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MHD Nanofluid Bioconvection Due To Gyrotactic Microorganisms Past A Convectively Heated Vertical Plate

Chebii Agnes Jeptoo
Corresponding Author: Chebii Agnes Jeptoo

Abstract: *The effects of magnetic field on boundary layer flow with heat and mass transfer of a water-based nanofluid containing gyrotactic microorganisms over a vertical plate are numerically investigated.The governing boundary layer equations are formulated and transformed into ordinary differential equations using a suitable similarity transformation.The resulting ordinary differential equations are solved numerically using the fourth order Runge-kutta method with shooting technique. Pertinent results are presented graphically and discussed quantitatively with respect to variation in the controlling parameters such as; bioconvection lewis number (Lb),traditional lewis number (Le),bioconvection peclet lewis number (Pe),buoyancy ratio parameter (Nr),bioconvection Rayleigh number (Rb),Brownian motion parameter (Nb),thermophoresis parameter (Nt),Hartmann number Ha, Grashof number (Gr) and Eckert number (Ec) on dimensionless velocity, temperature, nanoparticle concentration and microorganisms conservation. It is observed that increasing the magnetic field strength retards the thermal boundary layer thickness which from application point of view ,it is obvious the surface cooling effect is enhanced thus nanofluids are appropriate as heat transfer fluids.*

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I. Introduction

In this chapter, the main terminologies used in this dissertation are introduced and defined. Furthermore, the problem statement, objectives of the study and significance are discussed

1.1 Magneto hydrodynamics (MHD)

Magneto hydrodynamic (MHD) is the academic discipline concerned with the dynamics of electrically conducting fluid in a magnetic field. These fluids include salt water, liquid metals (such as mercury, gallium, molten iron) and ionized gases or plasma (such as solar atmosphere). The term MHD is comprised of the words magneto- meaning magnetic, hydro- meaning fluids, and dynamics-meaning movement. Synonyms of MHD that are less frequently used are the terms magneto fluid, dynamics, and hydro magnetics.The field of MHD was initiated by the Swedish physicist Hannes Alfven (1942), who received the Nobel prize in physics in 1970 for fundamental work and discoveries in the magneto hydrodynamics with fruitful applications in different parts of plasma physics.MHD covers those phenomena where in an electrically conducting fluid, the velocity fluid V and the magnetic field B are coupled. The magnetic field induces an electric current of density j in the moving conductive fluid (electromagnetism). The induced current creates forces on the liquid and also changes the magnetic field. Each unit volume of the fluid having magnetic field B experience an MHD force jxB known as Lorentz force.The set of equation which describe MHD flow are combination of Navier stokes equation of fluid dynamics and Maxwell's equation of electromagnetism. These differential equations are solved simultaneously either analytically or numerically.

1.2 Nanofluids

Fluid is a substance that deforms continuously under the applications of shear stress (tangential force per unit area). Convectional heat transfer fluids such as water, minerals oil, and ethylene glycol play an important role in many industrial sectors including power generation, chemical production, air conditioning, transportation and microelectronics. Although various techniques have been applied to enhance their heat transfer, their performance is often limited by their low thermal conductivities which obstruct the performance enhancement and compactness of heat exchangers. With the rising demand of modern technology for process intensification and device miniaturization, there is need to develop new types of fluid that are more effective in terms of heat exchange performance. In order to achieve this, it has been recently proposed to disperse small amount of nanometer size (10- 50nm) solid particle (nanoparticles) in base fluid resulting into what is known as nanofluid.The term "nanofluid"was coined by Choi(1995) who was working with the group at the Argonne National Laboratory (ANL), USA. Nanofluids appear to behave more like single phase fluid than a solid –liquid mixture.The commonly used materials for nanoparticles made of chemically stable metals (Ag, Cu. Au, Fe), non-metals (graphite ,carbon , nanotubes) , oxides ceramic (Al_2O_3, CuO, TiO_2,SiO_2) carbides, Nitrides (AlN, SiN) layered (Al^+, Al_2O_3Cu+C), PCM and functionalized nanoparticles. The base fluid is usually a conductive fluid such as water (or other coolants), oil (and other lubricants), polymer solutions, bio fluids and other fluids such as paraffin.

1.3 Bioconvection

The term bioconvection refers to macroscopic convection motion of fluid caused by density gradient created by collective swimming of motile microorganisms.These self propelled motile microorganisms increase the density of the base fluid by swimming in a particular direction, thus causing bioconvection.Different bioconvection systems can be distinguished on the basis of mechanism of direction of motion of the different types of microorganisms. Oxytactic or chemotactic microorganisms swim up the oxygen concentration gradient as they require certain minimum concentration of oxygen to be active. Negative geotactic microorganisms swim against gravity and gyrotactic whose direction of swimming is determined by the balance of gravitational and viscous torques, swim due to displacement between the cell center of mass and buoyancy.

1.4 Heat transfer

Temperature variation may exist within a fluid due to temperature difference between boundaries or between boundary and an ambient fluid. Temperature variations can also a rise from a variety of causes such as radioactivity, absorption of thermal radiation and release of latent heat as fluid vapour condenses. Heat transfer is the generation, use, conversion and exchange of thermal energy and heat between the physical systems which occurs as a result of temporary gradient. This energy transfer is defined as heat. There are three modes of heat transfer; convection, conduction and radiation .In our study we consider convection.

1.4.1 Convection

Convection refers to heat transfer that will occur between a surface and a moving fluid when they are at different temperature. The convection heat transfer mode is sustained both by random molecular motion and by the bulk motion of a fluid within the boundary layer. The contribution due to random molecular motion (diffusion) generally dominates near the surface where the fluid velocity is low. The contribution due to bulk motion originates from the fact that boundary layer grows as flow progresses. Convection heat depends on viscosity, thermal conductivity, specific heat and density of the fluid.

1.5 Boundary layer

The concept of boundary layer in a fluid flow was introduced by Ludwig Prandtl in 1904.Due to friction, the velocity of the fluid immediately adjacent to the surface stick to the surface and the fluid velocity is zero. Prandtl discovered that for most applications, the influence of viscosity is confined to an extremely thin region very close to the body and that the remainder of the flow field could be treated as inviscid. This thin layer adjacent to the surface of a body or a solid wall in which viscous forces affect the flow is called the boundary layer and the thickness of the boundary layer is defined as the distance from the boundary to the height above the surface at which the velocity becomes 99% of the free stream velocity.

1.6 Gyrotactic microorganisms

Gyrotaxis is the term that describes the biased swimming of microorganism such as chlamydomonas, dunaliella and volvox in the presence of shear in the surrounding fluid (Kessler 1984) These microorganisms are structurally bottom heavy i.e. their Centre of mass is located behind the center of buoyancy .Therefore when a micro-organism of this type is aligned with the vertical, the bottom heaviness generates a gravitational torque which reorients the cells to the vertical. This mechanism leads to the microorganism to be naturally gravitactic: it swims against gravity in the absences of shear .However, when a shear is imposed in the surrounding fluid the microorganism also experience a viscous torque due to the shear therefore the orientation of the cell is determined by the balance between gravitational and viscous torques, and the term gyrotaxis, refers to this process (Kessler 1984, 1985 a, b, 1986).

1.7 Thermophoresis

The term thermophoresis which is also known as soret effect is a phenomenon observed in a mixture of mobile particles where the different particle types exhibit different response to the force of a temperature gradient. The term soret effect normally applies to liquid mixtures which behave according to different, less well understood mechanism than gaseous mixtures. According to Maiga *et al* (2005), thermophoresis plays an important role in forced and natural convection in channels and enclosure when nanofluids are used instead of pure fluid. Among the transport phenomena, thermophoresis (soret effect), which couples mass and heat fluxes, lead to an increase in the rate of heat transfer, but a decrease in the rate of mass transfer.

1.8 Brownian motion

Brownian motion is a term that refers to the random motion of particles suspended in a fluid resulting from their collision with the quick atom or molecules in the fluid. Brownian motion is one of the important factors for the enhancement of the thermal conductivity of nanofluids. Nanoparticles in base fluid easy experience Brownian force, hence, it is plausible that the observed thermal conductivity of nanofluids is from the combined static (thermal properties and interfacial layer) and dynamic (e.g. Brownian motion) mechanism of dispersed nanofluid particles. Studies by Motsumi and Makinde (2012) have shown that the smaller the particle the larger the dynamic (Brownian motion-based) thermal conductivity contribution .Mainly for smaller size and low volume fraction of Nanoparticles, the dynamic contribution of thermal conductivity is significant. The reason being the smaller the particle size the greater the movement of particles in the fluid. Brownian motion-based dynamic mechanism is significant for nanofluids with smaller size and low concentration of nanoparticles.

1.9 Problem statement

Different convectional heat transfer fluid and nanoparticle have different electrical conductivities and behave differently in the presence of a magnetic field. It is then expected that Nanofluid (which is a mixture of base fluid and nanoparticles) to be electrically conducting and hence susceptible to MHD. The flow of electrically conducting fluid has many industrial and engineering application .Thus the influence of a magnetic field on Nanofluid flow cannot be overlooked and requires investigation.

1.10 General research objectives

To analyze numerically the MHD Nanofluid bioconvection due to gyrotactic microorganisms past a convectively heated vertical plate

1.10.1 Specific Research objectives

1. To formulate mathematical models for nanofluids flowing along a vertical surface under the influence of magnetic field.
2. To investigate the effect of magnetic field and nanoparticles on fluid velocity and temperature.
3. To investigate the effects of convective heating (Biot number) on the fluid velocity and temperature.
4. To investigate the effects magnetic field on nanoparticle concentration and microorganism concentration.

1.11 Significance of the study

Magneto hydrodynamics (MHD) natural convection heat transfer flow in a porous medium past a flat surface is of considerable interest in the technical fields due to its application in industrial technology. These applications include micro MHD pumps, high temperature plasma, liquid metal fluid and biological transportation. The study of MHD flow with heat transfer has also received considerable attention due to its wide application in astrophysical problems such as sun-spot theory, motion of inter-stellar gas, re-entry problem of intercontinental ballistic missiles.

Laws governing heat transmission are important to the engineer for metal-working processes, operation of heat exchange apparatus, nuclear reactor, just to mention a few. A more recent promising discovery is the application of MHD in drug targeting for the treatment of cancer. A suspension of metal nanoparticles is being developed for medical application including cancer therapy. One of the main problems of chemotherapy is often not lack of efficient drug but the inability to precisely deliver and concentrate these drugs in affected areas. Failure to provide localized targeting result is an increase of toxic effects on neighboring organs and tissues. Medicines are bound to magnetic particles (Ferro fluids) which are biologically compatible and injected into the blood stream. The targeted areas are subjected to an external magnetic field that is able effect the blood stream by reducing flow rate. In these regions the drugs are slowly reduced from the significantly higher compared to the ones delivered of standard (systemic) delivery method. Interactions between the magnetic particles passing through the blood with the external field are studied using MHD equation and finite element analysis. Thus efficacy of the treatment can be estimated.

Miniaturization has been a major trend in modern science and technology. This trend has gone from earlier millimeter scale to the present atomic scale. This trend is being actualized by the rapidly emerging microelectromechanical system (MEMS) technology. Macro scale products such as miniaturized sensors, motors, heat exchangers, pumps, medical devices have great advantage over the conventional systems. Since they are extremely compact and light weight, their manufacturing costs are lower, their fuel consumption is low and they need less space in building and engineering plants. These devices require ultra-high performance cooling which is crucial technical challenge facing much industrial and engineering application. The conventional method for increasing heat transfer is to increase the area available for exchanging heat with a heat transfer fluid. Unfortunately the convectional heat transfer fluids used have low thermal conductivity hence there is an urgent need for new and innovative coolants with improved performance. The recent discovery of nanofluids provides a solution to cooling technology. This is because nanofluids have fascinating features: high thermal conductivity at very low nanoparticles concentration and considerable enhancement of forced convective heat transfer. Their cooling properties are used in many industrial applications.

The main driving force for nanofluids research lies in a wide range of application. Most of these industrial and engineering application using fluids have magnetic field acting on the fluid. Current trend clearly shows that the convectional fluids are rapidly being replaced by nanofluids. The nanofluids will have to be used in magnetic fields since most of these applications have magnetic field within them.

II. Literature Review

The MHD heat and mass transfer processes over a moving surface are of great engineering and geophysical applications such as geothermal reservoirs, thermal insulation ,enhanced oil recovery, packed–bed ,catalytic reactors, cooling of nuclear reactors among others .Many chemical engineering processes like metallurgical and polymer extrusion processes involve cooling of a molten liquid moving over a cooling system .The fluid mechanical properties of the penultimate product depends mainly on the cooling liquid used and the rate of moving. Some polymer fluids like polyethylene oxide and polyisobutylene solution in certain ,having better electromagnetic properties, are normally used as cooling liquid as their flow can be regulated by external magnetic fields in order to improve quality of final product in engineering and industrial processes largely depends upon the rate of cooling. The simultaneous effects of heat transfer and MHD are useful in order to achieve the final product of desired characteristics. Experimental and theoretical investigations on conventional electrically conducting fluids indicate that magnetic field markedly changes their transport and heat transfer characteristics. In a pioneering work, B.C.Sakiadis, (1961) investigated the boundary layer flow induced by moving plate in a quiescent ambient fluid. E.M. Sparrow and R.D. Cess, (1961) investigated the effect of magnetic field on the natural convection of heat transfer. Conventional fluids such as water, ethylene glycol mixture and some types of oil have low heat transfer coefficients. Thus the performance of heat transfer systems can be significantly improved if regular fluids are replaced by nanofluids. An innovative technique which involves dispersing small amounts of nanometer-sized (10-50mm) particles and fibers in conventional base fluids was introduced by Choi, S.U.S (1995) in order to enhance their heat transfer performance.

Several theoretical and experimental results have shown that nanofluids possess enhanced thermo physical properties such as thermal conductivity, thermal diffusivity, viscosity and convective heat transfer coefficients as compared to those of conventional base fluids .Studies carried out by Maiga *et a*l (2005), found that the heat transfer co-efficient and the wall shear stress increase with increasing nanoparticle volume fraction. Most of the conventional base fluids used for producing nanofluids is liquids. Their electrical conductivity properties are lower than those of metallic or non-metallic materials. These nanofluids possess enhanced electrical conductivity property and are therefore susceptible to the base fluid. Makinde *et al*. (2013) investigated numerically the effect of buoyancy on MHD stagnation point flow and heat transfer of a Nanofluid past a convectively heated stretching/shrinking sheet. They found out that the skin friction coefficient and the local Sherwood number decreases while the local Nusselt number increases with increasing intensity of buoyancy.

Olanrewaju A.M and Makinde O.D (2013) reported a similarity solution for boundary layer flow of a Nanofluid over a permeable flat surface with Newtonian heating. A numerical solution for natural convection and double-diffusive boundary layer flow over a vertical surface was presented by Kuznetzov A.V and Nield D.A (2011). In a recent study, Mutuku Njane and Makinde (2014) numerically investigated the combined effects of magnetic field, buoyancy force, viscous dissipation and ohmic heating on the boundary layer flow of Nanofluid over a convectively heated vertical plate. They highlighted the complex interactions of the electrical conductivities of metallic and non-metallic materials with that of conventional base fluid.

Bioconvection has many applications in biological systems and biotechnology. The concept of Nanofluid bioconvection which is the focus of the study therein describes the spontaneous pattern formation and density stratification caused by simultaneous interaction of denser self-propelled microorganisms, nanoparticles and buoyancy forces. These microorganisms may include gravitactic, gyrotactic and oxytactic organisms. The benefits of adding motile microorganisms to the suspension include enhanced mass transfer, micro scale mixing especially in micro volumes and improved Nanofluid stability .The hydrodynamic convection caused by oxytactic microorganisms leads to a flow system which transport cells and oxygen from the upper region to the lower fluid region. Unlike motile microorganisms, the nanoparticles are not self-impelled and their motion is driven by Brownian motion and Thermophoresis taking place within the Nanofluid.It is thus apparent that motion of motile microorganisms is independent of motion of nanoparticles. A combination of Nanofluids and bioconvection is consequently alluring for novel micro fluidic devices. It is assumed that the presence of nanoparticles has no effect on the directional locomotion of self-propelled microorganisms. Nanofluid bioconvection is predicted to be possible if the concentration of nanoparticles is small so that the nanoparticles do not cause any significant increase in viscosity of the base fluid .The concept of Nanofluid bioconvection was first introduced by Kuznetsov (2010,2011].Kuznetzov (2010) focused on nanofluids containing gyrotactic microorganisms and reaffirmed that the resultant large scale motion of fluid caused by self-propelled motile microorganisms, enhances mixing and prevent nanoparticle agglomeration in nanofluids. Aziz *et al* (2012) using Buongiorno model (2006), studied boundary layer flow in porous medium filled with nanofluids and gyrotactic microorganisms. They presented a similarity solution and showed the influences of the dimensionless parameters on the behavior of microbes.

Mutuku Njane and Makinde (2014) extended the work of Olanrewaju and Makinde (2013) and investigated the hydro magnetic flow of a novel type of a water based nanofluid containing nanoparticles and microorganisms past a permeable vertical moving plate. Later, Khan W.A *et a*l (2014) investigated the combined effects of Navier slip and magnetic field on boundary layer flow with heat and mass transfer of a water-based Nanofluid containing gyrotactic microorganism over a vertical plate. They found out that the reduced Nusselt, Sherwood and density numbers of microorganisms depend strongly upon the magnetic, buoyancy, Nanofluid and bioconvection parameters. Das K.et al (2015), investigated numerically the hydro magnetic bioconvection of gyrotactic microorganisms past a permeable vertical plate embedded in a porous medium filled with a water − based nanofluid. In a recent study, Mehryan S.A.M et al (2016) studied numerically the behavior of a water−based nanofluid containing motile microorganisms passing an isothermal nonlinear stretching sheet in the presence of a non − uniform magnetic field. The current study aims at investigating MHD Nanofluid bioconvection due to gyrotactic microorganisms past a convectively heated vertical plate taking into consideration the effects of Brownian motion and thermophoresis.

III. General Equations Governing MHD Nanofluids Flow

3.1 Continuity Equation

This equation is based on the law of conservation of mass which states that "mass cannot be created nor destroyed" .This implies that the rate of change of a particle is zero. The differential form of equation of continuity is,

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.\rho\boldsymbol{v} = 0$$

For incompressible fluid flow, $\rho = constant$ hence $\frac{\partial \rho}{\partial t}$ =0, therefore the continuity equation for an incompressible flow becomes,

$$\boldsymbol{\nabla}.\boldsymbol{v} = 0$$

Where; $\boldsymbol{\nabla} = i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}$

The above equation can be written as;

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0 \quad \text{……………………………………………………………………………….3.1.1}$$

Where $\boldsymbol{\nabla}.\boldsymbol{v}$ is called divergence of the velocity which physically is the rate of volume of a moving fluid element per unit volume.

3.2 Momentum Equation

The momentum equation is based on Newton's second law of motion ($\Sigma F = ma = m\frac{du}{dt}$).

This equation is also known as the Navier Stokes equation. The equation of momentum governing the flow of a Nanofluid is given by,

$$\frac{\partial u}{\partial t} + (\vec{V}.\nabla)\mathrm{V} = \frac{1}{\rho_{nf}}\left[-\nabla_p + \mu_{nf}\nabla^2 V\right] + F \quad \text{……………………………………………….3.2.1}$$

Where F represents other forces acting on the flow.

Taking into account force due to gravity(g), thermal expansion and the force per unit volume when an electric current density **j** flows through the fluid ($lorentz\ force\ \boldsymbol{j} \times \boldsymbol{B}$), since the fluid flow is in a magnetic field, then the Navier stokes equation becomes;

$$\frac{\partial V}{\partial t} + (\vec{V}.\nabla)\mathrm{V} = \frac{1}{\rho_{nf}}\left(-\nabla_p + \mu_{nf}\nabla^2 V\right) + (\rho\beta)_{nf}\, g\Delta T + \frac{1}{\rho_{nf}}\boldsymbol{j} \times \boldsymbol{B} \quad \text{………….. ……… 3.2.2}$$

Where

- V is velocity
- P is pressure
- □ is the density of fluid
- □ is the dynamic viscosity of the fluid
- □ is force due to gravity
- □ is thermal expansion coefficient of the fluid
- □ is magnetic flux or magnetic field

- j is electric current density
- $j \times B$ is Lorentz force

3.3 Energy Equation

This equation is derived from the first law of thermodynamics which states that the amount of heat added to a system dQ equals to the change in internal energy dE plus the work done dW ,i.e. dQ=dE +dW. In other words if a net energy transfer to a system occurs, the energy stored in the system must increase by an amount equal to the energy transferred. The first law of thermodynamics requires that;

$$\left(\rho c_p\right)\left(\frac{\partial T}{\partial t}\right) + (\nu.\nabla)T) = K\nabla^2 T + q \ldots\ldots 3.3.1$$

Where ρc_p is heat capacitance of a nanofluid, T is local temperature of Nanofluid,ρ is density of the nanofluid, k is thermal conductivity of the nanofluid, v is density, q is heat flux.

The Nanofluid properties; density, dynamic viscosity, thermal conductivity, electric conductivity and heat capacitance are defined in terms of base fluid and nanoparticle properties as below;

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_f$$

$$\beta_{nf} = (1-\varphi)\beta_f + \varphi\beta_s.$$

$$u_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}.$$

$$\left(\rho c_p\right)_{nf} = (1-\varphi)\left(\rho c_p\right)_f + \varphi\left(\rho c_p\right)_s.$$

$$\frac{k_{nf}}{k_f} = \frac{\left(k_s+2k_f\right)-2\varphi\left(k_f-k_s\right)}{\left(k_s+2k_f\right)+\varphi\left(k_f-k_s\right)}.$$

Where ρ_f is density of base fluid,ρ_s is nanoparticle,φ is the volume fraction of the nanoparticle,β_f is the base fluid thermal expansion coefficient,β_s is the nanoparticle thermal expansion coefficient,μ_f is dynamic viscosity of the base fluid, k_f is the thermal conductivity of the base fluid, k_s is the conductivity of the nanoparticle,$\left(\rho c_p\right)_s$ is the capacitance of the nanoparticle and $\left(\rho c_p\right)_f$ is the heat capacitance of the base fluid.

3.4 Maxwell's Equations

These are related through Maxwell's equations governing the evolution of electric and magnetic fields are;

$\nabla \times B = \mu_0$j (amperes law)……3.4.2

$\frac{\partial B}{\partial t} = -\nabla \times E(Faradays\ law)$……..3.4.3

J== $\theta(E \times V \times B)(Ohms\ law)$……3.4.4

Where μ_0 is magnetic permeability, B is magnetic field, j is electric current density and E is the electric field.

3.5 Computational Approach

The boundary value partial differential equations are first transformed into a system of non-linear ordinary differential equations which are then solved numerically using the fourth order Runge-Kutta method .Additional computations are obtained using MAPLE software program which solves the model problem into numerical results. The pertinent parameters are then analyzed and presented graphically.

IV. Modelling Of Analysis of MHD Nanofluid Bioconvection Due To Gyrotactic Microorganisms Past A Convectively Heated Vertical Plate

4.1 Introduction

During the last few decades, the study of boundary layer and heat transfer over a stretching surface has attracted many researchers due to its numerous applications in industrial manufacturing processes such as aerodynamic extrusion of plastic sheets, hot rolling ,metal spinning ,artificial fibers, wire drawing, paper production, among others. The rates of stretching and cooling have a significant influence on the quality of the final product with desired characteristics. The main challenge was the thermal conductivity of the ordinary heat transfer fluids was not adequate to meet the required cooling rates, hence a need to develop fluids with better cooling or heating performance. Choi (1995), at Argone National Laboratory of USA coined the novel concept of 'nanofluids' to meet the cooling challenges facing many high-tech industries. Nanofluids are the suspension of nanometer-sized solid particles and fibers, which have been proposed as a means for enhancing the performance of heat transfer liquids currently available such as water, oil and ethylene glycol mixture. Nanofluids have been shown to increase the thermal conductivity and convective heat transfer performance of the base liquids. Nanofluids, have received the interest of many researchers because of their greatly enhanced thermal conductivity property. The concept of convective transport in nanofluids taking into consideration physical mechanisms responsible for the slip velocity between the nanoparticles and the base fluid such as Brownian Motion and thermophoresis has been widely investigated by Buongiorno (2006), and Olanrewaju and Makinde (2013) .One of the possible mechanisms for the anomalous increase in the thermal conductivity of nanofluids is the Brownian motion of the nanoparticles inside the base fluids. The addition of small particles causes scattering of the incident radiation allowing higher levels of absorption within the fluid.

The aim of the present study is to investigate MHD nanofluid bioconvection of nanofluids due to gyrotactic microorganisms past a convectively heated vertical plate. In the subsequent sections of this chapter, the governing boundary layer equations are formulated and transformed into ordinary differential equations using similarity transformation.The resulting differential equations are then solved numerically using the fourth order Runge-Kutta method. Graphical results are then presented and discussed taking into consideration the industrial applications.

4.2 Model formulation

In this study, a two dimensional steady flow of a water based electrically conducting nanofluid containing oxytactic microorganisms over a vertical plate is considered. The presence of nanoparticles is assumed to have no effect on the direction in which the microorganisms swim and on their swimming velocity. Bioconvection induced flow only take place in a dilute suspension of nanoparticles. A transverse magnetic field of strength B_0 is applied in the positive y-direction, normal to the plate surface. There is no applied voltage and the magnetic Reynolds number is small, hence the induced magnetic field and Hall effects are negligible .The left side of the plate is assumed to be heated by convection from a hot fluid at temperature which provides heat transfer coefficient h_f ,while the right surface is subjected to a stream of an electrically conducting cold nanofluid at temperature T_∞.The co−ordinate system is chosen such that x-axis is along the plate surface and y-axis is normal to it. The physical flow model and the co-ordinate system is shown in figure 1 below.

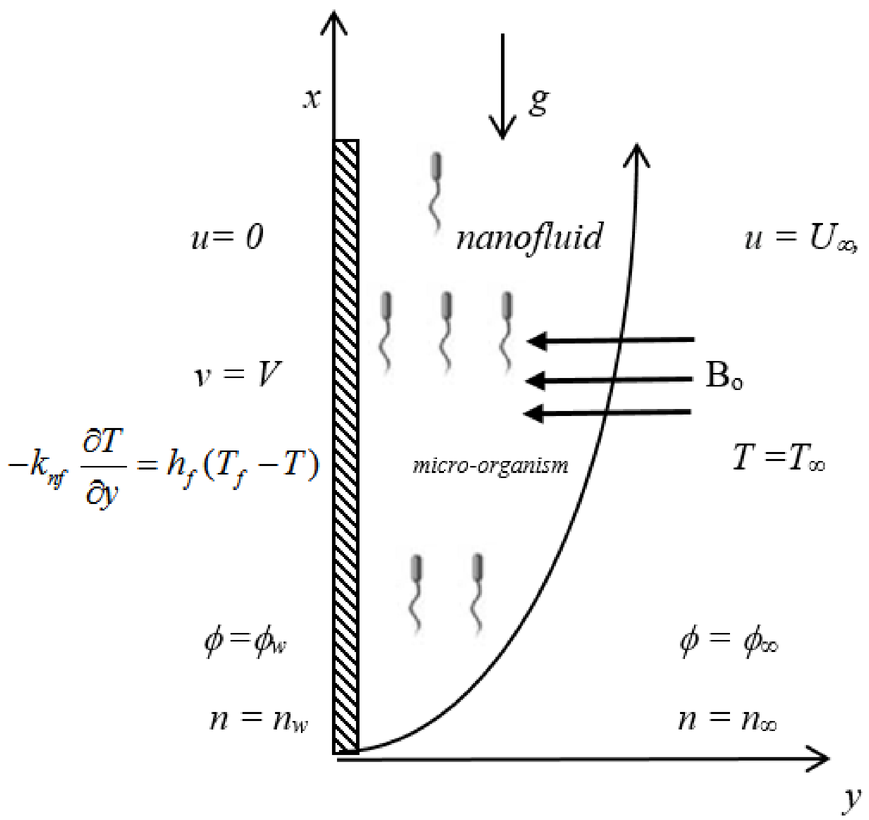

Fig 4.1: Schematic diagram of the flow

The basic steady governing equations related to conservation of mass, momentum ,energy, oxygen and microorganisms can be written as;

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$……………………………………………………………………………………4.1

$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x} + \nu_f\frac{\partial^2 u}{\partial y2} - \frac{\sigma B_0^2}{\rho_f} + \frac{1}{\rho_f}\left[(1-\phi_\infty)\rho_f\beta_g(T-T_\infty) - \left(\rho_{P-}\rho_f\right)g(\phi-\phi_\infty) - \right.$ $(n -$
$n\infty g\gamma\rho m - \rho f$………………………………………………………..4.2

$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha(\frac{\partial^2 T}{\partial x2} + \frac{\partial^2 T}{\partial y2}) + \tau\left\{D_B\frac{\partial\phi}{\partial y}\frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right\} + \frac{\mu\alpha}{K}\left(\frac{\partial u}{\partial v}\right)^2 + \frac{\alpha\sigma B_0^2 u^2}{k}$…...4.3

$$u\frac{\partial\phi}{\partial x}+v\frac{\partial\phi}{\partial y}=D_B\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y}\right)\ldots\ldots 4.4$$

$$u\frac{\partial n}{\partial x}+v\frac{\partial n}{\partial y}+\frac{bW_c}{(\phi_w-\phi_\infty)}\left[\frac{\partial}{\partial y}\left(n\frac{\partial\phi}{\partial y}\right)+\frac{\partial}{\partial x}\left(n\frac{\partial\phi}{\partial x}\right)\right]=D_m\left(\frac{\partial^2n}{\partial x^2}+\frac{\partial^2n}{\partial y^2}+2\frac{\partial^2\mathrm{n}}{\partial\mathrm{x}\partial y}\right)\ldots\ldots 4.5$$

Where u and v are the velocity components along the x and y directions respectively,ρ_f is the density of the base fluid,p is the fluid pressure,T is the local temperature,α is the thermal diffusivity of the base fluid,D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, D_M is the diffusivity of microorganisms,g is gravity vector,β is the volume expansion coefficient of the fluid,σ is the electrical conductivity of the fluid,μ is the viscosity of the fluid,$\tau=\frac{(\rho C)_p}{(\rho C)_f}$ is the ratio of the effective heat capacitance of the nanoparticle to that of the base fluid.,γ is the average volume of the microorganism, b is the chemotaxis constant ,W_c is maximum cell swimming speed (bW_c is assumed to be constant),n is the concentration of the microorganisms,ρ_m is the microorganisms density,ρ_f is the base fluid density,ϕ is the nanoparticle volume fraction.

The following boundary conditions are taken into consideration

$$u=0\ ,\ v=V,\ -k_{nf}\frac{\partial T}{\partial y}=h_f\left(T_f-T\right),\ \phi=\phi_{w,}\ \ n=n_w\qquad at\ \ y=0$$

$$u=u_\infty,\quad T=T_\infty\ ,\ \phi=\phi_\infty,\ \mathrm{n}=n_\infty\ \ at\ \ y\to\infty\ldots\ldots 4.6$$

Where $\phi_{w,}n_w$ are nanoparticle volume fraction and density of the motile microorganisms at the plate surface respectively.The corresponding ambient values are denoted $\phi_{\infty,}n_\infty$. k_{nf} is the effective thermal conductivity.

Using the stream function $\psi=\psi(x,y)$ the velocity components u and v are defined as;

$$u=\frac{\partial\psi}{\partial y}\ \ and\ \ v=-\frac{\partial\psi}{\partial x}\ldots\ldots 4.7$$

To attain similarity solution of equations 1-5, the stream function and dimensionless variables can be posited in the following form,

$$\eta=\left(\frac{a}{v_f}\right)^{\frac{1}{2}}y\ ,\ \ \psi=\left(av_f\right)^{\frac{1}{2}}xf(\eta)\ ,\ \ \theta(\eta)=\frac{\mathrm{T}-\mathrm{T}_\infty}{\mathrm{T_f}-\mathrm{T}_\infty}$$

$$\chi(\eta)=\frac{n-n_\infty}{n_w-n_\infty},\ \phi(\eta)=\frac{C-C_W}{C_W-C_\infty}\ \ ,\ \xi(\eta)=\frac{\boldsymbol{\phi}-\boldsymbol{\phi}_\infty}{\boldsymbol{\phi}_{\boldsymbol{W}}-\boldsymbol{\phi}_\infty}\qquad\ldots\ldots 4.8$$

With the above definitions, equation 4.1 is satisfied identically. The pressure outside the boundary layer (i.e the inviscid part of the flow) is constant. Hence the flow occurs only due to stretching of the sheet, therefore the pressure gradient can be neglected. Then by applying the similarity transforms on the remaining governing equations [i.e. 4.2-4.5], the similarity equations are obtained as follows;

$$u=\frac{\partial\psi}{\partial y}=\frac{\partial\psi}{\partial\eta}.\frac{\partial\eta}{\partial y}$$

$$=axf'(\eta)\ldots\ldots 4.9a$$

$$v=-\frac{\partial\psi}{\partial x}=-\left(av_f\right)^{1/2}f(\eta)\ldots\ldots 4.9b$$

$$\frac{\partial u}{\partial x}=\frac{\partial}{\partial x}\left(axf'(\eta)\right)=af'(\eta)\ldots\ldots 4.9c$$

$$\Longrightarrow u\frac{\partial u}{\partial x}=axf'.af'=a^2xf'^2\ldots\ldots 4.9d$$

Also;

$$\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}(axf')=axf''.\left(\frac{a}{v}\right)^{1/2}$$

$$\Longrightarrow v\frac{\partial u}{\partial y}=-a^2xff''\ldots\ldots 4.9e$$

$$\frac{\partial^2u}{\partial y^2}.=\frac{xa^2f'''}{v_f}\ldots\ldots 4.9f$$

$v_f \frac{\partial^2 u}{\partial y^2} = a^2 x f'''$……………………………………………………………………4.9g

By substituting equations 4.9a-4.9h into 4.2 we have;

$$a^2 x f''' + a^2 x f f'' - a^2 x f' - \frac{\sigma B_0{}^2 a x f'}{\rho_f} + \left(\frac{(\phi-\phi_\infty)\rho_f Bg(T-T_\infty)}{\rho_f}\right) - \frac{(\rho_p-\rho_f)g(\phi-\phi_\infty)-(n-n_\infty)g\gamma(\rho_m-\rho_f)}{\rho_f}$$ …………………...4.10

Factorizing equation 4.10 and introducing the following dimensionless quantities;

$$Ha = \frac{\sigma B_0{}^2}{a\rho_f}, \quad Gr = \frac{\beta\rho_f(1-\phi_\infty)(T_W-T_\infty)}{au_0}, \qquad Nr = \frac{(\rho_f-\rho_\infty)(\phi_w-\phi_\infty)}{\beta\rho_f(1-\phi_\infty)(T_W-T_\infty)}$$

$$Rb = \frac{\gamma(n_w-n_\infty)(\rho_m-\rho_f)}{\beta\rho_f(1-\phi_\infty)(T_W-T_\infty)}$$ ……………………………………………………………………..4.11

And rearranging, the following similarity equation is obtained

$$f''' + ff'' - (f')^2 - Haf' + Gr(\theta - Nr\xi - Rb\chi) = 0$$ …………………………………4.12

Making T the subject in the quantity $\theta(\eta) = \frac{T-T_\infty}{T_f-T_\infty}$, we obtain

T= $(T_f - T_\infty)\theta(\eta) + T_\infty$………………………………………………………………4.13

Differentiating equation 13 with respect to x and y we have;

$\frac{\partial T}{\partial x} = 0$……………………………………………………………………………..4.14a

$\frac{\partial T}{\partial y} = \left(\frac{a}{v}\right)^{1/2}(T_f - T_\infty)\theta'$………………………………………………………………..4.14c

$\frac{\partial^2 T}{\partial y^2} = \left(\frac{a}{v}\right)^{1/2}(T_f - T_\infty)\theta''$…………………………………………………………...4.14d

Substituting equ.4.14a-4.14c to equation 4.3 we have;

$$a^{\frac{1}{2}}(T_f - T_\infty)\theta' = \alpha\left(\left(\frac{a}{v}\right)^{\frac{1}{2}}(T_f - T_\infty)\theta''\right) + \tau\left\{D_B\frac{\partial\phi}{\partial y}\left(\frac{a}{v}\right)^{\frac{1}{2}}(T_f - T_\infty)\theta'\right\} + \frac{D_T}{T_\infty}\left[a/v(T_f - T_\infty)2\theta'^2\right] + \frac{\mu\alpha}{K}\left(a^2x^2f''^2.\frac{a}{v}\right) + \frac{\alpha u\sigma B_0{}^2}{k}\frac{\alpha u\sigma B_0{}^2}{k}$$ ……………………………………………4.15

Introducing the boundary conditions;

$$Pr = \frac{v}{\infty}, \quad Nb = \frac{\tau D_B(\phi_w-\phi_w)}{\infty}, \quad Nt = \frac{\tau D_T(T_W-T_\infty)}{\infty},$$

$$Ec = \frac{u_0{}^2}{c\rho_f(T_f-T_\infty)}, \xi(\eta) = \frac{\boldsymbol{\phi}-\boldsymbol{\phi}_\infty}{\boldsymbol{\phi}_W-\boldsymbol{\phi}_\infty}$$ ……………………………………………………………….......4.16

On rearranging, equation 4.15 reduces to

$\theta'' + \theta'(Prf + Nb\xi') + Nt(\theta')^2 + PrEc((f'')^2 + Ha(f')^2) = 0$………………………….4.17

From, $\xi(\eta) = \frac{\phi-\phi_\infty}{\phi_w-\phi_\infty}$,

$\phi = (\phi_w - \phi_\infty)\xi(\eta) + \phi_\infty$…………………………………………………………........4.18

Differentiating w.r.t .x and y gives;

$\frac{\partial\phi}{\partial x} = 0$……………………………………………………………………………..4.19a

$\frac{\partial\phi}{\partial y} = \left(\frac{a}{v}\right)^{\frac{1}{2}}(\phi_w - \phi_\infty)\xi'$…………………………………………………………………………………4.19b

$\frac{\partial^2\phi}{\partial y^2} = \left(\frac{a}{v}\right)^{\frac{1}{2}}(\phi_w - \phi_\infty)\xi''$……………………………………………………………………………..4.19c

Substituting equations 4.19a-4.19c to governing equation 4.4 we have;

$$(av)^{1/2}((\phi_w - \phi_\infty)\xi'') = D_B\left(0 + \left(\frac{a}{v}\right)^{\frac{1}{2}}(\phi_w - \phi_\infty)\xi''\right) + \left(\frac{D_T}{T_\infty}\right)\left(\frac{a}{v}\right)^{\frac{1}{2}}(T_f - T_\infty)\theta''$$ …………4.20

Introducing the following dimensionless quantity

$Le = \frac{v}{D_B}$, and rearranging, equation 4.20 becomes,

ξ" $+Lef\xi' + \frac{Nt}{Nb}\theta'' =$
0……………………………………………………………………………………………4.21
From $\chi(\eta) = \frac{n-n_\infty}{n_w-n_\infty}$, n can be written as;
n= $(n_w - n_\infty)\chi(\eta) + n_\infty$…………………………………………………………………….4.22
Differentiating 4.22 with respect to x and y gives
$\frac{\partial n}{\partial x} = 0$……………………………………………………………………………......4.23a
$\frac{\partial n}{\partial y} = \left(\frac{a}{v}\right)^{\frac{1}{2}}(n_w - n_\infty)\chi'$…………………………………………………………………4.23b
$v\frac{\partial n}{\partial y}$=$-af(n_w - n_\infty)\chi'$………………………………………………………………...4.23c
$\frac{\partial^2 n}{\partial y^2} = \frac{a}{v}(n_w - n_\infty)\chi''$ ………………………………………………………………4.23d
Substituting 4.23a-4.23d to 4.5 we obtain
-a$f(n_w - n_\infty)\chi'$+$\frac{bw_c}{\phi_w-\phi_\infty}\left[\frac{\partial}{\partial y}\left(n\frac{\partial\phi}{\partial y}\right) + \frac{\partial}{\partial x}\left(n\frac{\partial\phi}{\partial x}\right)\right] = D_m\left(\frac{a}{v}\right)(n_w - n_\infty)\chi''$…………......4.24
Introducing the following dimensionless quantities;
$Lb = \frac{v}{D_m}$, $\xi(\eta) = \frac{\phi-\phi_\infty}{\phi_w-\phi_\infty}, Pe = \frac{bw_c}{D_m}$,$\Omega = \frac{n_\infty}{n_w-n_\infty}$………………………………………………..4.25
Factorizing and rearranging, equation 4.24 reduces to;
$\chi'' + Lbf\chi' - Pe[\xi''(\chi + \Omega) + \xi'\chi']$=0…………………………………………………4.26
The corresponding boundary conditions
$u = 0,\ v = V,\ -k_{nf}\frac{\partial T}{\partial y} = h_f(T_f - T),\ \phi = \phi_w,$ n= n_w at y= 0 can be transformed as follows;
$u = 0 = axf'(\eta)$………………………………………………………………………4.27a
Simplifying equation 4.40 we get
$f'(0) = 0$…………………………………………………………………………………4.27b
At $y = 0$,$v = 0$
$0 = -(av_f)^{\frac{1}{2}}\ f\ (0)$ ……………………………………………………………………………… ...……..4.28a
$\Longrightarrow f(0) = 0$………………………………………………………………………………4.28b
At $y = 0$,$-k_{nf}\frac{\partial T}{\partial y} = h_f(T_f - T)$
$T_f - T = -k_{nf}\frac{\partial T}{\partial y}/h_f$…………………………………………………………………………4.29a
Introducing, $Bi = \frac{h_f}{k_f}\sqrt{\frac{v_f}{a}}$, $\frac{\partial T}{\partial y} = \left(\frac{a}{v}\right)^{1/2}(T_f - T_\infty)\theta',\ \phi(\eta) = \frac{T-T_\infty}{T_f-T_\infty}$ and simplifying,
Equation 4.29a reduces to;
$\theta'(0) = Bi(\theta -$
1…………………………………………………………………………………………………..4.29b
At y =0 n=n_w
Then,$\chi(0) = \frac{n_w-n_\infty}{n_w-n_\infty} = 1$
$\therefore\ \chi(0)$=1……………………………………………………………………………...4.30
Also at y=0, $\phi = \phi_W$
$\Longrightarrow\xi(0) = \frac{\phi_W-\phi_\infty}{\phi_W-\phi_\infty} = 1$
$\therefore\ \xi(0)$=1…………………………………………………………………………………4.31
At y=∞, $\eta = \infty$, the boundary conditions transform as follows;
$u = u_\infty = axf'(\infty)$……………………………………………………………...........4.32a
u_∞= $axf'(\infty)$……………………………………………………………………......4.32b
i.e. $f'(\infty) = \frac{u_\infty}{ax} = 1$………………………………………………………………4.32c
At y=0 ,$\eta = \infty$, T=T_∞
Implying that;
$\theta(\infty)$=$\frac{T_\infty-T_\infty}{T_f-T_\infty}$=0
$\Longrightarrow\theta(\infty)$=0……………………………………………………………………………4.33
At $y = \infty, \eta = \infty, \phi = \phi_\infty$
$\xi(\infty) = \frac{\phi_\infty-\phi_\infty}{\phi_w-\phi_\infty} = 0$…………………………………………………………………..4.34
Again at $y = \infty, \eta = \infty, n = n_w$

$\chi(\infty) = \frac{n_\infty - n_\infty}{n_w - n_\infty} = 0$…………………………………………………………………………...4.35

From the above formulation, the required equations to be used in getting the numerical solutions are equs.4.12, 4.17, 4.21 and 4.26 i. e.

$f''' + ff'' - (f')^2 - Haf' + Gr(\theta - Nr\xi - Rb\chi) = 0$…………………………………4.36a

$\theta'' + \theta'(Prf + Nb\xi') + Nt(\theta')^2 + PrEc((f'')^2 - Ha(f')^2) = 0$………………………4.36b

$\xi'' + Lef\xi' + \frac{Nt}{Nb}\theta'' =$

0………………………………………………………………………………………4.36c

$\chi'' + Lbf\chi' - Pe[\xi''(\chi + \Omega) + \xi'\chi'] = 0$……………………………………………...4.36d

Subject to the dimensionless boundary conditions;

$f(0) = 0 \;, f'(0) = 0, \; \theta'(0) = Bi(\theta - 1), \; \xi(0) = 1, \chi(0) = 1$ at $\eta = 0$ …………...4.37

$f'(\infty) = 0, \theta(\infty) = 0, \xi(\infty) = 0 \;, \chi(\infty) = 0$ as $\eta \to \infty$…………………………………4.38

Where prime denotes differentiation with respect to η and Ha is Hartman number, Nr is buoyancy ratio parameter, Rb is the Rayleigh number, Pr is the Prandtl number, Nt is thermophoresis parameter, Nb Brownian motion parameter, Ec is Eckert number, Gr is grashof number, Le is traditional or regular lewis number, Lb is bioconvection lewis number, Pe is bioconvection peclet number, Ω is the microorganisms concentration difference parameter.

4.3 Numerical Procedure

To obtain the numerical solutions, equations 4.36a−4.36d are solved subject to the boundary conditions 4.37 and 4.38 using the fourth order Runge−kutta integration scheme with the shooting technique.The Marple computer programme which uses symbolic and computational language was used in carrying out the computations. In this method, the coupled differential equations 4.36a−$4.36d$ are transformed into first order equations.

New variables are defined as follows

$f_1 = f$, $f_2 = f'$, $f_3 = f''$, $f_4 = \theta$, $f_5 = \theta'$, $f_6 = \xi$, $f_7 = \xi'$, $f_8 = \chi$, $f_9 = \chi'$…………………………………………………………………………………4.39

Where prime represents differentiation of f, θ, ξ and χ with respect to η.The set of higher order non− linear boundary value problem with their respective boundary conditions are reduced to first order differential equations with appropriate initial conditions respectively as follows

$f_1' = f_2,$ ……………………………………………………………………………4.40a

$f_2' = f_3,$…………………………………………………………………………….4.40b

$f_3' = -f_1f_3 + f_2{}^2 + Haf_2 - Gr(f_4 - Nrf_6 - Rbf_8)$……………………………………4.40c

$f_4' = f_5$…………………………………………………………………………....4.40d

$f_5' = -f_5(Prf_1 + Nbf_6) - Nt(f_5)^2 - PrEc(f_3)^2 - Ha(f_2)^2$…………………………..4.40e

$f_6' = f_7$……………………………………………………………………………4.40f

$f_7' = -Lef_1f_7 + \frac{Nt}{Nb}f_5'$………………………………………………………………....4.40g

$f_8' = f_9$……………………………………………………………………………4.40h

$f_9' = -Lbf_9 + Pe[f_7'(f_8 + \Omega) + f_9f_8]$…………………………………………………. 4.40i

Subject to the boundary conditions

$f_1(0) = 0, f_2(0) = 0, f_5(0) = Bi(f_4(0) - 1), f_6(0) = 1, f_8(0) = 1$

$f_2(\infty) = 0, f_4(\infty) = 0, f_6(\infty) = 0 \;, f_8(\infty) = 0$

V. Results and Discussion

Numerical computations were performed for various values of the physical parameters involved namely Hartman number Ha, Prandtl number Pr, Eckert number Ec, bioconvection lewis number Lb, traditional lewis number Le, bioconvection peclet number Pe, buoyancy ratio parameter Nr, bioconvection Rayleigh number Rb, Brownian motion parameter Nb, thermophoresis parameter Nt, and grashof number Gr. For the illustration of results, numerical values were plotted in figures 5.2−5.18 and a detailed discussion on the effects of the governing physical parameters on the velocity, temperature, nanoparticles volume fraction, and microorganisms' conservation profiles was done. The Prandtl number of the base fluid was kept constant at 6.2.

5.2 Effect of Parameter Variation on velocity Profiles

The effects of various thermo physical parameters on the nanofluids velocity profiles are illustrated in figures 5.1−5.5 . The velocity of the nanofluids increases gradually until it attains the free stream value far away from the plate, satisfying the prescribed boundary conditions. In fig.5.2 and 5.3, it is noted that an increase in Ha and Nr decreases both the momentum boundary thickness and the fluid velocity. This is due to the fact that their increase impetuses the fluid towards the plate surface. The reverse effect is observed with an increase in Gr, Nb and Nt as seen in figures 5.2, 5.4 and 5.5.This result agrees with the fact that bioconvection plumes which are characterized by downward motion of the fluid oppose the upward motion of the nanofluid due to buoyancy and also the presence of magnetic field results in a Lorentz force which tends to retard the motion of the fluid.

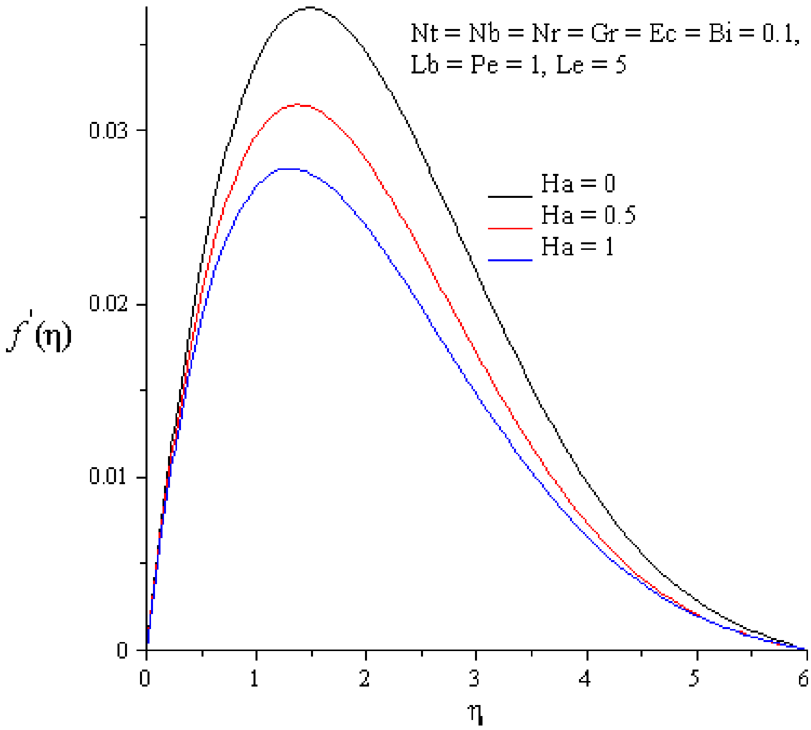

Figure 5.1: Effects of Hartman number on velocity

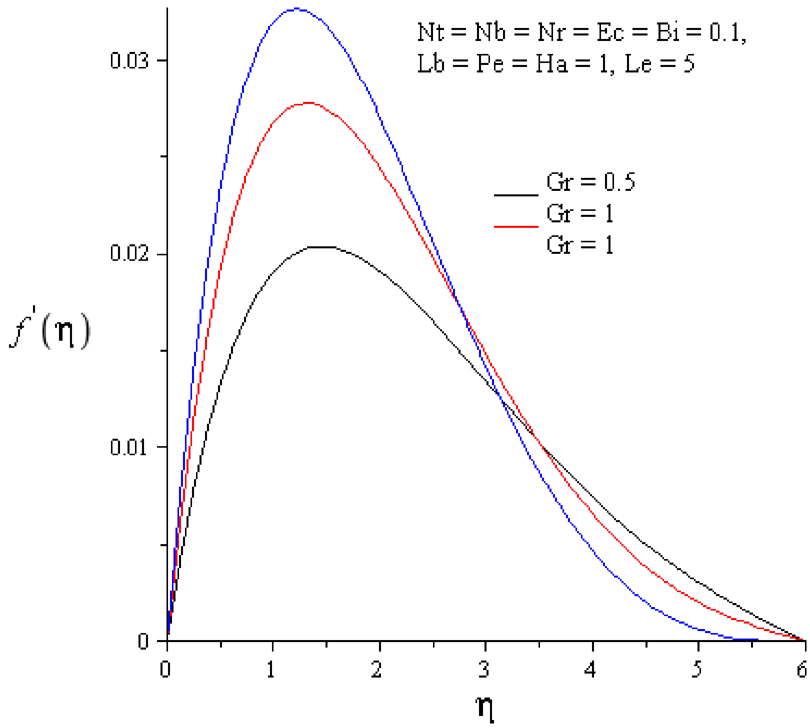

Figure 5.2: Effects of Grashof number on velocity

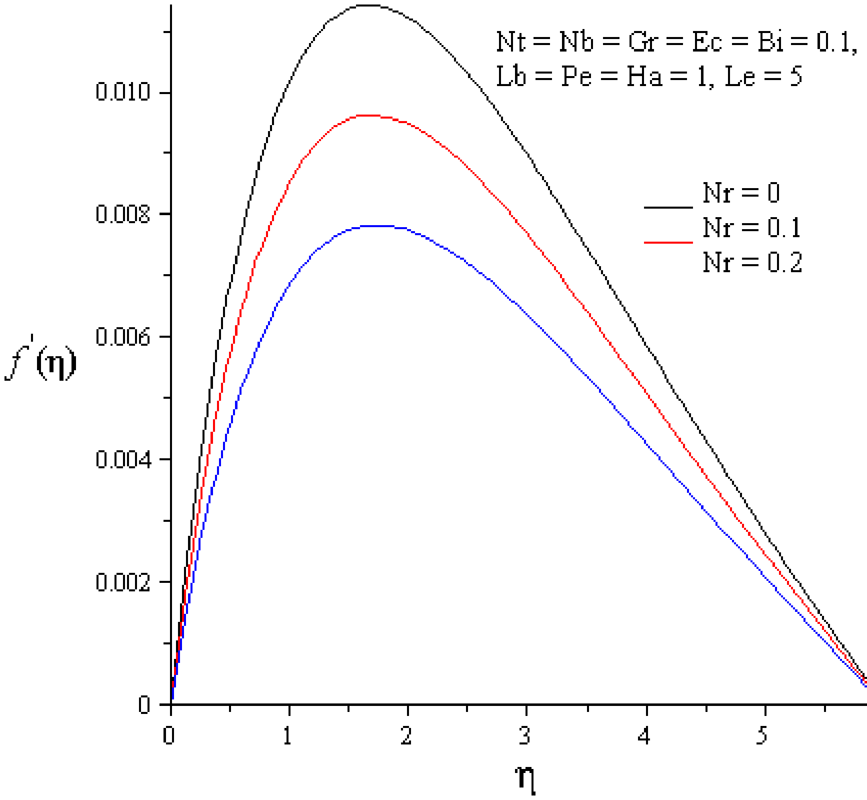

Figure 5.3: Effects of buoyancy ratio on velocity

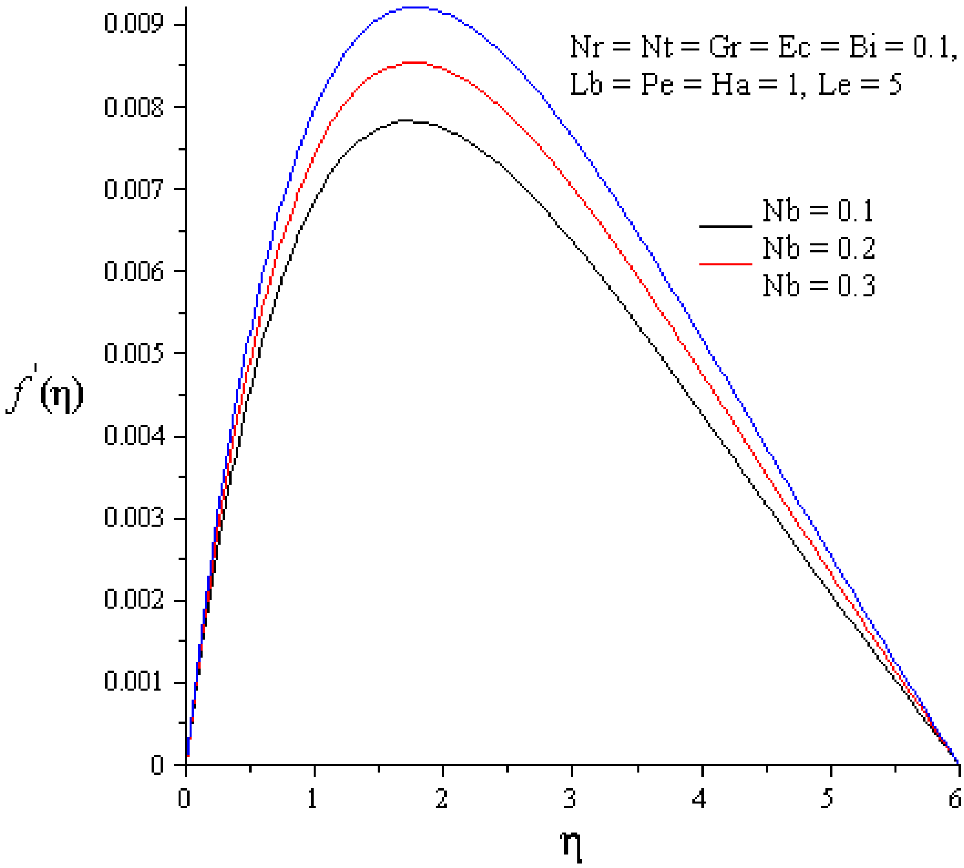

Figure 5.4: Effects of Brownian motion on velocity

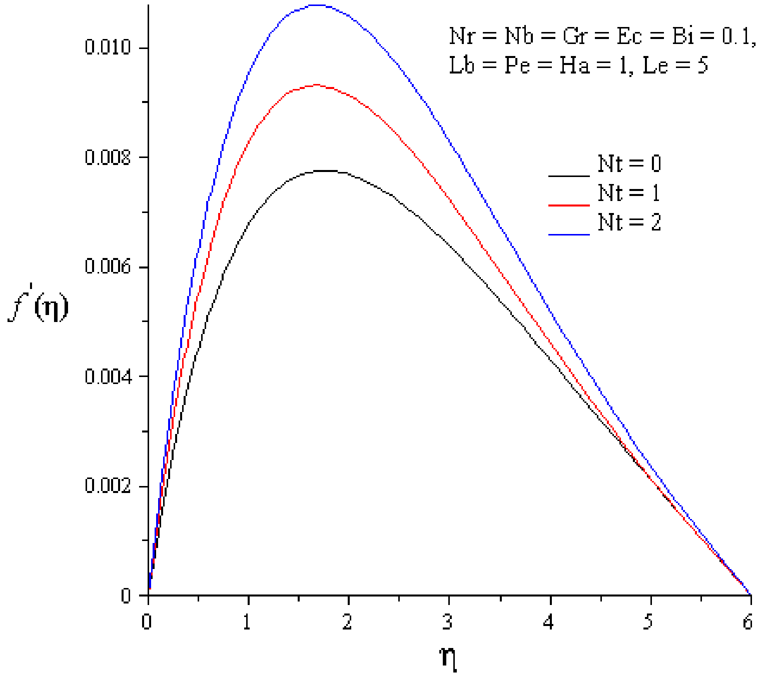

Figure 5.5: Effects of thermophoresis on velocity

5.2 Effects of Parameter Variation on Temperature Profiles

Figures 5.6 − 5.9 illustrate the effects of various thermo physical parameters on the temperature profiles. The figures indicate that the temperature is maximum at the plate surface due to convectional heating but decreases to zero far away from the plate surface satisfying the free stream conditions. From the figures 5.6 − 5.9, it can be depicted that increasing Ha,Nb,Nt and Ec increases both the temperature and thermal boundary layer thickness.This is attributed to the additional heating due to resistance of fluid flow as a result of magnetic field in the presence of the nanoparticle and additional heating as a result of viscous dissipation. The presence of nanoparticles in the base fluid increases the thermal conductivity thus resulting into an increased thermal boundary layer thickness and temperature. Furthermore it can be noted that in some of the graphs, the curves intersect at some point. The values at these intersection points give the best operating conditions for the fluid flow that in turn result in best efficiency point.

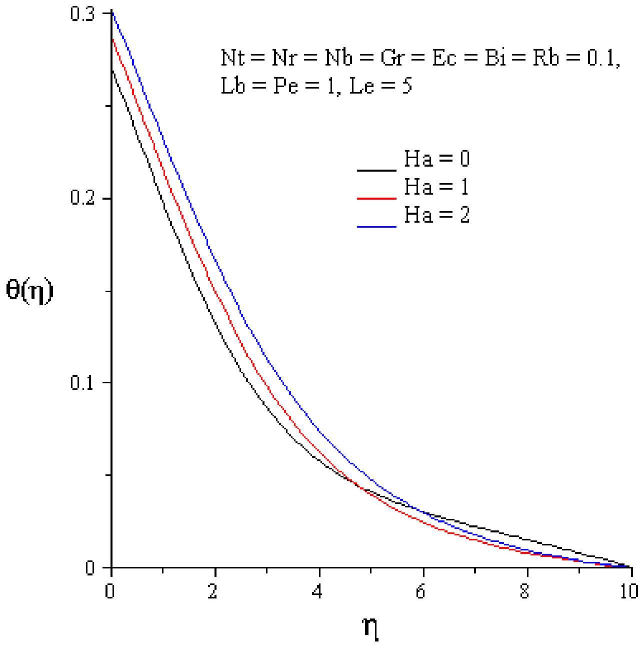

Fig 5.6: Effects of Hartman number on Temperature

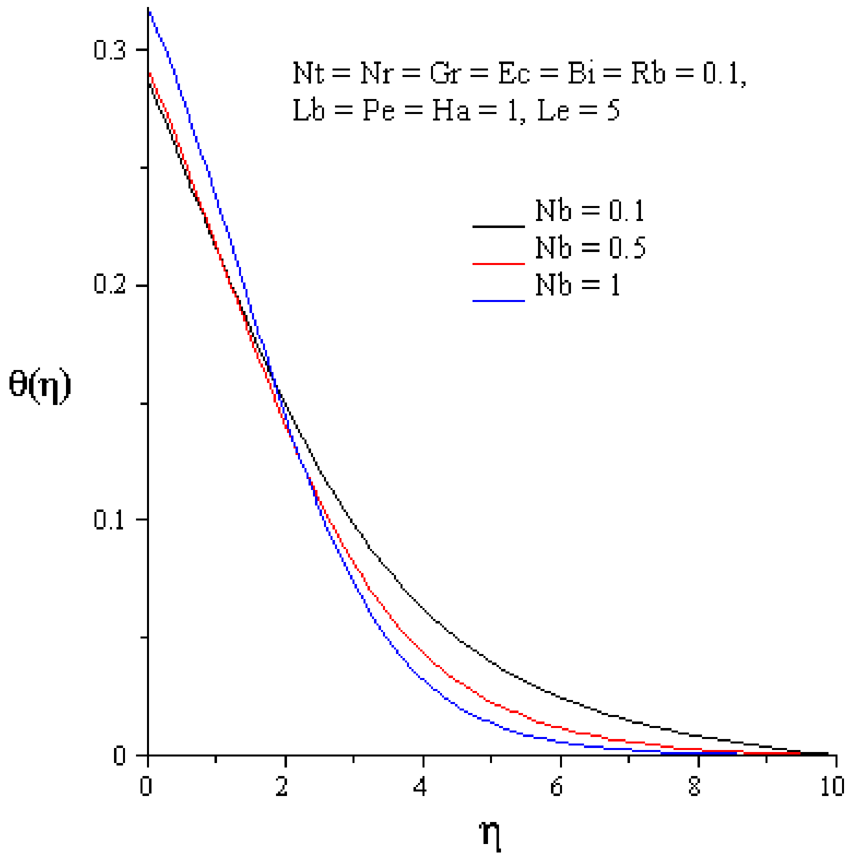

Fig 5.7: Effects of Brownian motion on Temperature

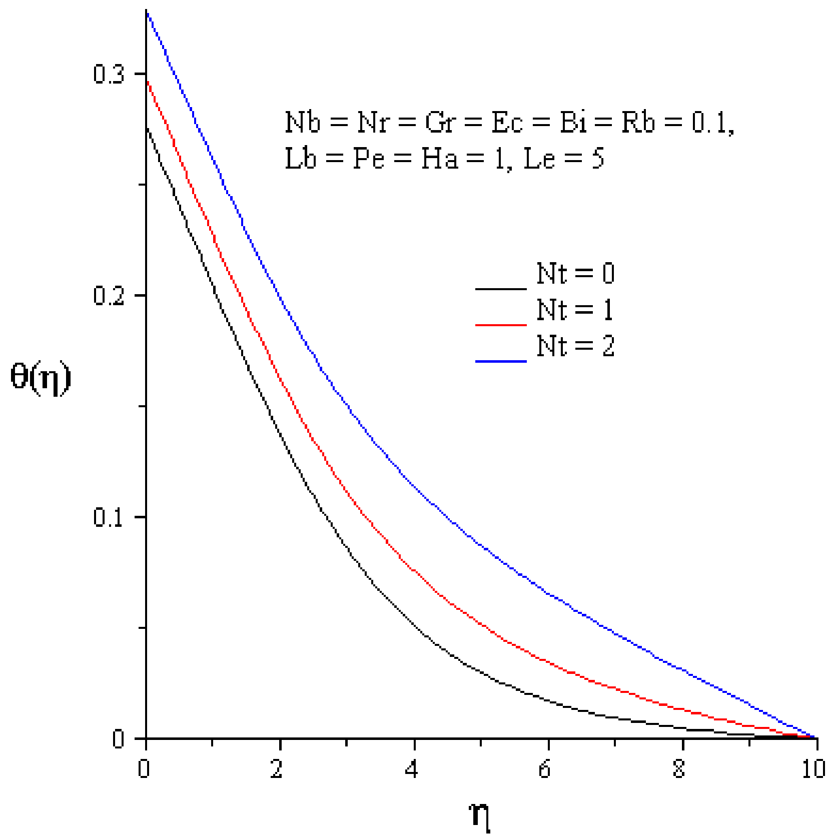

Figure 5.8: Effects of Thermophoresis on Temperature

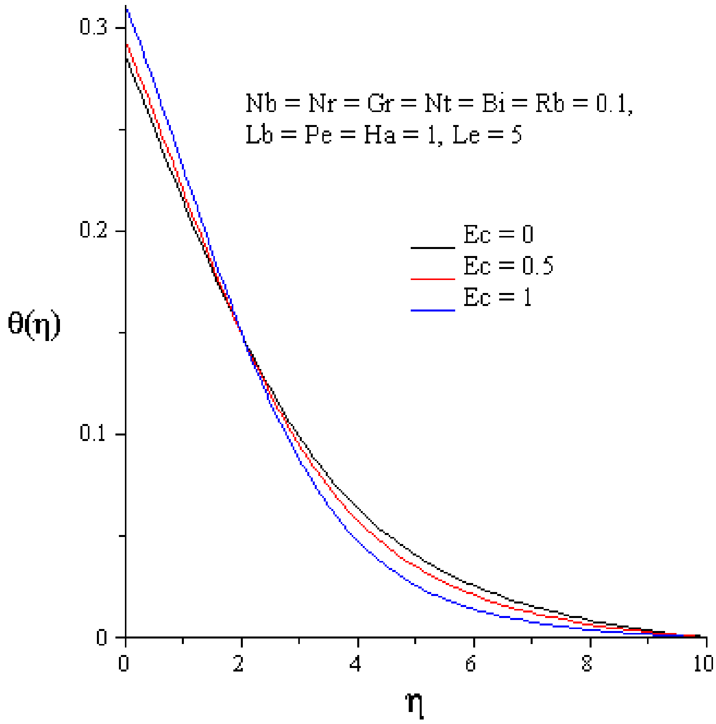

Figure 5.9: Effects of Eckert number on Temperature

5.3 Effects of parameter variation on Nanoparticle concentration Profiles

From figures 5.10−5.11 it can be observed that increasing Ha and Nt increases the nanoparticle concentration layers. It is also noted from figures 5.12 and 5.13 that an increase in Nb and Le results in a decrease in nanoparticle concentration. This is due to combined effect of migration of a colloidal particle, buoyancy, magnetic effect and bioconvection plumes that result in a reduced momentum boundary layer thickness and as such there will be more particles near the boundary layer region hence increased nanoparticle concentration at the plate surface. The reduction in the nanoparticle boundary layer concentration can be due to the decrease in mass diffusivity and the Brownian motion of nanoparticles in the boundary layer region.

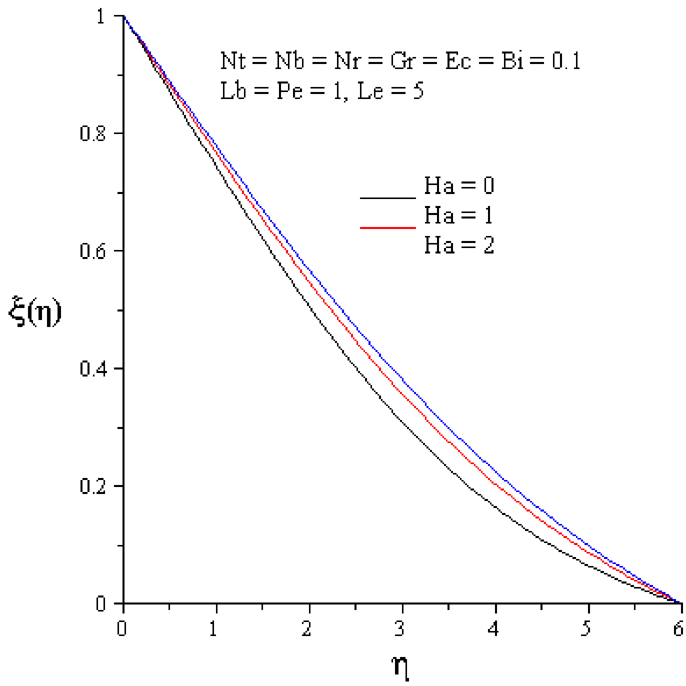

Figure 5.10: Effects of Hartman number on nanoparticle concentration

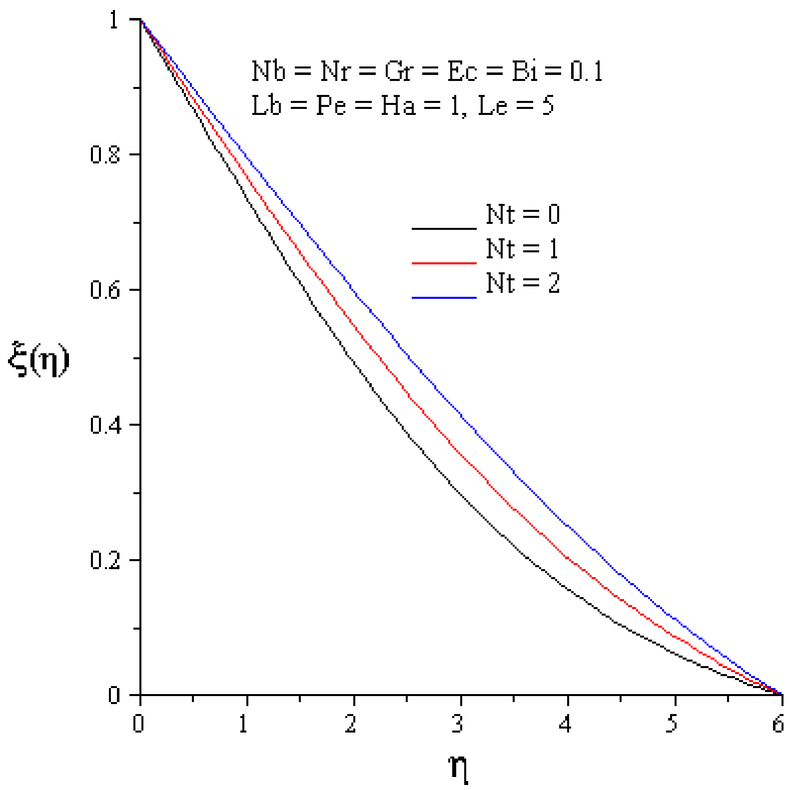

Figure 5.11: Effects of thermophoresis on nanoparticle concentration

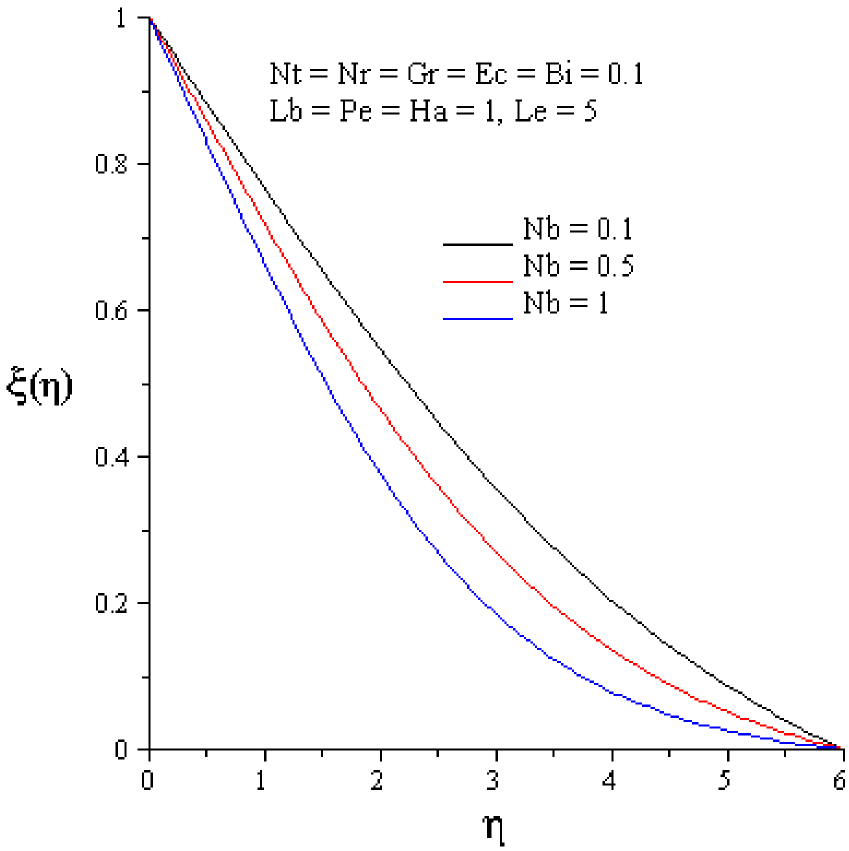

Figure 5.12: Effects of Brownian motion on nanoparticle concentration

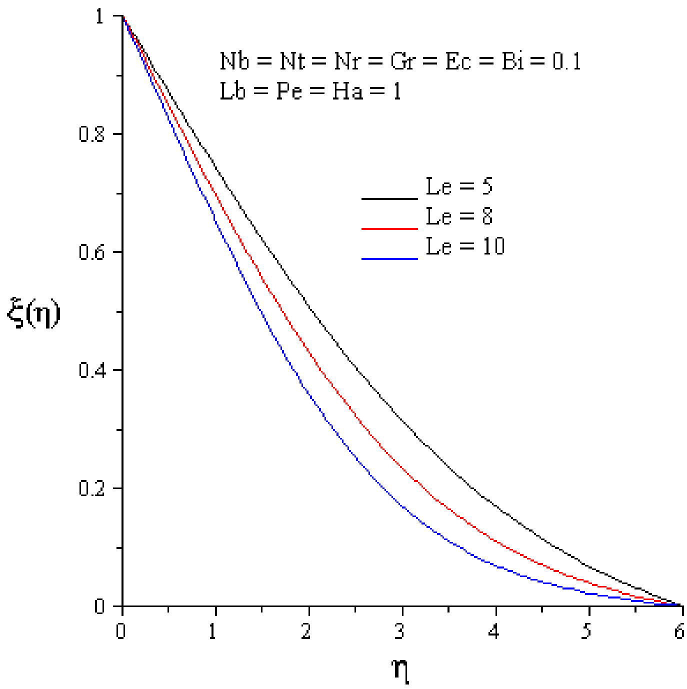

Figure 5.13: Effects of traditional lewis number on nanoparticle concentration

5.4 Effects of parameter variation on Microorganisms Density Profiles

Figures 5.14-5.17 show the effects of various thermo physical parameters on the microorganisms' concentration profiles. The dimensionless density of motile microorganisms is strongly affected by Lb and Pe. From figures 5.14 and 5.15, it can be noted that an increase in Lb and Pe decreases the concentration thickness for the motile microorganisms as well as the density of the motile microorganisms. On the other hand, the dimensionless velocity of motile microorganisms increases with increase in Nr and Nb. This is because buoyancy, bioconvection and magnetic field intensity drive the fluid towards the plate surface thus the motile microorganisms' density at the boundary layer will increase.

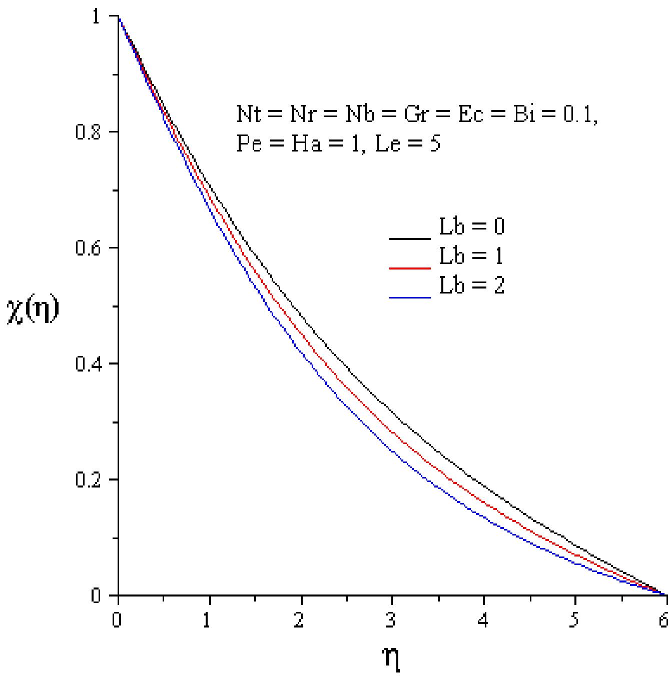

Figure 5.14: Effects of bioconvection lewis number on microorganism concentration

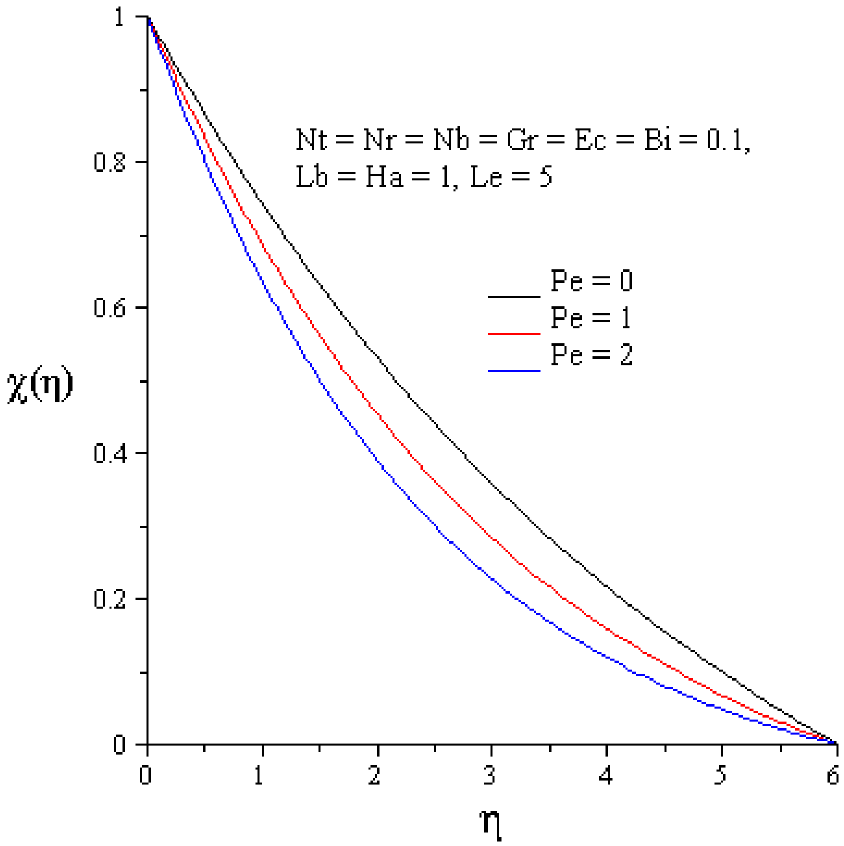

Figure 5.15: Effects of bioconvection peclet number on microorganism concentration

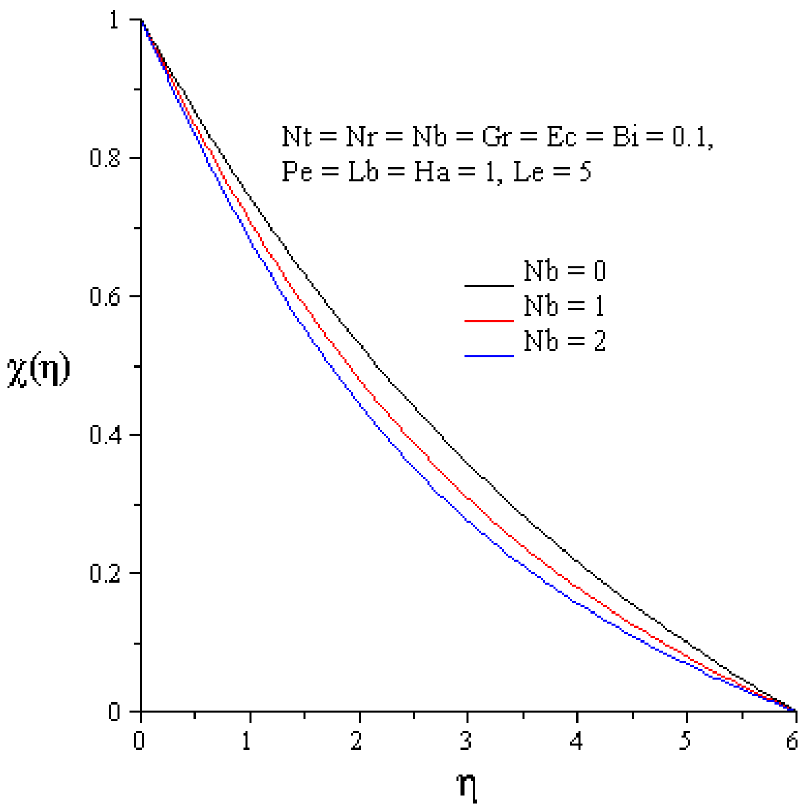

Figure 5.16: Effects of Brownian motion on microorganism concentration

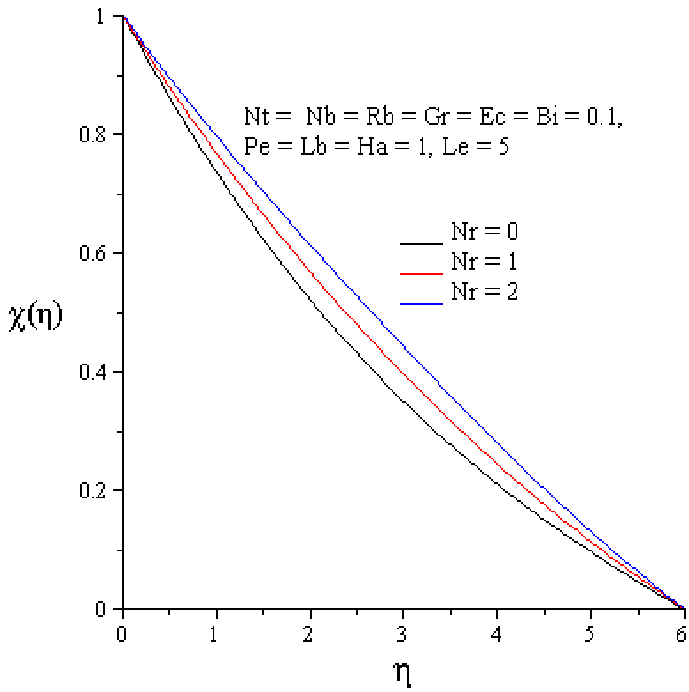

Figure 5.17: Effects of buoyancy on microorganism concentration

VI. Conclusions And Recommendations

In this study, the boundary layer flow of a water-based nanofluid containing motile microorganisms past a vertical plate is investigated numerically. The governing non-linear partial differential equations were transformed into ordinary differential equations using the similarity transformation and solved numerically using the Runge-Kutta fourth order algorithm with a systematic guessing of shooting technique until the boundary condition at infinity were satisfied. The conventional fluid was considered, this was also referred to as base fluid (water) and the Prandtl number was kept constant at 6.2.The numerical results were obtained for dimensionless parameters and graphical analysis was done and analyzed quantitatively. Based on the graphical representation, the following conclusions were drawn:

- The fluid velocity decreases with an increase in the magnetic parameter(Ha)
- Increasing the Ec, Nt and Ha leads to an increase in both the fluid temperature and the thermal boundary layer thickness.
- The dimensionless nanoparticle concentration increases with an increase in Ha, Nt and Nr. The opposite is observed with increased Gr, Nb and Ec.
- Increasing Lb and Pe reduces the dimensionless microorganism conservation while an increase is observed on the same with increased Nr and Ha.

Generally from these results it can be concluded that there is an increase in temperature and a decrease in velocity due to the increased magnetic strength, which implies that nanofluids in the presence of magnetic field are important in cooling and heating processes.

5.1 Suggestions for Further Work.

Further studies can be carried out on simultaneous effects of magnetic field and Navier-slip on boundary layer flow with nanofluids containing gyrotactic microorganisms over an inclined surface.

Acknowledgement

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NOMENCLATURE

(u,v) Velocity Components
(x,y) co-ordinates
$\boldsymbol{B_0}$ Constant Applied magnetic field
$\boldsymbol{C_p}$ Specific heat at constant pressure
Nu Local Nusselt number
Pr Prandtl number
Ec Eckert number
Gr Grashof number
Bi Local Biot number
Ha Hartmann number
T Temperature
$\boldsymbol{T}_\infty$ Free stream temperature
$\boldsymbol{u}_\infty$ Free stream velocity
K Thermal conductivity
M Magnetic Parameter
g acceleration due to gravity
h_f heat transfer coefficient

Greek Symbols

ψ Stream function
θ Dimensionless Temperature
η Similarity variable
β Thermal expansion coefficient
α Thermal diffusivity
φ Solid volume fraction
υ Kinematic viscosity
μ Dynamic viscosity
ρ Density
T_W Skin friction or Shear stress
λ Slip coefficient
ρ_f Fluid density
ρ_p nanoparticle mass density
$(\rho c)_f$ heat capacity of the fluid
$(\rho c)_p$ effective heat capacity of the nanoparticle material
τ ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid
χ Rescaled density of motile microorganisms
γ Average volume of a microorganism

Subscripts

f fluid
s solid
nf nanofluid

Abbreviations

MHD Magneto hydrodynamics
nm nanometer

References

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