IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 15, Issue 2 Ser. II (Mar – Apr 2019), PP 46-51
www.iosrjournals.org

Contra α Generalized Continuous Mappings in Neutrosophic Topological Spaces

Prishka F[1] and Jayanthi D[2]
Department of Mathematics, Avinashilingam (Deemed to be) University, Coimbatore.

Abstract*: In this paper we introduce the concepts of contra α generalized continuous mappings and contra α generalized irresolute mappings in neutrosophic topological spaces. We also discuss various properties and relations between the newly introduced continuous mappings and the other existing contra continuous mappings in neutrosophic topological spaces.*
Mathematics Subject Classification (2010): *54A40, 03E72*
Keywords: *Neutrosophic topology, neutrosophic contra α generalized continuous mapping, neutrosophic contra α generalized irresolute mapping.*

Date of Submission: 29-03-2019 Date of acceptance: 13-04-2019

I. Introduction

Fuzzy set was introduced by L.A.Zadeh[14] in 1965. K.Atanassov[1] in 1983 introduced the concept of intuitionistic fuzzy sets using the notion of fuzzy sets. The concept of fuzzy topology was introduced by C.Chang[2] in 1968. Coker[3] introduced intuitionistic fuzzy topological spaces. Neutrosophic set theory was proposed in 1998 by Florentin Smarandache[7], who also developed the concept of single-valued neutrosophic set oriented towards real world scientific and engineering applications. Ekici and Etienne kerre [6] introduced contra continuous mapping in fuzzy topological spaces in 2006. Intuitionistic fuzzy contra continuous mapping was introduced by Kresteska and Ekici [10]. F.Prishka and D.Jayanthi [11] introduced α generalized continuous mappings in neutrosophic topological spaces.

In this paper, we have introduced contra α generalized continuous mapping and contra α generalized irresolute mapping in neutrosophic topological spaces and obtained some of their properties.

II. Preliminaries

Here in this paper the neutrosophic topological space is denoted by (X,τ). Also the neutrosophic interior, neutrosophic closure of a neutrosophic set A are denoted by NInt(A) and NCl(A). The complement of a neutrosophic set A is denoted by C(A) and the empty and whole sets are denoted by 0_N and 1_N respectively.

Definition 2.1: [7] Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form A = $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x), \nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element x ϵ X to the set A.

A neutrosophic set A = $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ can be identified as an ordered triple $\langle \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ in $]^-0, 1^+[$ on X.

Definition 2.2: [7] Let A= $\langle \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ be a neutrosophic set on X, then the complement C(A) may be defined as

1. C(A) = $\{\langle x, 1- \mu_A(x), 1- \nu_A(x)\rangle : x \in X\}$
2. C(A) = $\{\langle x, \nu_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$
3. C(A) = $\{\langle x, \nu_A(x), 1- \sigma_A(x), \mu_A(x)\rangle : x \in X\}$

Note that for any two neutrosophic sets A and B,

4. C(A∪ B) = C(A) ∩ C (B)
5. C(A ∩ B) = C(A) ∪ C(B)

Definition 2.3: [7] For any two neutrosophic sets A = $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ and B = $\{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle : x \in X\}$ we may have

1. A⊆ B ⇔ $\mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x) and\ \nu_A(x) \ge \nu_B(x)\ \forall\ x \in X$
2. A⊆ B ⇔ $\mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x) and\ \nu_A(x) \ge \nu_B(x)\ \forall\ x \in X$
3. A∩ $B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x)\rangle$
4. A∩ $B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \vee \nu_B(x)\rangle$
5. A∪ $B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x)\rangle$
6. A∪ $B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \wedge \nu_B(x)\rangle$

Definition 2.4: [12] A neutrosophic topology (NT) on a non-empty set X is a family τ of neutrosophic subsets in X satisfies the following axioms:

(NT_1) $0_N, 1_N \in \tau$

(NT_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(NT_3) $\cup G_i \in \tau \; \forall \{G_i : i \in J\} \subseteq \tau$

In this case the pair (X, τ) is a neutrosophic topological space (NTS) and any neutrosophic set in τ is known as a neutrosophic open set (NOS) in X. A neutrosophic set A is a neutrosophic closed set (NCS) if and only if its complement C(A) is a neutrosophic open set in X. Here the empty set (0_N) and the whole set (1_N) may be defined as follows:

(0_1) $0_N = \{\langle x, 0,0,1\rangle : x \in X\}$

(0_2) $0_N = \{\langle x, 0,1,1\rangle : x \in X\}$

(0_3) $0_N = \{\langle x, 0,1,0\rangle : x \in X\}$

(0_4) $0_N = \{\langle x, 0,0,0\rangle : x \in X\}$

(1_1) $1_N = \{\langle x, 1,0,0\rangle : x \in X\}$

(1_2) $1_N = \{\langle x, 1,0,1\rangle : x \in X\}$

(1_3) $1_N = \{\langle x, 1,1,0\rangle : x \in X\}$

(1_4) $1_N = \{\langle x, 1,1,1\rangle : x \in X\}$

Definition 2.5:[12] Let (X, τ) be a neutrosophic topological space and let A= $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ be a neutrosophic set in X. Then the neutrosophic interior and the neutrosophic closure of A are defined by

NInt(A) = ∪ { G: G is a neutrosophic open set in X and G ⊆ A}

NCl(A) = ∩ { K: K is a neutrosophic closed set in X and A⊆ K}

Note that for any neutrosophic set A, NCl(C(A)) = C(NInt(A)) and NInt(C(A)) = C(NCl(A)).

Definition 2.6: [8] A neutrosophic set A of a neutrosophic topological space X is said to be

(i) a neutrosophic pre-open set if A⊆NInt(NCl(A))

(ii) a neutrosophic semi-open set if A⊆NCl(NInt(A))

(iii) a neutrosophic α- open set if A⊆NInt(NCl(NInt(A)))

(iv) a neutrosophic semi- α-open set if A⊆NCl(αNInt(A))

Definition 2.7: [8] A neutrosophic set A of a neutrosophic topological space X is said to be

(i) a neutrosophic pre-closed set if NCl(NInt(A)) ⊆ A

(ii) a neutrosophic semi-closed set if NInt(NCl(A)) ⊆ A

(iii) a neutrosophic α- closed set if NCl(NInt(NCl(A))) ⊆ A

(iv) a neutrosophic semi- α-closed set if NInt(αNCl(A)) ⊆ A

Definition 2.8: [9] Let A be a neutrosophic set of a neutrosophic topological space (X, τ).Then the neutrosophic α interior and the neutrosophic α closure are defined as

N_α Int(A) = ∪ {G: G is an neutrosophic α open set in X and G ⊆ A}

N_α Cl(A) = ∩ { K: K is an neutrosophic α closed set in X and A⊆ K}

Definition 2.9: [9] A neutrosophic set A in a neutrosophic topological space X is said to be a neutrosophic α generalized closed set if N_α Cl(A) ⊆ U whenever A⊆ U and U is a neutrosophic open set in X.

The complement C(A) of a neutrosophic α generalized closed set A is a neutrosophic α generalized open set in X.

Definition 2.10: [13] Let (X,τ) and (Y,σ) be two neutrosophic topological spaces. A mapping f : $(X,\tau) \to (Y,\sigma)$ is called neutrosophic continuous if the inverse image of every neutrosophic closed set in (Y,σ) is a neutrosophic closed set in (X,τ).

Definition 2.11: [13] Let (X,τ) and (Y,σ) be two neutrosophic topological spaces. A mapping f: $(X,\tau) \to (Y,\sigma)$ is called neutrosophic α continuous if the inverse image of every neutrosophic closed set in (Y,σ) is a neutrosophic α closed set in (X,τ).

Definition 2.12: [4] Let f be a mapping from a neutrosophic topological space (X,τ) into a neutrosophic topological space (Y,σ). Then f is said to be a neutrosophic generalized irresolute mapping if the inverse image of every neutrosophic generalized closed set in (Y,σ) is a neutrosophic generalized closed set in (X,τ).

Definition 2.13: [5] Let (X,τ) and (Y,σ) be any two neutrosophic topological spaces. A mapping f : $(X,\tau) \to (Y,\sigma)$ is called neutrosophic contra continuous if the inverse image of every neutrosophic open set in (Y,σ) is a neutrosophic closed set in (X,τ).

1. Neutrosophic contra α continuous mappings

In this section we have introduced neutrosophic contra α continuous mappings and investigated some of their properties. Also we have established the relation between the newly introduced mapping and already existing mappings.

Definition 3.1: A mapping f : $(X,\tau) \rightarrow (Y,\sigma)$ is called a neutrosophic contra α continuous mapping if $f^{-1}(B)$ is a neutrosophic α closed set in (X,τ) for every neutrosophic open set B in (Y,σ) .

Definition 3.2: A mapping f : $(X,\tau) \rightarrow (Y,\sigma)$ is called a neutrosophic contra pre continuous mapping if $f^{-1}(B)$ is a neutrosophic pre closed set in (X,τ) for every neutrosophic open set B in (Y,σ) .

Proposition 3.3: Every neutrosophic contra continuous mapping is a neutrosophic contra α continuous mapping but not conversely in general.

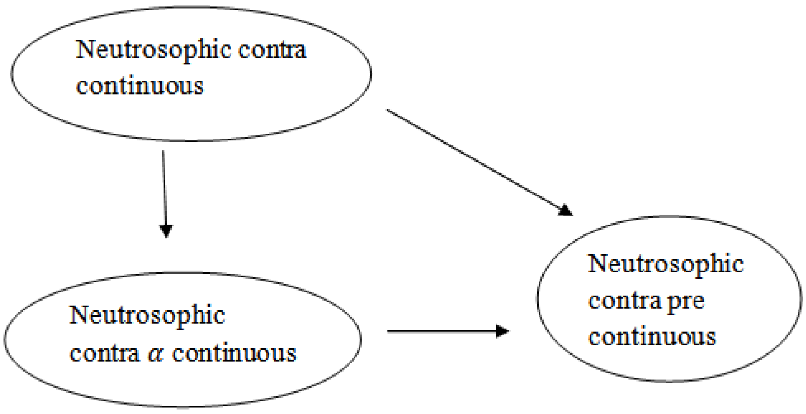
Proof: Let f : $(X,\tau) \rightarrow (Y,\sigma)$ be a neutrosophic contra continuous mapping. Let A be a neutrosophic open set in Y. By hypothesis, $f^{-1}(A)$ is a neutrosophic closed set in X. Since every neutrosophic closed set is a neutrosophic α-closed set in X, f is a neutrosophic contra α continuous mapping.

Example 3.4: Let X={a,b}and Y={u,v}. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x,(0.4,0.6),(0.1,0.1),(0.3,0.3) \rangle$ and $G_2 = \langle y,(0.5,0.6),(0.2,0.2),(0.1,0.1) \rangle$. Define a mapping f : $(X,\tau) \rightarrow (Y,\sigma)$ by f(a) = u and f(b) = v. Here the neutrosophic set $G_2 = \langle y,(0.5,0.6),(0.2,0.2),(0.1,0.1) \rangle$ is a neutrosophic open set in Y. Then $f^{-1}(G_2) = \langle x,(0.5,0.6),(0.2,0.2),(0.1,0.1) \rangle$ is a neutrosophic α-closed set in (X,τ), as $NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \subseteq f^{-1}(G_2)$ .Therefore f is a neutrosophic contra α continuous mapping but since $f^{-1}(G_2)$ is not a neutrosophic closed set in X, f is not a neutrosophic contra continuous mapping.

Proposition 3.5: Every neutrosophic contra α-continuous mapping is a neutrosophic contra pre-continuous mapping but not conversely in general.

Proof: Let f : $(X,\tau) \rightarrow (Y,\sigma)$ be a neutrosophic contra α-continuous mapping. Let A be a neutrosophic open set in Y. By hypothesis, $f^{-1}(A)$ is a neutrosophic α-closed set in X. Every neutrosophic α-closed set is a neutrosophic pre-closed set in X, f is a neutrosophic contra pre-continuous mapping.

Example 3.6: Let X={a,b}and Y={u,v}. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x,(0.4,0.6),(0.3,0.3),(0.3,0.3) \rangle$ and $G_2 = \langle y,(0.5,0.6),(0.2,0.2),(0.1,0.1) \rangle$. Define a mapping f : $(X,\tau) \rightarrow (Y,\sigma)$ by f(a) = u and f(b) = v. Here the neutrosophic set $G_2 = \langle y,(0.5,0.6),(0.2,0.2),(0.1,0.1) \rangle$ is a neutrosophic open set in Y. Then $f^{-1}(G_2) = \langle x,(0.5,0.6),(0.2,0.2),(0.1,0.1) \rangle$ is a neutrosophic pre-closed in (X,τ) as $NCl(NInt(f^{-1}(G_2)) = G_1^c \subseteq f^{-1}(G_2)$.Therefore f is a neutrosophic pre-continuous mapping but since $f^{-1}(G_2)$ is not a neutrosophic α closed set in X, f is not a neutrosophic contra α-continuous mapping.



4. Neutrosophic contra α generalized continuous mappings

In this section we have introduced neutrosophic contra α generalized mapping and studied some of its properties.

Definition 4.1: A mapping f : $(X,\tau) \rightarrow (Y,\sigma)$ is called a neutrosophic contra generalized continuous mapping if the inverse image of every neutrosophic open set in (Y,σ) is a neutrosophic generalized closed set in (X,τ).

Definition 4.2: A mapping f : $(X,\tau) \rightarrow (Y,\sigma)$ is called a neutrosophic contra α generalized continuous mapping if $f^{-1}(B)$ is a neutrosophic α generalized closed set in (X,τ) for every neutrosophic open set B of (Y,σ).

Example 4.3: Let X={a,b}and Y={u,v}. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x,(0.2,0.3),(0.1,0.1),(0.8,0.7) \rangle$ and $G_2 = \langle y,(0.4,0.4),(0.2,0.2),(0.6,0.6) \rangle$. Define a mapping f : $(X,\tau) \rightarrow (Y,\sigma)$ by f(a) = u and f(b) = v. Here the neutrosophic set $G_2 = \langle y,(0.4,0.4),(0.2,0.2),(0.6,0.6) \rangle$ is a neutrosophic open set in Y. Then $f^{-1}(G_2) = \langle$

x,(0.4,0.4),(0.2,0.2),(0.6,0.6) ⟩ is a neutrosophic α generalized closed set in (X,τ) as $f^{-1}(G_2) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2)) = f^{-1}(G_2) \cup NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \subseteq 1_N$. Therefore f is a neutrosophic contra α generalized continuous mapping.

Proposition 4.4: Every neutrosophic contra continuous mapping is a neutrosophic contra α generalized continuous mapping but not conversely in general.

Proof: Let $f : (X,\tau) \to (Y,\sigma)$ be a neutrosophic contra continuous mapping. Let A be a neutrosophic open set in Y. By hypothesis, $f^{-1}(A)$ is a neutrosophic closed set in X. Since every neutrosophic closed set is a neutrosophic α generalized closed set in X, f is a neutrosophic contra α generalized continuous mapping.

Example 4.5: Let X={a,b}and Y={u,v}. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where G_1=⟨ x,(0.1,0.2),(0.1,0.1),(0.7,0.6) ⟩ and G_2= ⟨ y,(0.3,0.3),(0.1,0.1),(0.5,0.5) ⟩. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by f(a) = u and f(b) = v. Here the neutrosophic set G_2= ⟨y,(0.3,0.3),(0.1,0.1),(0.5,0.5) ⟩ is a neutrosophic open set in Y. Then $f^{-1}(G_2)$ = ⟨ x,(0.3,0.3),(0.1,0.1),(0.5,0.5) ⟩ is a neutrosophic α generalized closed set in (X,τ) as $f^{-1}(G_2) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2)) = f^{-1}(G_2) \cup NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \subseteq 1_N$. Therefore f is a neutrosophic contra α generalized continuous mapping but since $f^{-1}(G_2)$ is not a neutrosophic closed set in X as $NCl(f^{-1}(G_2)) = G_1^c \neq f^{-1}(G_2)$, f is not a neutrosophic contra continuous mapping.

Proposition 4.6: Every neutrosophic contra α continuous mapping is a neutrosophic contra α generalized continuous mapping but not conversely in general.

Proof: Let $f : (X,\tau) \to (Y,\sigma)$ be a neutrosophic contra α continuous mapping. Let A be a neutrosophic open set in Y. Then by hypothesis, $f^{-1}(A)$ is a neutrosophic α closed set in X. Since every neutrosophic α closed set is a neutrosophic α generalized closed set, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X. Hence f is a neutrosophic contra α generalized continuous mapping.

Example 4.7: Let X={a,b}and Y={u,v}. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where G_1= ⟨x,(0.2,0.3),(0.3,0.3),(0.7,0.6) ⟩ and G_2= ⟨y,(0.3,0.3),(0.2,0.2),(0.5,0.6) ⟩. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by f(a) = u and f(b) = v. Here the neutrosophic set G_2= ⟨ y, (0.3,0.3),(0.2,0.2),(0.5,0.6) ⟩ is a neutrosophic open set in Y. Then $f^{-1}(G_2)$ = ⟨x, (0.3,0.3),(0.2,0.2),(0.5,0.6) ⟩ is a neutrosophic α generalized closed set in (X,τ) as $f^{-1}(G_2) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2)) = f^{-1}(G_2) \cup NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \subseteq 1_N$. Therefore f is a neutrosophic contra α generalized continuous mapping but since $NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \nsubseteq f^{-1}(G_2))$ is not a neutrosophic α closed set in X, f is not a neutrosophic contra α continuous mapping.

Proposition 4.8: Every neutrosophic contra pre-continuous mapping and neutrosophic contra α generalized continuous mapping are independent to each other in general.

Example 4.9: Let X={a,b}and Y={u,v}. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where G_1= ⟨ x,(0.1,0.1),(0.4,0.4),(0.7,0.3)⟩, G_2= ⟨y,(0.2,0.2),(0.1,0.1),(0.6,0.3)⟩. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by f(a)=u and f(b) =v. Here the neutrosophic set G_2 = ⟨y,(0.2,0.2),(0.1,0.1),(0.6,0.3)⟩ is a neutrosophic open set in Y. Then $f^{-1}(G_2)$ = ⟨x,(0.2,0.2),(0.1,0.1),(0.6,0.3) ⟩ is a neutrosophic α generalized closed set in (X,τ) as $f^{-1}(G_2) \subseteq 1_N$ and $N_\alpha Cl(f^{-1}(G_2)) = f^{-1}(G_2) \cup NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \subseteq 1_N$. Therefore f is a neutrosophic contra α generalized continuous mapping. Since $NCl(NInt(f^{-1}(G_2))) = G_1^c \nsubseteq f^{-1}(G_2)$, $f^{-1}(G_2)$ is not a neutrosophic pre-closed set in X. Hence f is not a neutrosophic contra pre-continuous mapping.

Example 4.10: Let X={a,b}and Y={u,v}. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where G_1= ⟨x,(0.3,0.4),(0.3,0.2),(0.7,0.6)⟩ and G_2= ⟨ y,(0.3,0.3),(0.2,0.2),(0.7,0.6) ⟩.Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by f(a) = u and f(b) = v. Then f is a neutrosophic contra pre continuous mapping. Since for neutrosophic open set G_2= ⟨y, (0.3,0.3),(0.2,0.2),(0.7,0.6) ⟩, $f^{-1}(G_2)$ is a neutrosophic pre-closed set in X as $NCl(NInt(f^{-1}(G_2))) = NCl(0_N) = 0_N \subseteq f^{-1}(G_2)$. But f is not a neutrosophic contra α generalized continuous mapping, since for a neutrosophic open set G_2= ⟨y, (0.3,0.3),(0.2,0.2),(0.7,0.6) ⟩ in Y, $f^{-1}(G_2)$ is not a neutrosophic α generalized closed set in X as $f^{-1}(G_2) \subseteq G_1$ where as $N_\alpha Cl(f^{-1}(G_2)) = f^{-1}(G_2) \cup NCl(NInt(NCl(f^{-1}(G_2)))) = G_1^c \nsubseteq G_1$.

Proposition 4.11: A mapping $f : (X,\tau) \to (Y,\sigma)$ is a neutrosophic contra α generalized continuous mapping if and only if the inverse image of each neutrosophic closed set in Y is a neutrosophic α generalized open set in X.

Proof: Necessity: Let A be a neutrosophic closed set in Y. This implies A^c is a neutrosophic open set in Y. Since f is a neutrosophic contra α generalized continuous mapping, $f^{-1}(A^c)$ is a neutrosophic α generalized closed set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic α generalized open set in X.

Sufficiency: Let A be a neutrosophic open set in Y. This implies A^c is a neutrosophic closed set in Y. By hypothesis, $f^{-1}(A^c)$ is neutrosophic α generalized open set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X. Hence f is a neutrosophic contra α generalized continuous mapping.

Proposition 4.12: Let f : $(X,\tau) \rightarrow (Y,\sigma)$ be a neutrosophic contra α generalized continuous mapping, then f is a neutrosophic contra continuous mapping if X is a $N_{\alpha a}T_{1/2}$ space.

Proof: Let A be a neutrosophic open set in Y. Then by hypothesis, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X. Since X is a $N_{\alpha a}T_{1/2}$ space, $f^{-1}(A)$ is a neutrosophic closed set in X. Hence f is a neutrosophic contra continuous mapping.

Proposition 4.13: If f : $(X,\tau) \rightarrow (Y,\sigma)$ be a neutrosophic contra α generalized continuous mapping and g : $(Y,\sigma) \rightarrow (Z,\delta)$ is a neutrosophic contra continuous mapping, then $g \circ f : (X,\tau) \rightarrow (Z,\delta)$ is a neutrosophic α generalized continuous mapping.

Proof: Let A be a neutrosophic open set in Z. Then by hypothesis, $g^{-1}(A)$ is a neutrosophic closed set in Y. Since f is a neutrosophic contra α generalized continuous mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized open set in X. That is $(g\circ f)^{-1}(A)$ is a neutrosophic α generalized open set in X. Hence g∘f is a neutrosophic α generalized continuous mapping.

Proposition 4.14: Let f : $(X,\tau) \rightarrow (Y,\sigma)$ be a neutrosophic α generalized continuous mapping and g : $(Y,\sigma) \rightarrow (Z,\delta)$ is a neutrosophic contra continuous mapping, then $g \circ f : (X,\tau) \rightarrow (Z,\delta)$ is a neutrosophic contra α generalized continuous mapping.

Proof: Let A be a neutrosophic open set in Z. Then $g^{-1}(A)$ is a neutrosophic closed set in Y, by hypothesis. Since f is a neutrosophic α generalized continuous mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized closed set in X. That is $(g\circ f)^{-1}(A)$ is a neutrosophic α generalized closed set in X. Hence g∘f is a neutrosophic contra α generalized continuous mapping.

Proposition 4.15: Let f : $(X,\tau) \rightarrow (Y,\sigma)$ and g : $(Y,\sigma) \rightarrow (Z,\delta)$ be two mappings. Then the following conditions are equivalent if X is a $N_{\alpha a}T_{1/2}$ space:

(i) g∘f is a neutrosophic contra α generalized continuous mapping

(ii) $NCl(NInt(NCl((g\circ f)^{-1}(B)))\subseteq (g\circ f)^{-1}(B)$ for every neutrosophic open set B in Z

Proof: (i) ⟹ (ii) Let B be any neutrsophic open set in Z. Then $(g\circ f)^{-1}(B)$ is a neutrosophic α generalized closed set in X, by hypothesis. Since X is a $N_{\alpha a}T_{1/2}$ space, $(g\circ f)^{-1}(B)$ is a neutrosophic closed set in X. Therefore, $NCl((g\circ f)^{-1}(B)) = (g\circ f)^{-1}(B)$. Now $NCl(NInt(NCl((g\circ f)^{-1}(B)))) = NCl(NInt((g\circ f)^{-1}(B))) \subseteq NCl((g\circ f)^{-1}(B)) = (g\circ f)^{-1}(B)$.

(ii) ⟹(i) Let B be a neutrosophic closed set in Z. Then its complement B^c is a neutrosophic open set in Z. By hypothesis, $NCl(NInt(NCl((g\circ f)^{-1}(B^c)))) \subseteq (g\circ f)^{-1}(B^c)$. Hence $(g\circ f)^{-1}(B^c)$ is a neutrosophic α closed set in X. Since every neutrosophic α closed set is a neutrosophic α generalized closed set, $(g\circ f)^{-1}(B^c)$ is a neutrosophic α generalized closed set in X and hence $(g\circ f)^{-1}(B)$ is a neutrosophic α generalized open set in X as $(g\circ f)^{-1}(B^c) = ((g\circ f)^{-1}(B))^c$. Thus (g∘f) is a neutrosophic contra α generalized continuous mapping.

Proposition 4.16: If f : $(X,\tau) \rightarrow (Y,\sigma)$ is a neutrosophic contra α generalized continuous mapping and g : $(Y,\sigma) \rightarrow (Z,\delta)$ is a neutrosophic continuous mapping, then $g \circ f : (X,\tau) \rightarrow (Z,\delta)$ is a neutrosophic contra α generalized continuous mapping.

Proof: Let A be a neutrosophic open set in Z. Then by hypothesis, $g^{-1}(A)$ is a neutrosophic open set in Y. Since f is a neutrosophic contra α generalized continuous mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized closed set in X. That is $(g\circ f)^{-1}(A)$ is a neutrosophic α generalized closed set in X. Hence g∘f is a neutrosophic contra α generalized continuous mapping.

Proposition 4.17: Let f : $(X,\tau) \rightarrow (Y,\sigma)$ and g : $(Y,\sigma) \rightarrow (Z,\delta)$ be any two mappings. Then the following conditions are equivalent if X is a $N_{\alpha a}T_{1/2}$ space.

(i) g∘f is a neutrosophic contra α generalized continuous mapping

(ii) $(g\circ f)^{-1}(B) \subseteq NInt(NCl(NInt((g\circ f)^{-1}(B)))$ for each neutrosophic closed set B of Z.

Proof: (i) ⟹(ii)Let B be any neutrosophic closed set in Z. By hypothesis, $(g\circ f)^{-1}(B)$ is a neutrosophic α generalized open set in X. Since X is a $N_{\alpha a}T_{1/2}$ space, $(g\circ f)^{-1}(B)$ is a neutrosophic open set in X. Therefore, $(g\circ f)^{-1}(B) = NInt((g\circ f)^{-1}(B))$. But $NInt((g\circ f)^{-1}(B)) \subseteq NInt(NCl(NInt((g\circ f)^{-1}(B)))$.This implies $(g\circ f)^{-1}(B) \subseteq NInt(NCl(NInt((g\circ f)^{-1}(B)))$ for every neutrosophic closed set B in Z.

(ii) ⟹(i) Let B be any neutrosophic closed set in Z. By hypothesis, $(g\circ f)^{-1}(B) \subseteq NInt(NCl(NInt((g\circ f)^{-1}(B))))$. This implies $(g\circ f)^{-1}(B)$ is a neutrosophic α open set in X and hence $(g\circ f)^{-1}(B)$ is a neutrosophic α generalized open set in X. Therefore f is a neutrosophic contra α generalized continuous mapping.

5. Neutrosophic contra α generalized irresolute mapping

In this section we have introduced neutrosophic contra α generalized irresolute mapping and discussed some of its properties.

Definition 5.1: A mapping $f : (X,\tau) \to (Y,\sigma)$ is called a neutrosophic contra α generalized irresolute mapping if $f^{-1}(A)$ is a neutrosophic α generalized closed set in (X,τ) for every neutrosophic α generalized open set A of (Y, σ).

Proposition 5.2: If $f : (X,\tau) \to (Y,\sigma)$ is called a neutrosophic contra α generalized irresolute mapping, then f is a neutrosophic contra α generalized continuous mapping but not conversely in general.

Proof: Let f be a neutrosophic contra α generalized irresolute mapping. Let A be any neutrosophic open set in Y. Since every neutrosophic open set is a neutrosophic α generalized open set, A is a neutrosophic α generalized open set in Y. By hypothesis, $f^{-1}(A)$ is a neutrosophic α generalized closed set in X. Hence f is a neutrosophic contra α generalized continuous mapping.

Example 5.3: Let X={a,b} and Y={u,v}. Then $\tau = \{0_N, G_1, 1_N\}$ and $\sigma = \{0_N, G_2, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $G_1 = \langle x,(0.2,0.3),(0.1,0.1),(0.5,0.4)\rangle$ and $G_2 = \langle y,(0.6,0.2),(0.2,0.2),(0.1,0.5)\rangle$. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by f(a) = u and f(b) = v. Then f is a neutrosophic contra α generalized continuous mapping but not a neutrosophic contra α generalized irresolute mapping. Since the neutrosophic set $A = \langle y,(0.1,0.3),(0.2,0.2),(0.6,0.4)\rangle$ is a neutrosophic α generalized open set in Y but $f^{-1}(A)$ is not a neutrosophic α generalized closed set in X as $f^{-1}(A) = \langle x,(0.1,0.3),(0.2,0.2),(0.6,0.4)\rangle \subseteq G_1$ but $N_\alpha Cl(f^{-1}(A)) = f^{-1}(A) \cup NCl(NInt(NCl(f^{-1}(A)))) = G_1^c \nsubseteq G_1$.

Proposition 5.4: If $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\delta)$ are neutrosophic contra α generalized irresolute mapping then $g \circ f : (X,\tau) \to (Z,\delta)$ is a neutrosophic α generalized irresolute mapping.

Proof: Let A be a neutrosophic α generalized open set in Z. Then $g^{-1}(A)$ is a neutrosophic α generalized closed set in Y. Since f is a neutrosophic contra α generalized irresolute mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized open set in X. That is $(g\circ f)^{-1}(A)$ is a neutrosophic α generalized open set in X. Hence $g\circ f$ is a neutrosophic α generalized irresolute mapping.

Proposition 5.5: If $f : (X,\tau) \to (Y,\sigma)$ is a neutrosophic contra α generalized irresolute mapping and $g : (Y,\sigma) \to (Z,\delta)$ is a neutrosophic contra α generalized continuous mapping, then $g \circ f : (X,\tau) \to (Z,\delta)$ is a neutrosophic α generalized continuous mapping.

Proof: Let A be a neutrosophic open set in Z. Then by hypothesis, $g^{-1}(A)$ is a neutrosophic α generalized closed set in Y. Since f is a neutrosophic contra α generalized irresolute mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic α generalized open set in X. That is $(g\circ f)^{-1}(A)$ is a neutrosophic α generalized open set in X. Hence $g\circ f$ is a neutrosophic α generalized continuous mapping.

References

[1]. **Atanassov, k**., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 1986, 87-96.
[2]. **Chang, C**., Fuzzy topological spaces, J.Math.Anal.Appl., 1968, 182-190.
[3]. **Coker, D**., An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 1997, 81-89.
[4]. **Dhavaseelan, R**., **and Jafari, S**., "Generalized Neutrosophic closed sets", New trends in neutrosophic theory and its applications, 2017, Volume II.
[5]. **Dhavaseelan.R**, **Jafari.S**, **Ozel.C** and **Al-Shumrani.M.A**, Generalized Neutrosophic Contra –Continuity, New trends in neutrosophic theory and applications, 2017, volume II.
[6]. **Ekici.E** and **Etienne Kerre.K**, On fuzzy contra continuities, Advances in Fuzzy Mathematics, 2006, 35-44.
[7]. **Floretin Smarandache**, Neutrosophic Set:- A generalization of intuitionistic fuzzy set, Jour. of Defense Resources Management, 2010, 107-116.
[8]. **Ishwarya, P**., **and Bageerathi, K**., On Neutrosophic semi-open sets in neutrosophic topological spaces, International Jour. of Math.Trends and Tech., 2016, 214-223.
[9]. **Jayanthi, D**., "Alpha generalized closed sets in neutrosophic topological spaces", International conference on Recent trends in Mathematics and information Technology (ICRMIT 2018).
[10]. **Kresteska, B., and Ekici, E.**, Intuitionistic fuzzy contra strong precontinuity, Faculty of sciences and Mathematics, University of Nis, Siberia, 2007, 273284.
[11]. **Prishka, F., and Jayanthi, D.,** α generalized continuous mappings in neutrosophic topological spaces (submitted).
[12]. **Salma, A. A., and Alblowi, S. A.**, Neutrosophic set and Neutrosophic topological spaces, IOSR Jour. of Mathematics, 2012, 31-35.
[13]. **Salma, A.A., Floretin Smarandache and Valeri Kromov**, Neutrosophic closed set and Neutrosophic Continuous Functions, Neutrosophic Sets and Systems, 2014, 4-8.
[14]. **Zadeh, L.A**., Fuzzy sets, Information and control, 1965, 338-353.