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Regular α Generalized Closed Set in Neutrosophic Topological Spaces

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Abstract: *The purpose of this paper is to introduce and study about neutrosophic regular α generalized closed sets in neutrosophic topological spaces. Some interesting propositions based on this set are presented and established with suitable examples. Their properties are discussed.*
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I. Introduction

The concept of fuzzy sets was first introduced by L.Zadeh [9] in 1965. It shows the degree of membership of the element in a set. Later, fuzzy topology was introduced by Chang [2] in 1968. In 1983 Atanassov [1] introduced the concept of intuitionistic fuzzy set, where the degree of membership and non-membership are discussed. Coker [3] introduced intuitionistic fuzzy topological spaces. Florentin Smarandache [4] introduced and developed the concept of neutrosophic set from the fuzzy sets and intuitionistic fuzzy sets in 1997, who also developed the concept of single-valued neutrosophic set oriented towards real world scientific and engineering applications. A.A. Salama and S.A. Alblowi [8] introduced neutrosophic topological spaces by using the neutrosophic sets. In this paper, we introduce one of the concepts namely regular α generalized closed set in neutrosophic topological spaces.

II. Preliminaries

Here in this paper the neutrosophic topological space is denoted by (X,τ). Also the neutrosophic interior, neutrosophic closure of a neutrosophic set A are denoted by NInt(A) and NCl(A). The complement of a neutrosophic set A is denoted by C(A) and the empty and whole sets are denoted by 0_N and 1_N respectively.

Definition 2.1: [4] Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form A = $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \epsilon X\}$ where $\mu_A(x), \sigma_A(x), \nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element x∈X to the set A.

A neutrosophic set A = $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \epsilon X\}$can be identified as an ordered triple $\langle \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ in $]^-0, 1^+[$ on X.

Definition 2.2: [4] Let A= $\langle \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ be a NS on X, then the complement C(A) may be defined as

1. C(A) = $\{\langle x, 1- \mu_A(x), 1- \nu_A(x)\rangle : x \epsilon X\}$
2. C(A) = $\{\langle x, \nu_A(x), \sigma_A(x), \mu_A(x)\rangle : x \epsilon X\}$
3. C(A) = $\{\langle x, \nu_A(x), 1- \sigma_A(x), \mu_A(x)\rangle : x \epsilon X\}$

Note that for any two neutrosophic sets A and B,

4. C(A∪ B) = C(A) ∩ C (B)
5. C(A ∩ B) = C(A) ∪ C(B)

Definition 2.3: [8] For any two neutrosophic sets A = $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \epsilon X\}$ and B = $\{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle : x \epsilon X\}$ we may have

1. A⊆ B ⇔ $\mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x) and\ \nu_A(x) \ge \nu_B(x)\ \forall x \epsilon X$
2. A⊆ B ⇔ $\mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x) and\ \nu_A(x) \ge \nu_B(x)\ \forall x \epsilon X$
3. A∩ $B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x)\rangle$
4. A∩ $B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \vee \nu_B(x)\rangle$
5. A∪ $B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x)\rangle$
6. A∪ $B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \wedge \nu_B(x)\rangle$

Definition 2.4: [8] A neutrosophic topology on a non-empty set X is a family τ of neutrosophic subsets in X satisfies the following axioms:

(NT1) $0_N, 1_N \in \tau$

(NT2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(NT3) $\cup G_i \in \tau \; \forall \{G_i : i \in J\} \subseteq \tau$

In this case the pair (X, τ) is a neutrosophic topological space and any neutrosophic set in τ is known as a neutrosophic open set (NOS) in X. A neutrosophic set A is a neutrosophic closed set (NCS) if and only if its complement C(A) is a neutrosophic open set in X. Here the empty set (0_N) and the whole set (1_N) may be defined as follows:

(0_1) $0_N = \{\langle x, 0,0,1\rangle : x \in X\}$
(0_2) $0_N = \{\langle x, 0,1,1\rangle : x \in X\}$
(0_3) $0_N = \{\langle x, 0,1,0\rangle : x \in X\}$
(0_4) $0_N = \{\langle x, 0,0,0\rangle : x \in X\}$
(1_1) $1_N = \{\langle x, 1,0,0\rangle : x \in X\}$
(1_2) $1_N = \{\langle x, 1,0,1\rangle : x \in X\}$
(1_3) $1_N = \{\langle x, 1,1,0\rangle : x \in X\}$
(1_4) $1_N = \{\langle x, 1,1,1\rangle : x \in X\}$

Definition 2.5: [8] Let (X, τ) be a NTS and A= $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \epsilon X\}$ be a NS in X. Then the neutrosophic interior and the neutrosophic closure of A are defined by

NInt(A) = ∪ { G: G is an NOS in X and G ⊆ A}
NCl(A) = ∩ { K: K is an NCS in X and A⊆ K}

Note that for any NS A, NCl(C(A)) = C(NInt(A)) and NInt(C(A)) = C(NCl(A)).

Definition 2.6: [5] A neutrosophic set A of a neutrosophic topological space X is said to be
(i) a neutrosophic pre-open set if A⊆NInt(NCl(A))
(ii) a neutrosophic semi-open set if A⊆NCl(NInt(A))
(iii) a neutrosophic α- open set if A⊆NInt(NCl(NInt(A)))
(iv) a neutrosophic semi- α-open set if A⊆NCl(αNInt(A))

Definition 2.7: [5] A neutrosophic set A of a neutrosophic topological space X is said to be
(i) a neutrosophic pre-closed set if NCl(NInt(A)) ⊆ A
(ii) a neutrosophic semi-closed set if NInt(NCl(A)) ⊆ A
(iii) a neutrosophic α- closed set if NCl(NInt(NCl(A))) ⊆ A
(iv) a neutrosophic semi- α-closed set if NInt(αNCl(A)) ⊆ A

Definition 2.8: [6] A neutrosophic set A in a neutrosophic topological space (X, τ) is said to be a neutrosophic regular closed set, if A = NCl(NInt(A)) and neutrosophic regular open set if NInt(NCl(A)) = A.

Definition 2.9: [7] A neutrosophic set A in a neutrosophic topological space (X, τ) is said to be a neutrosophic generalized closed set, if NCl(A) ⊆ U whenever A ⊆ U and U is a neutrosophic open set in X.

Definition 2.10: [6] A neutrosophic set (NS) A in a neutrosophic topological space (X, τ) is a neutrosophic α generalized closed set, if NαCl(A) ⊆ U whenever A ⊆ U and U is a neutrosophic open set in X.

III. Neutrosophic Regular Generalized Closed Set

In this section, we introduce neutrosophic regular generalized closed sets and analyse some of their properties.

Definition 3.1: A neutrosophic set A in a neutrosophic topological space (X, τ) is said to be a neutrosophic regular generalized closed set, if NCl(A) ⊆ U whenever A ⊆ U and U is a neutrosophic regular open set in X. The complement A^c of the neutrosophic regular generalized closed set is a neutrosophic regular generalized open set

Example 3.2: Let X={a,b}and τ ={0_N,U,V,1_N}where U=$\langle x, (0.6,0.7)(0.1,0.1)(0.4,0.2)\rangle$ and V=$\langle x, (0.1,0.2)(0.1,0.1)(0.8,0.8)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x, (0.2,0.2)(0.1,0.1)(0.6,0.7)\rangle$ is a neutrosophic regular generalized closed set in X, since A ⊆ U and U is a neutrosophic regular open set, we have NCl(A) = U^c ⊆ U.

Proposition 3.3: Every neutrosophic closed set is a neutrosophic regular generalized closed set in X, but not conversely in general.

Proof: Let U be a neutrosophic regular open set in X such that A ⊆ U. Since A is a neutrosophic closed set, NCl(A) = A. By hypothesis, NCl(A) ⊆ U. Thus A is a neutrosophic regular generalized closed set in X.

Example 3.4: Let X ={a,b}andτ={0_N,U,V,1_N},where U=$\langle x,(0.6,0.7)(0.1,0.1)(0.4,0.2)\rangle$ and V=$\langle x,(0.1,0.2)(0.1,0.1)(0.8,0.8)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.2,0.2)(0.1,0.1)(0.6,0.7)\rangle$ is a neutrosophic regular generalized closed set in X. Since for A ⊆ U and U is a neutrosophic regular open set, we have NCl(A) = U^c ⊆ U. But A is not a neutrosophic closed set in X as NCl(A) = U^c ≠A.

Proposition 3.5: The union of two neutrosophic regular generalized closed set is a neutrosophic regular generalized closed set in X.

Proof: Let A and B be neutrosophic regular generalized closed sets in X. Let A ∪ B ⊆ U and U be a neutrosophic regular open set in X, where A ⊆ U and B ⊆ U. Then NCl(A ∪ B) = NCl(A) ∪ NCl(B) ⊆ U, by hypothesis. Hence A ∪ B is also a neutrosophic regular generalized closed set in X.

IV. Neutrosophic Regular α Generalized Closed Set

In this section, we introduce neutrosophic regular α generalized closed set and analyse some of their properties.

Definition 4.1: A neutrosophic set A in a neutrosophic topological space (X, τ) is said to be neutrosophic regular α generalized closed set, if NαCl(A) ⊆ U whenever A ⊆ U and U is a neutrosophic regular open set in X.

Example 4.2: LetX={a,b}and τ ={0_N,U,V,1_N},where U=$\langle x,(0.4,0.2)(0.1,0.1)(0.6,0.7)\rangle$ and V=$\langle x,(0.8,0.8)(0.1,0.1)(0.2,0.2)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.2,0.2)(0.1,0.1)(0.8,0.8)\rangle$ is a neutrosophic regular α generalized closed set in X. Since A ⊆ U and U is a neutrosophic regular open set, we have NαCl(A) =A∪NCl(NInt(NCl(A))) = 0_N ⊆ U .

Proposition 4.3: Every neutrosophic closed set is a neutrosophic regular α generalized closed set in X , but not conversely in general.

Proof: Let U be a neutrosophic regular open set in X such that A ⊆ U. Since A is neutrosophic closed set, NCl(A) = A. By hypothesis, NαCl(A) = A∪NCl(NInt(NCl(A))) = A∪NCl(NInt(A)) ⊆ A∪NCl(A) = A∪A = A ⊆ U. Thus A is a neutrosophic regular α generalized closed set in X.

Example 4.4: Let X={a,b}and τ={0_N,U,V,1_N},where U=$\langle x,(0.6,0.7)(0.1,0.1)(0.4,0.2)\rangle$ and V=$\langle x,(0.1,0.2)(0.1,0.1)(0.8,0.8)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.2,0.2)(0.1,0.1)(0.6,0.7)\rangle$ is a neutrosophic regular α generalized closed set in X. Since A ⊆ U and U is a neutrosophic regular open set, we have NαCl(A) = A∪NCl(NInt(NCl(A))) = U^c ⊆ U . But A is not a neutrosophic closed set in X as NCl(A) = U^c ≠A.

Proposition 4.5: Every neutrosophic regular closed set is a neutrosophic regular α generalized closed set in X, but not conversely in general.

Proof: Let U be a neutrosophic regular open set in X such that A ⊆ U. Since every neutrosophic regular closed set is a neutrosophic closed set, NCl(A) = A. By hypothesis, NαCl(A) = A∪NCl(NInt(NCl(A))) = A∪NCl(NInt(A)) ⊆ A∪NCl(A) = A∪A = A ⊆ U. Hence NαCl(A) ⊆ U. Thus A is a neutrosophic regular α generalized closed set in X.

Example 4.6: LetX={a,b} and τ={0_N,U,V,1_N},where U=$\langle x,(0.6,0.7)(0.1,0.1)(0.4,0.2)\rangle$ and V=$\langle x,(0.1,0.2)(0.1,0.1)(0.8,0.8)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.2,0.2)(0.1,0.1)(0.6,0.7)\rangle$ is a neutrosophic regular α generalized closed set since NαCl(A) = A∪NCl(NInt(NCl(A))) = U^c ⊆ U, whenever A ⊆ U. But since NCl(NInt(A)) = U^c ≠ A, A is not a neutrosophic regular closed set in X.

Proposition 4.7: Every neutrosophic α closed set is a neutrosophic regular α generalized closed set in X, but not conversely in general.

Proof: Let U be a neutrosophic regular open set in X such that A ⊆ U. Since A is a neutrosophic α closed set, NCl(NInt(NCl(A))) ⊆A. By hypothesis, NαCl(A) = A∪NCl(NInt(NCl(A))) ⊆ A∪A = A ⊆ U. Hence NαCl(A) ⊆ U and A is a neutrosophic regular α generalized closed set in X.

Example 4.8: Let X={a,b} andτ={0_N,U,V,1_N},where U=$\langle x,(0.6,0.7)(0.1,0.1)(0.4,0.2)\rangle$ and V=$\langle x,(0.1,0.2)(0.1,0.1)(0.8,0.8)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.2,0.2)(0.1,0.1)(0.6,0.7)\rangle$ is a neutrosophic regular α generalized closed set, since NαCl(A) = A∪NCl(NInt(NCl(A))) = U^c ⊆ U, whenever A ⊆ U and U is a neutrosophic regular open set in X. But since NCl(NInt(NCl(A))) = U^c ⊈ A, A is not a neutrosophic α closed set in X.

Proposition 4.9: Every neutrosophic generalized closed set is a neutrosophic regular α generalized closed set in X , but not conversely in general.

Proof: Let A ⊆ U and U be a neutrosophic regular open set in X. By hypothesis, NCl(A) ⊆ U, whenever A ⊆ U. This implies, NαCl(A) = A∪NCl(NInt(NCl(A))) ⊆ A∪NCl(A) ⊆ U. Therefore A is a neutrosophic regular α generalized closed set in X.

Example4.10: LetX={a,b}andτ={0_N,U,V,1_N},whereU=$\langle x,(0.6,0.7)(0.1,0.1)(0.4,0.2)\rangle$ and V=$\langle x,(0.1,0.2)(0.1,0.1)(0.8,0.8)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.2,0.2)(0.1,0.1)(0.6,0.7)\rangle$ is a neutrosophic regular α generalized closed set in X. Since A ⊆ U and U is a neutrosophic regular open set, we have NαCl(A) = A∪NCl(NInt(NCl(A))) = U^c ⊆ U . But A is not a neutrosophic generalized closed set in X as NCl(A) = U^c ⊈ U, whereas A ⊆ U.

Remark 4.11: Every neutrosophic regular α generalized closed set and neutrosophic pre-closed set in X are independent to each other in general.

Example 4.12: Let X = {a,b} and τ={0_N,U,V,1_N}, where U=$\langle x,(0.6,0.7)(0.1,0.1)(0.4,0.2)\rangle$ and V=$\langle x,(0.1,0.2)(0.1,0.1)(0.8,0.8)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.2,0.2)(0.1,0.1)(0.6,0.7)\rangle$ is a neutrosophic regular α generalized closed set, since NαCl(A) = A∪NCl(NInt(NCl(A))) = U^c ⊆ U, whenever A ⊆ U. But since NCl(NInt(A)) = U^c ⊈ A, A is not a neutrosophic pre-closed set in X.

Example 4.13: Let X={a,b} and τ = {0_N,U,V,1_N} where U=$\langle x,(0.2,0.3)(0.1,0.1)(0.7,0.7)\rangle$ and V=$\langle x,(0.8,0.7)(0.1,0.1)(0.2,0.2)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.1,0.2)(0.1,0.1)(0.7,0.8)\rangle$ is a neutrosophic pre-closed set in X, since NCl(NInt(A)) = 0_N ⊆ A. But since NαCl(A) = A∪NCl(NInt(NCl(A))) = U^c ⊈ U where A ⊆ U and U is a neutrosophic regular open set in X, A is not a neutrosophic regular α generalized closed set in X.

Remark 4.14: Every neutrosophic regular α generalized closed set and neutrosophic semi-closed set in X are independent to each other in general.

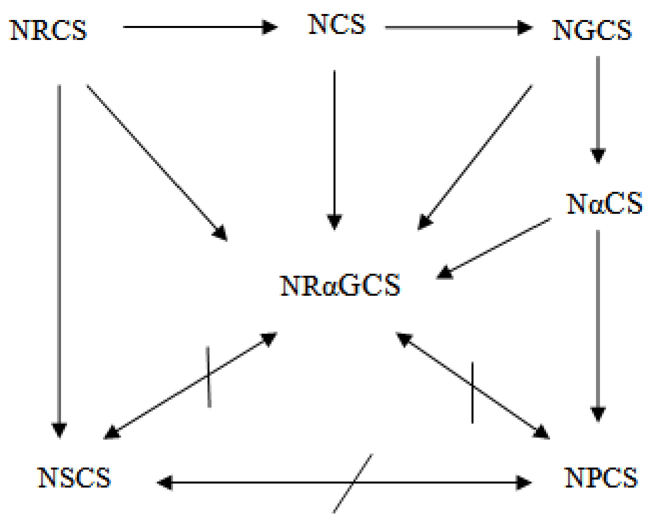
Example 4.15: Let X={a,b} and τ = {0_N,U,V,1_N}, where U=$\langle x,(0.6,0.7)(0.1,0.1)(0.4,0.2)\rangle$ and V=$\langle x,(0.1,0.2)(0.1,0.1)(0.8,0.8)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.2,0.2)(0.1,0.1)(0.6,0.7)\rangle$ is a neutrosophic regular α generalized closed set, since NαCl(A) = A∪NCl(NInt(NCl(A))) = U^c ⊆ U, whenever A ⊆ U. But since NInt(NCl(A)) = V ⊈ A, A is not a neutrosophic semi-closed set in X.

Example 4.16: Let X={a,b} and τ = {0_N,U,V,1_N},where U=$\langle x,(0.5,0.2)(0.1,0.1)(0.5,0.8)\rangle$ and V=$\langle x,(0.2,0.2)(0.1,0.1)(0.8,0.8)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.5,0.2)(0.1,0.1)(0.5,0.8)\rangle$ is a neutrosophic semi-closed set in X, since NInt(NCl(A)) = U ⊆ A. But since NαCl(A) = A∪NCl(NInt(NCl(A))) = U^c ⊈ U where A ⊆ U and U is a neutrosophic regular open set in X, A is not a neutrosophic regular α generalized closed set in X.

Proposition 4.17: The union of two neutrosophic regular α generalized closed set is a neutrosophic regular α generalized closed set in X.

Proof: Let A and B be the neutrosophic regular α generalized closed set in X. Let A ∪ B ⊆ U where U is a neutrosophic regular open set in X, Then A ⊆ U and B ⊆ U. NαCl(A∪ B) = (A∪B)∪NCl(NInt(NCl(A∪B))) ⊆ (A∪B)∪NCl(A∪B) ⊆ NCl(A∪B) = NCl(A) ∪ NCl(B) ⊆ U. Hence A ∪ B is also a neutrosophic regular α generalized closed set in X.

The relation between various types of neutrosophic closed sets are given in the following diagram:



Proposition 4.18: If A is both a neutrosophic regular open set and neutrosophic regular α generalized closed set in X, then A is a neutrosophic regular generalized closed set in X.

Proof: Let A ⊆ U and U be a neutrosophic regular open set in X. By hypothesis, we have NαCl(A) ⊆ U and NCl(A) = NCl(NInt(NCl(A))) ⊆ A∪NCl(NInt(NCl(A))) = NαCl(A) ⊆ U. Hence A is a neutrosophic regular generalized closed set in X.

Proposition 4.19: If A is both a neutrosophic pre-open set and neutrosophic regular α generalized closed set in X, then A is a neutrosophic regular generalized closed set in X.

Proof: Let A ⊆ U and U be a neutrosophic regular open set in X. By hypothesis we have NαCl(A) ⊆ U and NCl(A) ⊆ NCl(NInt(NCl(A))) ⊆ A ∪ NCl(NInt(NCl(A))) = NαCl(A) ⊆ U. Hence A is a neutrosophic regular generalized closed set in X.

Proposition 4.20: If A is both a neutrosophic regular open set and a neutrosophic regular α generalized closed set in X, then A is a neutrosophic α closed set in X.

Proof: As A ⊆ A, by the hypothesis, NαCl(A) ⊆ A. But we have A ⊆ NαCl(A). This implies NαCl(A) = A. Hence A is a neutrosophic α closed set in X.

Proposition 4.21: Let A be a neutrosophic regular α generalized closed set in X and A ⊆ B ⊆ NαCl(A), then B is a neutrosophic regular α generalized closed set in X.

Proof: Let B ⊆ U and U is a neutrosophic regular open set in X. Then A ⊆ U since A ⊆ B. As A is a neutrosophic regular α generalized closed set in X, NαCl(A) ⊆ U and by hypothesis B ⊆ NαCl(A). This implies NαCl(B) ⊆ NαCl(A) ⊆ U. Therefore NαCl(B) ⊆ U and hence B is a neutrosophic regular α generalized closed set in X.

Proposition 4.22: If A is a neutrosophic regular generalized closed set in X and if A ⊆ B ⊆ NCl(A), then B is a neutrosophic regular α generalized closed set in X.

Proof: Let B ⊆ U and U is a neutrosophic regular open set in X. Then A ⊆ U since A ⊆ B. As A is a neutrosophic regular generalized closed set in X, NCl(A) ⊆ U and by hypothesis B ⊆ NCl(A). This implies NαCl(B) ⊆ NCl(B) ⊆ NCl(A) ⊆ U. Therefore NαCl(B) ⊆ U and B is a neutrosophic regular α generalized closed set in X.

V. Neutrosophic Regular Generalized Open Set

In this section, we introduce neutrosophic regular generalized open sets and analyse some of their properties.

Definition 5.1: A neutrosophic set A in a neutrosophic topological space (X, τ) is said to be a neutrosophic regular generalized open set, if NInt(A) ⊇ U whenever A ⊇ U and U is a neutrosophic regular closed set in X.

Example 5.2: Let X={a,b}and τ ={0_N,U,V,1_N}where U=$\langle x,(0.4,0.5)(0.1,0.1)(0.5,0.7)\rangle$ and V=$\langle x,(0.8,0.8)(0.1,0.1)(0.2,0.3)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.6,0.7)(0.1,0.1)(0.2,0.2)\rangle$ is a neutrosophic regular generalized open set in X, since A ⊇ U^c and U^c is a neutrosophic regular closed set, we have NInt(A) = U ⊇ U^c.

Proposition 5.3: Every neutrosophic open set is a neutrosophic regular generalized open set in X, but not conversely in general.

Proof: Let U be a neutrosophic regular closed set in X such that A ⊇ U. Since A is a neutrosophic open set, NInt(A) = A. By hypothesis, NInt(A) ⊇ U. Thus A is a neutrosophic regular generalized open set in X.

Example 5.4: Let X ={a,b}andτ={0_N,U,V,1_N},where U=$\langle x,(0.4,0.2)(0.1,0.1)(0.5,0.7)\rangle$ and V=$\langle x,(0.8,0.8)(0.1,0.1)(0.2,0.2)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.6,0.7)(0.1,0.1)(0.2,0.2)\rangle$ is a neutrosophic regular generalized open set in X. Since for A ⊇ U^c and U^c is a neutrosophic regular closed set, we have NInt(A) = U ⊇ U^c. But A is not a neutrosophic open set in X as NInt(A) = U ≠A.

Proposition 5.5: The intersection of two neutrosophic regular generalized open set is a neutrosophic regular generalized open set in X.

Proof: Let A and B be neutrosophic regular generalized open sets in X. Let A ∩ B ⊇ U and U be a neutrosophic regular closed set in X, where A ⊇ U and B ⊇ U. Then NInt(A ∩ B) = NInt(A) ∩ NInt(B) ⊇ U, by hypothesis. Hence A ∩ B is also a neutrosophic regular generalized open set in X.

VI. Neutrosophic Regular α Generalized Open Set

In this section, we introduce neutrosophic regular α generalized open set and analyse some of their properties.

Definition 6.1: A neutrosophic set A in a neutrosophic topological space (X, τ) is said to be neutrosophic regular α generalized open set, if NαInt(A) ⊇ U whenever A ⊇ U and U is a neutrosophic regular closed set in X. The family of all neutrosophic regular α generalized open sets of an neutrosophic topological space (X,τ) is denoted by NRαGO(X).

Example 6.2: Let X={a,b}and τ ={0_N,U,V,1_N},where U=$\langle x,(0.5,0.4)(0.1,0.1)(0.5,0.4)\rangle$ and V=$\langle x,(0.1,0.2)(0.1,0.1)(0.7,0.7)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set

A= $\langle x,(0.6,0.7)(0.1,0.1)(0.3,0.1)\rangle$ is a neutrosophic regular α generalized open set in X. Since A ⊇ U^c and U^c is a neutrosophic regular closed set, we have NαInt(A) = A∩NInt(NCl(NInt(A))) = U ⊇ U^c.

Proposition 6.3: Every neutrosophic open set is a neutrosophic regular α generalized open set in X , but not conversely in general.

Proof: Let U be a neutrosophic regular closed set in X such that A ⊇ U. Since A is neutrosophic open set, NInt(A) = A. By hypothesis, NαInt(A) = A∩NInt(NCl(NInt(A))) = A∩NInt(NCl(A)) ⊇ A∩NInt(A) = A∩A = A ⊇ U. Thus A is a neutrosophic regular α generalized open set in X.

Example 6.4: Let X={a,b}and $\tau=\{0_N,U,V,1_N\}$,where U=$\langle x,(0.5,0.4)(0.1,0.1)(0.5,0.4)\rangle$ and V=$\langle x,(0.1,0.2)(0.1,0.1)(0.7,0.7)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.6,0.7)(0.1,0.1)(0.3,0.1)\rangle$ is a neutrosophic regular α generalized open set in X. Since A ⊇ U^c and U^c is a neutrosophic regular closed set, we have NαInt(A) = A∩NInt(NCl(NInt(A))) = U ⊇ U^c. But A is not a neutrosophic open set in X as NInt(A) = U ≠A.

Proposition 6.5: Every neutrosophic regular open set is a neutrosophic regular α generalized open set in X, but not conversely in general.

Proof: Let U be a neutrosophic regular closed set in X such that A ⊇ U. Since every neutrosophic regular open set is a neutrosophic open set, NInt(A) = A. By hypothesis, NαInt(A) = A∩NInt(NCl(NInt(A))) = A∩NInt(NCl(A)) = A∩A = A ⊇ U. Hence NαInt(A) ⊇ U. Thus A is a neutrosophic regular α generalized closed set in X.

Example 6.6: Let X={a,b}and $\tau=\{0_N,U,V,1_N\}$,where U=$\langle x,(0.5,0.4)(0.1,0.1)(0.5,0.4)\rangle$ and V=$\langle x,(0.1,0.2)(0.1,0.1)(0.6,0.6)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.6,0.7)(0.1,0.1)(0.3,0.1)\rangle$ is a neutrosophic regular α generalized open set since NαInt(A) = A∩NInt(NCl(NInt(A))) = U ⊇ U^c, whenever A ⊇ U^c. But since NInt(NCl(A)) = 1_N ≠ A, A is not a neutrosophic regular open set in X.

Proposition 6.7: Every neutrosophic α open set is a neutrosophic regular α generalized open set in X, but not conversely in general.

Proof: Let U be a neutrosophic regular closed set in X such that A ⊇ U. Since A is a neutrosophic α open set, A ⊆ NInt(NCl(NInt(A))). By hypothesis, NαInt(A) = A∩NInt(NCl(NInt(A))) ⊇ A∩A = A ⊇ U. Hence NαInt(A) ⊇ U and A is a neutrosophic regular α generalized open set in X.

Example 6.8: Let X={a,b} and$\tau=\{0_N,U,V,1_N\}$,where U=$\langle x,(0.5,0.4)(0.1,0.1)(0.5,0.4)\rangle$ and V=$\langle x,(0.1,0.2)(0.1,0.1)(0.7,0.7)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.6,0.7)(0.1,0.1)(0.3,0.1)\rangle$ is a neutrosophic regular α generalized open set, since NαInt(A) = A∩NInt(NCl(NInt(A))) = U ⊇ U^c, whenever A ⊇ U^c. But since NInt(NCl(NInt(A))) = U ⊉ A, A is not a neutrosophic α open set in X.

Proposition 6.9: Every neutrosophic generalized open set is a neutrosophic regular α generalized open set in X , but not conversely in general.

Proof: Let A ⊇ U and U be a neutrosophic regular closed set in X. By hypothesis, NInt(A) ⊇ U, whenever A ⊇ U.This implies, NαInt(A) = A∩NInt(NCl(NInt(A))) ⊇ A∩NCl(A) ⊇ U. Therefore A is a neutrosophic regular α generalized open set in X.

Example 6.10: LetX={a,b}and$\tau=\{0_N,U,V,1_N\}$,where U=$\langle x,(0.6,0.7)(0.1,0.1)(0.2,0.2)\rangle$ and V= $\langle x,(0.1,0.2)(0.1,0.1)(0.8,0.8)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.7,0.8)(0.1,0.1)(0.1,0.2)\rangle$ is a neutrosophic regular α generalized closed set in X. Since A ⊇ U^c and U^c is a neutrosophic regular closed set, we have NαInt(A) = A∪NInt(NCl(NInt(A))) = A ⊇ U^c . But A is not a neutrosophic generalized closed set in X as NCl(A) = U ⊉ U^c. Whereas A ⊇ U^c.

Remark 6.11: Every neutrosophic regular α generalized open set and neutrosophic pre-open set in X are independent to each other in general.

Example 6.12: Let X = {a,b} and $\tau=\{0_N,U,V,1_N\}$, where U=$\langle x,(0.5,0.7)(0.1,0.1)(0.4,0.2)\rangle$ and V=$\langle x,(0.2,0.2)(0.1,0.1)(0.8,0.8)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.6,0.7)(0.1,0.1)(0.2,0.2)\rangle$ is a neutrosophic regular α generalized open set, since NαInt(A) = A∩NInt(NCl(NInt(A))) = U ⊇ U^c, whenever A ⊇ U^c and U^c is a neutrosophic regular closed set in X. But since NInt(NCl(A)) = U ⊉ A, A is not a neutrosophic pre-open set in X.

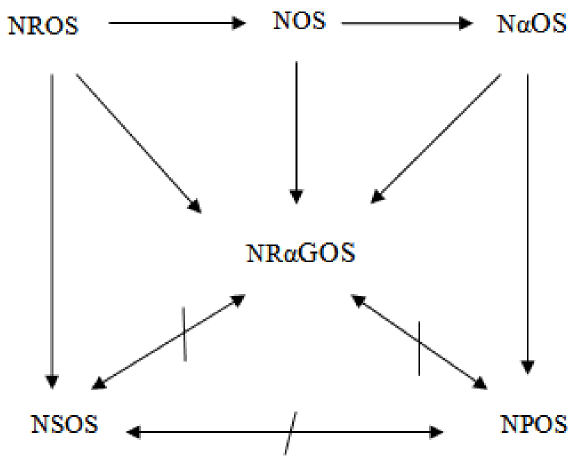
Example 6.13: Let X={a,b} and τ = $\{0_N,U,V,1_N\}$ where U=$\langle x,(0.2,0.3)(0.1,0.1)(0.6,0.7)\rangle$ and V=$\langle x,(0.8,0.7)(0.1,0.1)(0.1,0.1)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.7,0.8)(0.1,0.1)(0.1,0.2)\rangle$ is a neutrosophic pre-open set in X, since NInt(NCl(A)) = 1_N ⊇ A. But since NαInt(A) = A∩NInt(NCl(NInt(A))) = U ⊉ U^c where A ⊇ U^c and U^c is a neutrosophic regular closed set in X, A is not a neutrosophic regular α generalized open set in X.

Remark 6.14: Every neutrosophic regular α generalized open set and neutrosophic semi-open set in X are independent to each other in general.

Example 6.15: Let X={a,b} and τ = {0_N,U,V,1_N}, where U=$\langle x,(0.6,0.7)(0.1,0.1)(0.4,0.2)\rangle$ and V=$\langle x,(0.1,0.2)(0.1,0.1)(0.7,0.8)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.8,0.7)(0.1,0.1)(0.2,0.1)\rangle$ is a neutrosophic regular α generalized open set, since NαInt(A) = A∩NInt(NCl(NInt(A))) = U ⊇ U^c, whenever A ⊇ U^c. But since NCl(NInt(A)) = V^c ⊉ A, A is not a neutrosophic semi-open set in X.

Example 6.16: Let X={a,b} and τ = {0_N,U,V,1_N} where U=$\langle x,(0.5,0.2)(0.1,0.1)(0.5,0.8)\rangle$ and V=$\langle x,(0.2,0.2)(0.1,0.1)(0.8,0.8)\rangle$, then (X, τ) is a neutrosophic topological space. Here the neutrosophic set A= $\langle x,(0.5,0.8)(0.1,0.1)(0.5,0.2)\rangle$ is a neutrosophic semi-open set in X, since NCl(NInt(A)) = U^c ⊇ A. But since NαInt(A) = A∩NInt(NCl(NInt(A))) = U ⊉ U^c where A ⊇ U^c and U^c is a neutrosophic regular closed set in X, A is not a neutrosophic regular α generalized open set in X.

The relation between various types of neutrosophic open sets are given in the following diagram:



Proposition 6.17: The intersection of two neutrosophic regular α generalized open sets is a neutrosophic regular α generalized open set in X.

Proof: Let A and B be the neutrosophic regular α generalized open sets in X. Let A∩B ⊇ U and U be a neutrosophic regular closed set in X, where A ⊇ U and B ⊇ U. Then NαInt(A∩B) = (A∩B)∩NInt(NCl(NInt(A∩B))) ⊇ (A∩B)∩NInt(A∩B) = (A∩B)∩NInt(A)∩NInt(B) ⊇ U, by hypothesis. Hence A ∩ B is also a neutrosophic regular α generalized open set in X.

Proposition 6.18: If A is both a neutrosophic regular closed set and neutrosophic regular α generalized open set in X, then A is a neutrosophic regular generalized open set in X.

Proof: Let A ⊇ U and U be a neutrosophic regular closed set in X. By hypothesis, we have NαInt(A) ⊇ U and NInt(A) = NInt(NCl(NInt(A))) ⊇ A∩NInt(NCl(NInt(A))) = NαInt(A) ⊇ U. Hence A is a neutrosophic regular generalized open set in X.

Proposition 6.19: If A is both a neutrosophic pre-closed set and neutrosophic regular α generalized open set in X, then A is a neutrosophic regular generalized open set in X.

Proof: Let A ⊇ U and U be a neutrosophic regular closed set in X. By hypothesis we have NαInt(A) ⊇ U and NInt(A) ⊇ NInt(NCl(NInt(A))) ⊇ A∩NCl(NInt(NCl(A))) = NαInt(A) ⊇ U. Hence A is a neutrosophic regular generalized open set in X.

Proposition 6.20: Let A be a neutrosophic regular α generalized open set in X and A ⊇ B ⊇ NαInt(A), then B is a neutrosophic regular α generalized open set in X.

Proof: Let A be a neutrosophic regular α generalized open set in X and B be a neutrosophic set in X. Let A ⊇ B ⊇ NαInt(A). Then A^c is a neutrosophic regular α generalized closed set in X and A^c ⊆ B^c ⊆ NαCl(A^c). Then B^c is a neutrosophic regular α generalized closed set in X [5]. Hence B is a neutrosophic regular α generalized open set in X.

Proposition 6.21: If A is a neutrosophic regular closed set in X and a neutrosophic regular α generalized open set in X. Then A is a neutrosophic α open set in X.

Proof: As A ⊇ A, by the hypothesis, NαInt(A) ⊇ A. But we have A ⊇ NαInt(A). This implies NαInt(A) = A. Hence A is a neutrosophic α open set in X.

Proposition 6.22: Let (X,τ) be a neutrosophic topological space and every B be a neutrosophic regular closed set, B ⊆ A ⊆ NInt(NCl(B)). Then A is a neutrosophic regular α generalized open set in X.

Proof: Let B be a neutrosophic regular closed set in X. Then B = NCl(NInt(B)). By hypothesis, A⊆ NInt(NCl(B)) ⊆ NInt(NCl(NCl(NInt(B)))) ⊆ NInt(NCl(NInt(B))) ⊆ NInt(NCl(NInt(A))). Therefore A is a neutrosophic α open set. Since every neutrosophic α open set is a neutrosophic regular α generalized open set, A is a neutrosophic regular α generalized open set in X.

VII. Applications of Neutrosophic Regular α Generalized Closed Sets

In this section we introduce neutrosophic regular α $T_{1/2}$ space, neutrosophic regular $\alpha T^{*}_{1/2}$ space, neutrosophic regular α generalized $T_{1/2}$ space and proved their characterizations.

Definition 7.1: A neutrosophic topological space (X,τ) is said to be a neutrosophic regular α $T_{1/2}$ space, if every neutrosophic regular α generalized closed set is a neutrosophic α closed set in X.

Definition 7.2: A neutrosophic topological space (X,τ) is said to be a neutrosophic regular α $T^{*}_{1/2}$ space, if every neutrosophic regular α generalized closed set is a neutrosophic closed set in X.

Proposition 7.3: A neutrosophic topological space (X,τ) is a neutrosophic regular $\alpha T_{1/2}$ space if and only if, every neutrosophic α open set is a neutrosophic regular α generalized open set in X.

Proof: Necessity: Let A be a neutrosophic regular α generalized open set in X, then A^c is a neutrosophic regular α generalized closed set in X. By hypothesis, A^c is a neutrosophic α closed set in X. Therefore, A is a neutrosophic α open set in X.

Sufficiency: Let A be a neutrosophic regular α generalized closed set in X, then A^c is a neutrosophic regular α generalized open set in X. By hypothesis, A^c is a neutrosophic α open set in X. Therefore, A is a neutrosophicαclosed set in X. Hence (X,τ) is a neutrosophic regular $\alpha T_{1/2}$ space.

Proposition 7.4: For a neutrosophic regular $\alpha T_{1/2}$ space in (X,τ), the following properties are equivalent:

(i) A∈NRαGO(X)

(ii) A ⊆NInt(NCl(NInt(A)))

(iii) There exists neutrosophic open set G such that G ⊆ A ⊆NInt(NCl(G)).

Proof:(i) ⟹ (ii): is obvious.

(ii) ⟹ (iii): Let A ∊ NInt(NCl(NInt(A))). Then NInt(A) ⊆ A ⊆ NInt(NCl(NInt(A))). Therefore we have a neutrosophic open set G = NInt(A) in X such that G ⊆ A ⊆ NInt(NCl(G)).

(iii) ⟹ (i): Suppose there exists a neutrosophic open set G = NInt(A) such that G ⊆ A ⊆ NInt(NCl(G)), then $(NInt(NCl(G)))^c \subseteq A^c$. That is $(NInt(NCl(NInt(A))))^c \subseteq A^c$ which implies $NCl(NInt(NCl(A^c))) \subseteq A^c$. Therefore, A^c is a neutrosophic α closed set in X. Then A is a neutrosophic α open set in X and A is a neutrosophic α generalized open set in X. Hence A ∊ NRαGO(X).

Definition 7.5: A neutrosophic topological space (X, τ) is said to be a neutrosophic regular α generalized $T_{1/2}$ space, if every neutrosophic regular α generalized closed set in X is a neutrosophic α generalized closed set in X.

Proposition 7.6: If a neutrosophic topological space (X, τ) is a neutrosophic regular α generalized $T_{1/2}$ space, then every neutrosophic regular α generalized open set is a neutrosophic α generalized open set.

Proof: Let A be a neutrosophic regular α generalized open set in X. This implies A^c is a neutrosophic regular α generalized closed set in X. Since X is a neutrosophic regular α generalized $T_{1/2}$ space, then A^c is a neutrosophic α generalized closed set in X. Hence A is a neutrosophic α generalized open set in X.

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