducted. The results were analyzed using structural equation modeling and partial least square approach. And it revealed a statistically significant positive effect on academic competence. An investigation on the impact of online social networking on academic performance among high school students was made by [6]. The study focused on three respondents (students, parents and teachers). The research reveals that social media is negatively associated with academic performance of students. Hence, this study aims to model the impact of social media on academic performance of students in higher institution using the precise language of Mathematics.

III. Model Formulation

In higher institution, student is in either of these two categories; non users of social media or users of social media categories (less active users or active users). Therefore in this research, a compartmental approach is used to describe the total population $N(t)$ of the students. At any time t , the population is divided in to compartments as follows: non users of social media $S_N(t)$, less active users of social media $M_L(t)$, active users of social media $M_A(t)$, users performing high $H(t)$ and users performing low $L(t)$. Belonging from either of the two categories specified above does defined a probability and since total probability is equal to one, we can let the probability of students being in non-users class to be $(1 - mp)$, being in less active class to be $'np'$ so that the probability of students being in active class users of social media is $(m - n)p$. Here all the three classes should have recruitment rate as we logically categorized the students in term of social media usage. Students will be recruited to S_N at the rate $(1 - mp)\pi$, to M_L at $np\pi$ or to M_A at the rate $(m - n)p\pi$. The non-users of social media relate or commute with less active or active users of social media in higher institution, therefore as time goes on and as a result of day to day activities, the non-users will start developing interest in joining social media and this may lead them to join the network by becoming less active users with the rate of migration to less active class as $\frac{\beta S_N(M_L + M_A)}{N}$, this is clear because a student who joined social media today is assumed not to be addicted user and therefore can only migrate to less active class. Students from less active class move to active class at a rate $\theta_1 M_L$, also an active user social media migrate to less active class at a rate δM_A . Students in the less active class may either perform high at a rate $q\tau_1 M_L$ or perform low at a rate $(1 - q)\tau_2 M_L$, the probability of students being performing high and low respectively are q and $(1 - q)$. Also at any time t , students in the active class $M_A(t)$ may either perform high at a rate $r\alpha_1 M_A$ or perform low at a rate $(1 - r)\alpha_2 M_A$. Students with low performance if not withdrawn, may decide to migrate to less active class at a rate $\theta_2 L$. In each class, μ is the constant rate at which students quit using social media by death or losing internet connection with the assumption that: students with low performance may die naturally at a rate constant μ or be withdrawn at a rate $d_2 L$, students with high performance graduate at a rate $d_1 H$ or die naturally, and the students in the other classes may die naturally or lose internet connection if not withdrawn or graduated

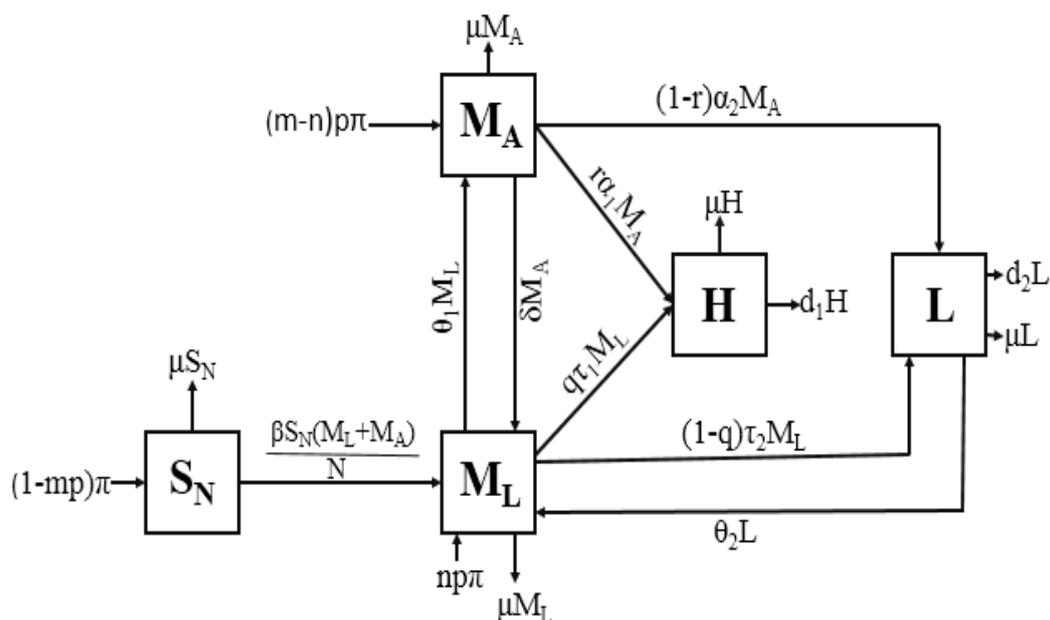


Fig. 1 Schematic diagram of the Model

Table I: Parameter Description

Parameters	Description
π	Recruitment rate
μ	Natural death rate
θ_1	Rate of moving from less active class to active class
δ	Rate of moving from active class to less active class
τ_1	Rate at which less active users perform high
τ_2	Rate at which less active users perform low
α_1	Rate at which active users perform high
α_2	Rate at which active users perform low
θ_2	Rate at which students with low performance move to less active class
d_1	Rate of graduation as a result of high performance
d_2	Rate of withdrawal as a result of low performance

Model Equations

From our proposed model, the following system of equations is obtained:

$$\begin{aligned}
 \frac{dS_N}{dt} &= (1 - mp)\pi - \frac{\beta S_N(M_A + M_L)}{N} - \mu S_N \\
 \frac{dM_L}{dt} &= \frac{\beta S_N(M_A + M_L)}{N} + np\pi + \delta M_A + \theta_2 L - q\tau_1 M_L - (1 - q)\tau_2 M_L - \theta_1 M_L - \mu M_L \\
 \frac{dM_A}{dt} &= (m - n)p\pi + \theta_1 M_L - \delta M_A - r\alpha_1 M_A - (1 - r)\alpha_2 M_A - \mu M_A \\
 \frac{dH}{dt} &= q\tau_1 M_L + r\alpha_1 M_A - d_1 H - \mu H \\
 \frac{dL}{dt} &= (1 - q)\tau_2 M_L + (1 - r)\alpha_2 M_A - \theta_2 L - d_2 L - \mu L
 \end{aligned}
 \tag{1}$$

III. Model Analysis

A. Existence and Uniqueness.

Theorem 1. [8, 10]

Let D denote the region $|t - t_0| \leq a$ and $\|x - x_0\| \leq b$, where $x = (x_1, x_2, x_3, \dots, x_n)$ and $x_0 = (x_{0,1}, x_{0,2}, x_{0,3}, \dots, x_{0,n})$. Suppose that $f(t, x)$ satisfies Lipchiz condition $\|f(t, x_1) - f(t, x_2)\| \leq k\|x_1 - x_2\|$ where the pair (t, x_1) and (t, x_2) belong to D and k is a positive constant.

Thus consider the following lemma.

Lemma 1. Let D denote the region $0 \leq N \leq k$, then system (1) has a unique solution.

Proof: Let $x_1 = S_N, x_2 = M_L, x_3 = M_A, x_4 = H, x_5 = L$ also f_1, f_2, f_3, f_4 and f_5 be respectively the equations in system (1) and let $\lambda = \frac{\beta(x_2 + x_3)}{N}$, our goal is to show that $\frac{\partial f_i}{\partial x_j}$ $i, j = 1, 2, 3, 4, 5$ are continuous and bounded in D. Now,

$$\begin{aligned}
 f_1 &= (1 - mp)\pi - \lambda x_1 - \mu x_1 \\
 \left| \frac{\partial f_1}{\partial x_1} \right| &= |-\lambda - \mu| < \infty & \left| \frac{\partial f_1}{\partial x_2} \right| &= \left| \frac{\partial f_1}{\partial x_3} \right| = \left| \frac{\partial f_1}{\partial x_4} \right| = \left| \frac{\partial f_1}{\partial x_5} \right| = 0 < \infty
 \end{aligned}$$

Similarly, going in same manner for f_2, f_3, f_4 and f_5 of model (1), we found that their partial derivatives exist for all the equations and are continuous and bounded. Hence the system (1) has a unique solution

B. Positivity of solution

Lemma 2. Let the initial solution set $\{S_N(0) > 0, M_L(0) > 0, M_A(0) > 0, H(0) > 0, L(0) > 0\} \in \mathbb{R}_+^5$, then the solution set $\{S_N(t), M_L(t), M_A(t), H(t), L(t)\}$ is positive for all time $t > 0$.

Proof: Let the force of infection be $\lambda = \frac{\beta(M_A + M_L)}{N}$. Consider the first equation of system (1)

$$\begin{aligned} \frac{dS_N}{dt} &= (1 - mp)\pi - \lambda S_N - \mu S_N \\ &\geq -(\lambda + \mu)S_N \\ \Rightarrow S_N(t) &\geq S_N(0)e^{-\int(\lambda + \mu)dt} > 0 \end{aligned}$$

Also for the second equation of system (1)

$$\begin{aligned} \frac{dM_L}{dt} &= \lambda S_N + np\pi + \delta M_A + \theta_2 L - q\tau_1 M_L - (1 - q)\tau_2 M_L - \theta_1 M_L - \mu M_L \\ &\geq -(q\tau_1 + (1 - q)\tau_2 + \theta_1 + \mu)M_L \\ \Rightarrow M_L(t) &\geq M_L(0)e^{-\int(q\tau_1 + (1 - q)\tau_2 + \theta_1 + \mu)dt} > 0 \end{aligned}$$

Going in same fashion for the third, fourth, and fifth equation of system (1), we found that $M_A(t) > 0, H(t) > 0$ and $L(t) > 0$ respectively. Hence the solution set is positive for all time $t > 0$.

C. Invariant Set

If a solution of a differential equation or a system of differential equations start on a given space, surface or curve and remains within it for all time t , then the set is said to be invariant [7]. Hence a positively invariant set will have solutions that are positive for all time. Now, let $N = S_N + M_L + M_A + H + L$ at any time t . Therefore,

$$\frac{dN}{dt} = \pi - \mu N - d_1 H - d_2 L$$

By comparison theorem [8],
$$\frac{dN}{dt} \leq \pi - \mu N \tag{2}$$

Now, integrating (2), implies

$$N \leq \frac{1}{\mu} (\pi - N_0 e^{-\mu t})$$

Where N_0 is the initial population at $t = 0$, thus as $t \rightarrow \infty$, we have $N \leq \frac{\pi}{\mu}$ and hence at any time t , the invariant set is

$$\Omega = \left\{ (S_N, M_L, M_A, H, L) \in \mathbb{R}_+^5 : N \leq \frac{\pi}{\mu} \right\}$$

D. Equilibrium Points

Two equilibria exist in the case of this model and are reveal by the infected infectious classes.

- Social Media Free Equilibrium (MFE): all social media users' class are zero.
- Addiction Equilibrium Point (AEP): social media users' classes are not zero.

Social Media Free Equilibrium (MFE)

This equilibrium point exists when there are no less active and active class. Students who applied for admission in the higher institution are not ad mitted or they have receive admission letter but not registered/on registration or they have registered but not started using social media for academic purposes. Thus we have $M_L = M_A = H = L = 0$ and hence,

$$MFE = \left(\frac{(1 - mp)\pi}{\mu}, 0, 0, 0, 0 \right)$$

Local Stability Analysis of MFE

The threshold quantity basic reproduction number (R_0) determines whether social media usage will invade students population, that is if $R_0 > 1$ or social media usage will not invade the population, that is if $R_0 < 1$. To calculate the basic reproduction number, we consider our model diagram which has two infected

classes as M_L and M_A with force of infection $\frac{\beta(M_A+M_L)}{N}$. Hence applying the next generation matrix approach by [9] at MFE we have

$$F = \begin{pmatrix} \beta & \beta \\ 0 & 0 \end{pmatrix} \quad V = \begin{pmatrix} q\tau_1 + (1-q)\tau_2 + \theta_1 + \mu & -\delta \\ -\theta_1 & \delta + r\alpha_1 + (1-r)\alpha_2 + \mu \end{pmatrix}$$

For simplicity let $C_1 = q\tau_1 + (1-q)\tau_2 + \theta_1 + \mu$ and $C_2 = \delta + r\alpha_1 + (1-r)\alpha_2 + \mu$ so that V becomes

$$V = \begin{pmatrix} C_1 & -\delta \\ -\theta_1 & C_2 \end{pmatrix}$$

$\det(V) = C_1C_2 - \delta\theta_1$. Now let $\det(V) = W$ and so

$$V^{-1} = \begin{pmatrix} \frac{C_2}{W} & \frac{\delta}{W} \\ \frac{\theta_1}{W} & \frac{C_1}{W} \end{pmatrix}$$

Thus

$$FV^{-1} = \begin{pmatrix} \frac{\beta(C_2 + \theta_1)}{W} & \frac{\beta(C_1 + \delta)}{W} \\ 0 & 0 \end{pmatrix}$$

The eigenvalues of FV^{-1} are given by $|FV^{-1} - \lambda I| = 0$. We obtained that $\lambda_1 = \frac{\beta(C_2 + \theta_1)}{W}$ and $\lambda_2 = 0$. But $R_0 = \max\{|\lambda_i| : i = 1, 2\}$. Hence,

$$R_0 = \frac{\beta(\delta + r\alpha_1 + (1-r)\alpha_2 + \mu + \theta_1)}{(q\tau_1 + (1-q)\tau_2 + \theta_1 + \mu)(\delta + r\alpha_1 + (1-r)\alpha_2 + \mu) - \delta\theta_1}$$

Applying theorem 2 of [9], we conclude the following lemma:

Lemma 3. The Media Free Equilibrium point of model (1) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

G. Addiction Equilibrium Point (AEP): this is a point where at least one of the infected classes is present.

Consider the force of infection for system (1) which is given by $\lambda^{**} = \frac{\beta(M_A^{**} + M_L^{**})}{N^{**}}$ where $N^{**} = S_N^{**} + M_L^{**} + M_A^{**} + H^{**} + L^{**}$.

Now, replacing the force of infection by λ^{**} in (1) and equating the right hand side to zero leads to

$$\begin{aligned} 0 &= (1 - mp) - \lambda^{**}S_N - \mu S_N \\ 0 &= \lambda^{**}S_N + npp + \delta M_A + \theta_1 L - q\tau_1 M_L - (1-q)\tau_2 M_L - \theta_1 M_L - \mu M_L \\ 0 &= (m - n)p\pi + \theta_1 M_L - \delta M_A - r\alpha_1 M_A - (1-r)\alpha_2 M_A - \mu M_A \\ 0 &= q\tau_1 M_L + r\alpha_1 M_A - d_1 H - \mu H \\ 0 &= (1-q)\tau_2 M_L + (1-r)\alpha_2 M_A - \theta_2 L - d_2 L - \mu L \end{aligned} \tag{3}$$

Thus from (3) above, we have $S_N^{**} = \frac{(1-mp)\pi}{\lambda^{**} + \mu}$, $H^{**} = \frac{q\tau_1 M_L^{**} + r\alpha_1 M_A^{**}}{d_1 + \mu}$ and $L^{**} = \frac{(1-q)\tau_2 M_L^{**} + (1-r)\alpha_2 M_A^{**}}{\theta_2 + d_2 + \mu}$. Substituting the value of S_N^{**} and L^{**} in the second equation of (3) and from the third equation we have respectively that

$$\begin{aligned} xM_A^{**} + yM_L^{**} &= -\psi_1 \\ -z_1 M_A^{**} + \theta_1 M_L^{**} &= -\psi_2 \end{aligned} \tag{4}$$

Where $x = \frac{\theta_2(1-r)\alpha_2}{\theta_2 + d_2 + \mu} + \delta = \frac{\theta_2 C_7}{C_8} + \delta$, $y = \frac{\theta_2(1-q)\tau_2}{\theta_2 + d_2 + \mu} - q\tau_1 - (1-q)\tau_2 - \theta_1 - \mu = \frac{\theta_2 C_8}{C_8} - C_1$, $\psi_1 = \frac{\lambda^{**}(1-mp)\pi}{\lambda^{**} + \mu} + npp$,

$$z = \delta + r\alpha_1 + (1-r)\alpha_2 + \mu = C_2 \text{ and } \psi_2 = (m - n)p\pi.$$

Applying Cramer's rule in solving (4) and making all the possible substitutions implies $M_L^{**} = \frac{\lambda^{**}A + B(\lambda^{**} + \mu)}{(\lambda^{**} + \mu)Q}$

and $M_A^{**} = \frac{\lambda^{**}D + E(\lambda^{**} + \mu)}{(\lambda^{**} + \mu)Q}$, where $A = C_2 C_8(1 - mp)\pi$, $B = [(m - n)(C_7 + \delta C_8) + n(C_2 C_8 - \theta_2) + m\theta_2]p\pi$,

$$D = \theta_1 C_8(1 - mp)\pi, \quad E = [(m - n)(C_1C_8 + \theta_2C_6) + C_8\theta_1]p\pi \quad \text{and}$$

$$Q = C_1C_2C_5C_8 - \delta\theta_1C_5C_8 - \theta_1\theta_2C_5C_7 - \theta_2C_2C_5C_6$$

Now, let $F = A + B$ and $G = D + E$, then it is easy to see that,

$$M_L^{**} = \frac{\lambda^{**}F + B\mu}{(\lambda^{**} + \mu)Q}, \quad M_A^{**} = \frac{\lambda^{**}G + E\mu}{(\lambda^{**} + \mu)Q}, \quad H^{**} = \frac{\lambda^{**}FC_3 + \lambda^{**}GC_4 + (BC_3 + EC_4)\mu}{(\lambda^{**} + \mu)C_5Q},$$

$$L^{**} = \frac{\lambda^{**}FC_6 + \lambda^{**}GC_7 + (BC_6 + EC_7)\mu}{(\lambda^{**} + \mu)C_5Q} \quad \text{and} \quad N^{**} = \frac{\lambda^{**}A_0 + T}{(\lambda^{**} + \mu)C_5C_8Q}$$

Where $A_0 = C_5C_8F + C_5C_8G + C_3C_8F + C_4C_8G + C_5C_6F + C_5C_7G$, and
 $T = C_5C_8Q(1 - mp)\pi + C_5C_8B\mu + C_5C_8E\mu + C_8(BC_3 + EC_4)\mu + C_8(BC_6 + EC_7)\mu$. We have that
 $\lambda^{**}N^{**} = \beta(M_A^{**} + M_L^{**})$ and so ,

$$\lambda^{**2}A_0 + \lambda^{**}[T - C_5C_8\beta(F + G)] - C_5\beta C_8\mu(B + E) = 0 \quad (5)$$

Observe that $C_5\beta C_8\mu(B + E) = \frac{\beta C_5 C_8 \mu W (B + E)}{\beta(C_2 + \theta_1) - W} (R_0 - 1)$. Let $A_1 = [T - C_5C_8\beta(F + G)]$ and

$$A_2 = \frac{\beta C_5 C_8 \mu W (B + E)}{\beta(C_2 + \theta_1) - W} (R_0 - 1)$$

Then (5) becomes

$$\lambda^{**2}A_0 + \lambda^{**}A_1 - A_2 = 0 \quad (6)$$

It is obvious to see that if $A_0 > 0$, then by Descartes rule of signs, the polynomial (6) has exactly one positive root if $R_0 > 1$.

Hence the following result

Lemma 4: The system (1) has a unique AEP if $R_0 > 1$.

IV. Numerical Simulation

In order to validate the proposed model (Fig. 1), we represent the analytic results obtained from system (1) by numerical simulation using the following parameter values. The goal here is to show that these solutions represent the real life situation.

We use the following parameter values:

$$\mu = 0.019, \alpha_1 = 0.78, \alpha_2 = 0.28, \beta = 0.7, \theta_2 = 0.4, \tau_1 = 0.68, \tau_2 = 0.32, q = 0.8, d_1 = 0.5, d_2 = 0.3, mp = 0.6, np = 0.2, r = 0.6, \pi = 1000, N = 3000$$

with three different values of θ_1 and δ , that is $\theta_1 = 0.1, 0.6, 0.9$ and $\delta = 0.1, 0.3, 0.8$.

A. Results

The results of the numerical simulation of system (1) are presented in the graphs below

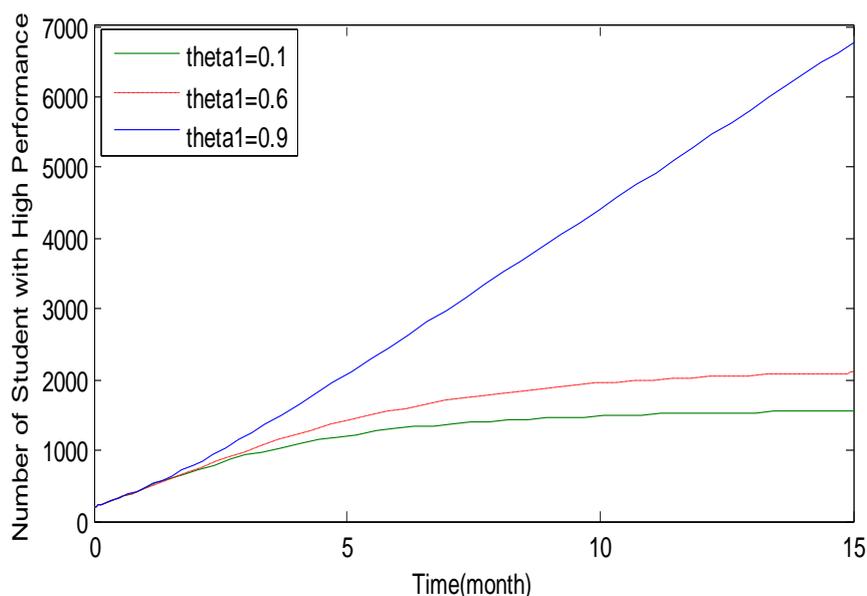


Fig. 2. Graph of Less active Class against time

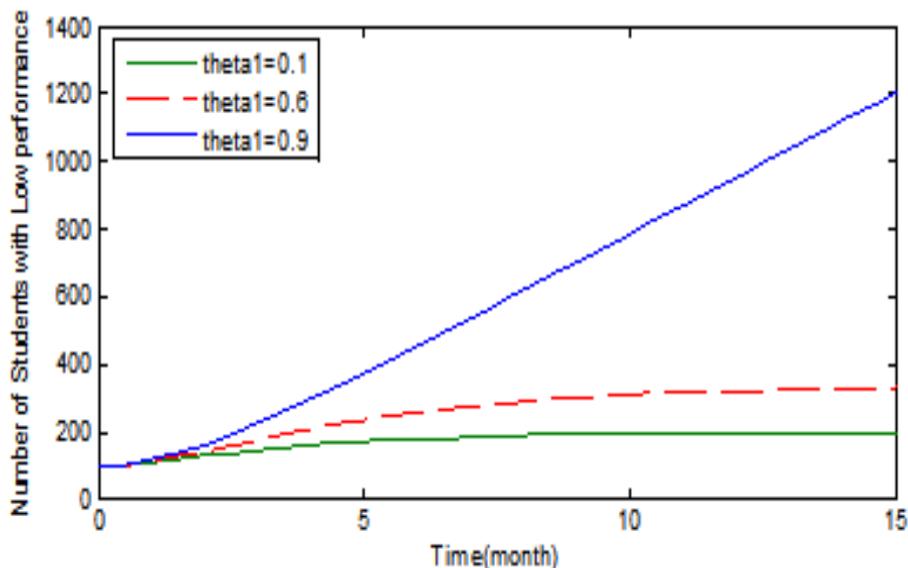


Fig. 3 Graph of Active Class against time

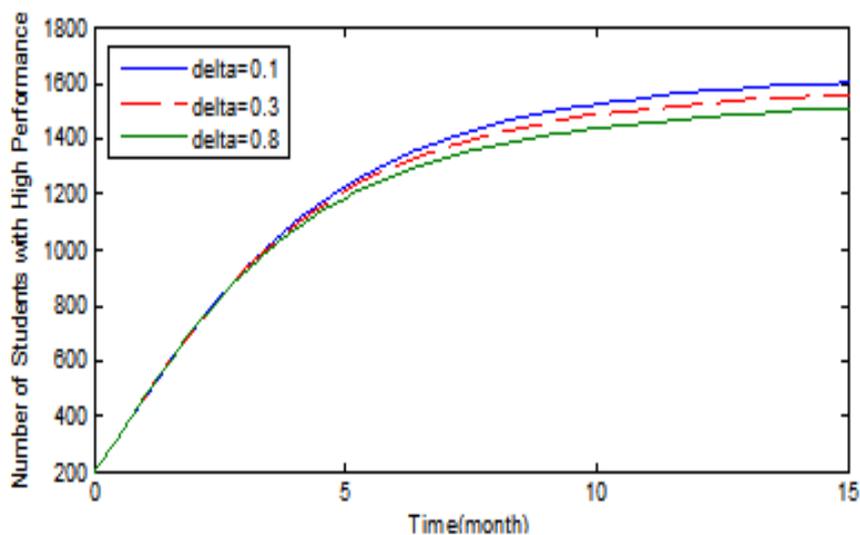


Fig. 4 Graph of High Performance Class against time

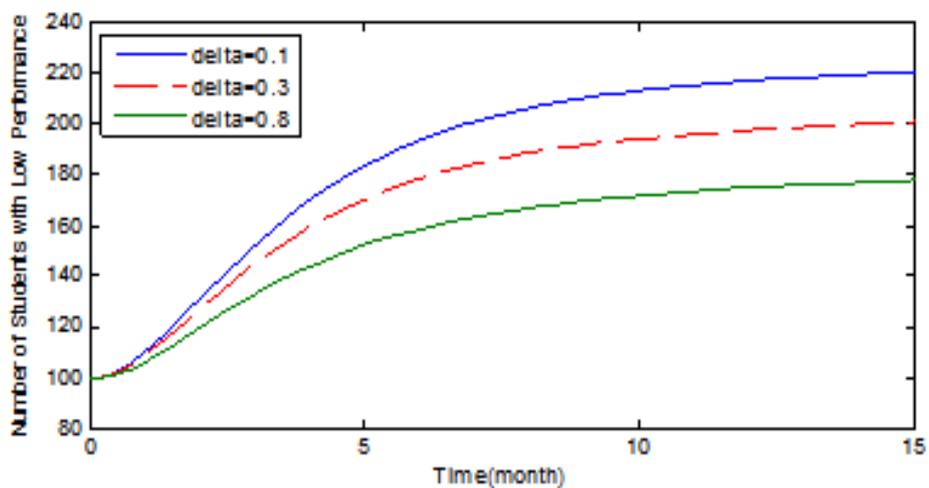


Fig. 5 Graph of Low Performance Class against time

V. Discussion

In Fig. 2 above, the number of students with high performance increases very slight and stabilize immediately when the movement from less active to active class is very minimal, but when the movement is increased to a certain point, the number of the students with high performance increases to a stable points and when the movement is very high, the number of the students with high performance increases exponentially. Similarly in Fig. 3, the number of students with low performance increases with increase in movement from less active to active class as in Fig. 2. But observe that, the range of the graph in Fig. 3 is from 0 to 1200 while in Fig.2 is from 0 to 7000, that is, the effect in Fig.2 takes over or occur immediately after the effect in Fig.3. Fig. 4 shows that the number of students with high performance increases when the movement of students from active to less active class is minimum, but when the movement is increased to a certain point, the number of students with high performance decreases to a stable point and when the movement is very high, the number of the students with high performance also decreases. Similarly, in Fig. 5 the number of students with low performance decreases with increase in the movement from active to less active class. But observe also that, the range of the graph in Fig. 5 is 100 to 220 and for that in Fig. 4 is 200 to 1600, that is Fig. 4 took effect above Fig. 5. Clearly, Fig. 2 and Fig. 3 show that increase in movement from less active to active class increases the rate of performance of students while it decrease or increase in movement from active to less active class in Fig. 4 and Fig. 5 decreases the rate of performance of students to a stabilize point, that is θ_1 and δ have significant effect on R_0 and hence the students' performance. Observe from all the graphs that θ_1 has more effect on students' performance than δ , that is active usage of social media has a greater impact than less active usage. Similarly increase in the number of students with high performance decreases the number of students with low performance and vice-versa, that is τ_1 and τ_2 depend on each other, so also α_1 and α_2 .

VI. Conclusion

In explaining the qualitative behavior of the effects of social media on academic performance of students, we used a compartmental model approach which from that a system of non linear differential equations was deduced and the analysis of the model seems to be in agreement with the real life situation. Consequently θ_1 and δ , α_1 , α_2 , τ_1 , and τ_2 , well explained the dynamic pattern of the model and the analytic results, that is the equilibria and the basic reproduction number. Finally we conclude that this model can be extended and reframed to be more realistic for better analysis.

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