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Investigating Turbulent Convection in a Rectangular Enclosure Using Shear Stress Transport K-ω Model

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Abstract: *Studies have been done on the aspect ratio effect on natural convection turbulence using standard k-ε model but further studies showed thatk-ω SST model performed better than both k-ε and k-ω model in the whole enclosure. Thus, there was need to do a numerical study on the natural convection fluid flow in a rectangular enclosure full of air using SST k-ω model. The left vertical wall of the enclosure was maintained at a steady high temperature T_h of 323K while the right wall at a steady cool temperature T_c of 303K with the remaining walls adiabatic. Time-averaged energy, momentum and continuity equations with the two equation SST k-omega turbulence model were used to generate isotherms, streamlines and velocity magnitudes for different aspect ratios of the enclosure so as to be able to investigate effect of aspect ratio on turbulence. It was shown that as the aspect ratio of increased from 2, 4, 6 and 8 of the enclosure, the velocity of elements decreased and the vortices became smaller and more parallel thus concluded that an increase in aspect ratio decreased the turbulence.*

Key terms

Convection: is heat transfer through movement of the heated sections of a fluid.
Aspect ratio: Proportion of length of isothermal wall to the gap between them
Turbulent Flow: A system of stream characterized by chaotic property changes flow for values of Reynolds number of above 4000
Streamlines: A path followed out by a massless component as it moves with the stream.
Isotherms: An isotherm is the curve on a graph that connects points of equal temperature.
Vortices: A region in the fluid medium where the flow is mostly rotating around an axis line.

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I. Introduction

The mode of heat transfer in fluids (liquids and gases) is known as convection. When fluids are heated, they expand and thus density decreases. According to Archimedes' principle, warmer and lighter part of the fluid will lead to rise through the neighbouring cooler fluid.

According to Matthew P, Wilcox (2013), fluid stream can be categorized into two; turbulent and laminar flow. Motion of fluid elementsin laminar flow is very organized and movement of fluid is in sheets that relatively slideon each other. The stream happens at very low speeds where there are just minor unsettling influences and low to no local speed variations.

Turbulence convection is an irregular or disturbed flow. It behaves with a chaotic and unpredictable motion. Turbulent convection in a fluid heated from a plane horizontal layer below, called Rayleigh-Bénard convection,is of great importance in severalindustrial and natural processes. The fluid becomes turbulent past a specific temperature difference.

Natural convection study in an enclosure has several engineering applications from natural space warming of household rooms to sections of engineering and atomic installations. Such as, this type of flows happens in material processing cooling of electronic equipment and building technology.

Turbulent flows are characterized by four main features: diffusion, dissipation, three-dimensionality and length scales. For numerical calculation of turbulent flows, an averaging of Navier-Stokes equations of motion is carried out with respect to time. This averaging leads to Reynolds Averaged Navier-Stokes equations (RANS). Additional terms with new variables occur in these partial differential equations because of the averaging. Consequently, there are suddenly more variables than equations. In order to close the motion equation system in this study, $k - \omega$ turbulence modeling will be used.

Objective

Toinvestigate turbulent convection in a rectangular enclosure using SST k-ω model

II. Governing Equations

The following are the set of governing equations

Mass conservation (continuity equation) equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad \text{......} 1$$

Momentum (Navier stokes) equations

$$\rho\left(\frac{\partial v}{\partial t}+v.\nabla v\right)=F_i-\nabla p+\mu\Delta v \quad \text{......} 2$$

Energy equation

$$\rho C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\mu\Phi\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2+\right.$$

∂w∂y+∂v∂z2+ ∂u∂z+∂w∂x2 3

Time averaged continuity equation

$$\frac{\partial\bar{u}}{\partial x}+\frac{\partial\bar{v}}{\partial y}+\frac{\partial\bar{w}}{\partial z}=0 \quad \text{......} 4$$

Time averaged momentum equation

$$\rho\left[\frac{\partial\bar{v}}{\partial t}+\bar{v}.\nabla\bar{v}\right]=F_i-\overline{\nabla p}+\mu\Delta\bar{v}-\overline{\nabla.\rho\grave{v}\grave{v}} \quad \text{......} 5$$

Time averaged energy equation

$$\rho C_p\left(\frac{\partial\bar{T}}{\partial t}+\bar{u}\frac{\partial\bar{T}}{\partial x}+\bar{v}\frac{\partial\bar{T}}{\partial y}+\bar{w}\frac{\partial\bar{T}}{\partial z}\right)=k\left(\frac{\partial^2\bar{T}}{\partial x^2}+\frac{\partial^2\bar{T}}{\partial y^2}+\frac{\partial^2\bar{T}}{\partial z^2}\right)-\frac{\partial c_p\overline{\grave{T}\grave{u}}}{\partial x_i}+\bar{\Phi} \quad \text{......} 6$$

$\frac{\partial c_p\overline{\grave{T}\grave{u}_i}}{\partial x_i}$represent the turbulent heat fluxes i.e.perturbations ofvelocity and temperature

The stress tensor in turbulent flow

Equation 5 can be written in tensor form as

$$\rho\frac{D\overline{u_i}}{Dt}=F_i-\frac{\partial\bar{p}}{\partial x_i}+\mu\Delta\overline{u_i}-\rho\left(\frac{\partial\overline{u_iu_j}}{\partial x_i}\right) \quad \text{......} 7$$

Where $\mu\Delta\overline{u_i}-\rho\left(\frac{\partial\overline{u_iu_j}}{\partial x_i}\right)=\mu\frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}\right)-\rho\frac{\partial}{\partial x_j}\overline{u_iu_j}$

$$=\frac{\partial}{\partial x_j}\left(\mu\frac{\partial u_i}{\partial x_j}-\rho\overline{u_iu_j}\right) \quad \text{......} 8$$

The term in brackets in the above equation is known as total shear stress expressed as τ_{ij} .

Equation 5 can be written as

$$\rho\frac{D\overline{u_i}}{Dt}=F_i-\frac{\partial\bar{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\tau_{ij} \quad \text{......} 9$$

With the approach of *Eddy Viscosity principle*, equation 9 is referred as Reynolds Averaged Navier Stokes equation (RANS).

And

$$\tau_{ij}=\mu\frac{\partial u_i}{\partial x_j}+\rho\left(V_T\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-{}^2/_3\,k\delta_{ij}\right) \quad \text{......} 10$$

Where δ_{ij} *is kronecker delta function*

V_T *is turbulent eddy viscosity*

Approach of Boussinesq

A relative old approach to this principle of eddy viscosity, which in 1877 was formulated by *Boussinesq* and is still the basis of many turbulence models (Rodi 1993).

$$-\overline{\grave{u}_i\grave{u}_j}=V_T\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-{}^2/_3\,k\delta_{ij} \ldots\ldots 11$$

Where k is kinetic energy turbulence defined as

$$k={}^1/_2\left(\overline{u^2}+\overline{v^2}+\overline{w^2}\right) \quad \text{......} 12$$

Turbulent eddy viscosity, V_T, depends on the degree of turbulencei.e. it varies within the fluid flow and depending on the flow condition. The approach of calculating eddy viscosity V_T is known as turbulence modeling.

Applications and Approaches for turbulence modeling

The zeroth models, following the approach of Boussinesq (1877) assume that flow of velocity is proportional to turbulent stresses. In one equation model additional p.d.e for velocity scale is used for turbulence. Another p.d.e for length scale is added for two equation models. This group also includes K-ε and K-ω models.Approaches to determine the turbulence eddy viscosity provides the described closer models zeroth, first and second order.

Shear Stress Transport k-ω Model

It's a two-equation eddy – viscosity model. It combines the standard k-ω and k- ε models. It activates k- ε model in the free stream and standard k-ω model near the wall. This makes sure that the suitable model is appliedall through the stream field.

The transport equations of SST k-ω model are;

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_j}(\rho k u_i)=\frac{\partial}{\partial x_j}\left(\Gamma_k \frac{dk}{dx_j}\right)+\widetilde{G_k}-Y_k+S_k$$

And

$$\frac{\partial}{\partial t}(\rho \omega)+\frac{\partial}{\partial x_j}(\rho \omega u_i)=\frac{\partial}{\partial x_j}\left(\Gamma_\omega \frac{d\omega}{dx_j}\right)+G_\omega-Y_\omega+S_\omega+D_\omega$$

$\widetilde{G_k}=min(G_k, 10\rho\beta^* k\omega)$-reproduction of turbulent kinetic energy owed to average velocity gradients where $G_k=-\rho\overline{u_i' u_j'}\frac{\partial u_j}{\partial x_i}$

$G_\omega=\frac{\alpha}{v_t}G_k$ is the generation of ω

D_ω denotes the cross-diffusion term.

Y_k and Y_ω denotes the dissipation of k and ω due to turbulence.

Γ_k and Γ_ω denotes the effective diffusivity of k and ω respectively.

For the SST k-ω model, the effective diffusivities are given by

$\Gamma_k=\mu+\frac{\mu_t}{\sigma_k}$ and $\Gamma_\omega=\mu+\frac{\mu_t}{\sigma_\omega}$

Where;

S_k and S_ω are user-defined source terms.

σ_ω & σ_k are turbulent Prandtl numbers for ω and k correspondingly.

Constants are determined from experiment and their values are as per the table1 below.

Table 1 Turbulence model constants

$\sigma_{k,1}$	1.176
$\sigma_{\omega,1}$	2.0
$\sigma_{k,2}$	1.0
$\sigma_{\omega,2}$	1.168
α_1	0.31
$\beta_{i,1}$	0.075
$\beta_{i,2}$	0.0828
$\boldsymbol{\alpha}_\infty^*$	1
α_∞	0.52
α_0	$\frac{1}{9}$
$\boldsymbol{\beta}_\infty^*$	0.09
R_β	8
R_k	6
R_ω	2.95
ζ^*	1.5
M_{to}	0.25

III. Mathematical Formulation

Figure 1 demonstrates a graphic outline of the issue under thought and the coordinate structure. Considering a 2D rectangular structure of width W and height H, where the left vertical temperature is kept at T_h and the right at T_c, $T_h>T_c$. No heat stream is accepted at the upper and lowerwall (adiabatic). The walls are unbending and no – slip circumstances are enforced at the limits.

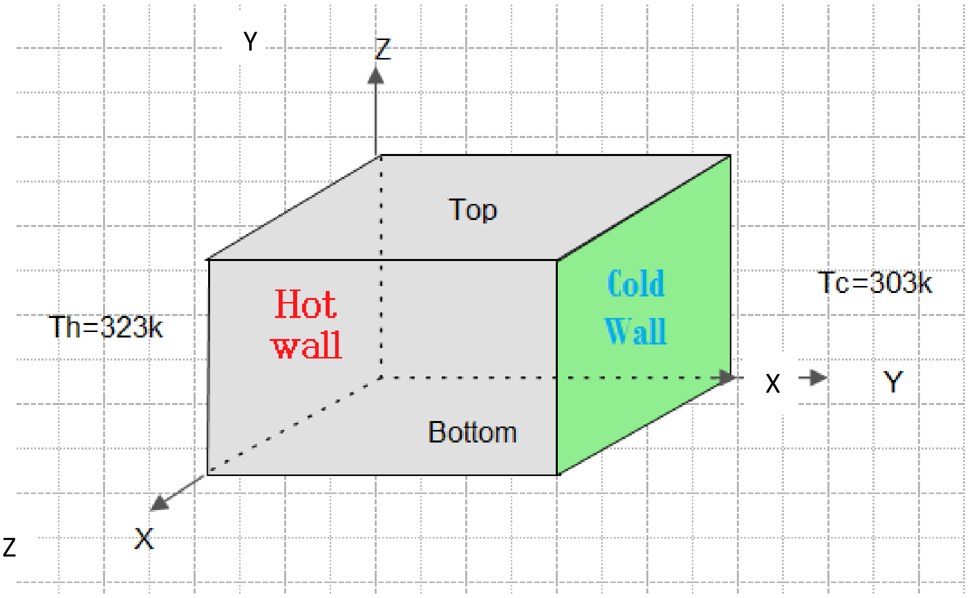


Fig. 1 Geometry of the problem

Dimensionless Energy, Momentum and Continuity Equations

Non – dimensionalizing governing equations makes equations simpler and highlights which terms are the most important. The main objective behind non–dimensionalization is to lessen number of variables. The set of governing equations ought to be resolved to acquire the unknowns p, v, T and u. By applying Boussinesq estimation and then bringing up dimensionless constraintsP, V, U, τ ,θ, Y and X;

$$X=\frac{x}{L}, Y=\frac{y}{L}, U=\frac{uL}{\alpha_f}, V=\frac{vL}{\alpha_f}, \theta_f=\frac{T_f-T_c}{T_h-T_c}., \tau=\frac{\alpha_f t}{L^2}, \text{p}=\frac{L^2 p}{\rho\alpha_f^2} \quad \text{.......} 13$$

The set of governing equations in dimensionless form becomes:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad \text{.......} 14$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \quad \text{.......} 15$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra.Pr.\theta_f \quad \text{.......} 16$$

$$\left(\frac{\partial \theta_f}{\partial \tau}+U\frac{\partial \theta_f}{\partial X}+V\frac{\partial \theta_f}{\partial Y}\right)=k\left(\frac{\partial^2 \theta_f}{\partial X^2}+\frac{\partial^2 \theta_f}{\partial Y^2}\right)+\Phi \quad \text{.......} 17$$

Where,Pr and Ra denotesPrantdland Rayleigh numberscorrespondingly; andθ_f the is dimensionless fluid temperature.

Streamfunction-Vorticity Relation and Vorticity Transport Equation

The equation below of streamfunction is demonstrating the connection between dimensionless streamfunction and dimensionless vorticity.

$$\frac{\partial^2 \psi}{\partial X^2}+\frac{\partial^2 \psi}{\partial Y^2}=-\Omega \quad \text{.......} 18$$

IV. Numerical Method

Finite Difference Solution Method

Using Taylor series expansion to approximate spatial derivatives with second order centered difference, we get

$$\frac{\partial^2 \emptyset}{\partial x^2}=\frac{\emptyset_{i-1,j}^{n+1}-2\emptyset_{i,j}^{n+1}+\emptyset_{i+1,j}^{n+1}}{\Delta x^2}+o(h^2) \quad \text{.......} 19$$

And

$$\frac{\partial^2 \emptyset}{\partial y^2}=\frac{\emptyset_{i,j-1}^{n+1}-2\emptyset_{i,j}^{n+1}+\emptyset_{i+1,j}^{n+1}}{\Delta y^2}+o(h^2) \quad \text{.......} 20$$

Finite Difference Solution Technique for Parabolic Differential Equations

Since energy equation and the vorticity equation are alike,Mobedi (1994), they can be written in form of a single generic equation

$$\frac{\partial \emptyset}{\partial \tau}+U\frac{\partial \emptyset}{\partial X}+V\frac{\partial \emptyset}{\partial Y}=C\left(\frac{\partial^2 \emptyset}{\partial X^2}+\frac{\partial^2 \emptyset}{\partial Y^2}\right)+f \qquad 21$$

Where $\emptyset$is a generic dependent variable representingΩ.

Equation 21 can be reduced to the following form;

$$\frac{\partial \emptyset}{\partial \tau}=\delta_X^2\emptyset+\delta_Y^2\emptyset+f \qquad 22$$

where

$$\delta_X^2\emptyset=C\frac{\partial^2 \emptyset}{\partial X^2}-U\frac{\partial \emptyset}{\partial X} \qquad 23$$

$$\delta_Y^2\emptyset=C\frac{\partial^2 \emptyset}{\partial Y^2}-V\frac{\partial \emptyset}{\partial Y} \qquad 24$$

These can be referred as diffusion-convection terms since the terms $\delta_Y^2\emptyset$ and $\delta_X^2\emptyset$ denoteconvection and diffusion transport in Y and X directioncorrespondingly. Several finite difference approaches can be used to solve the parabolic PDE. These approaches are commonlycategorized into 3categories, i.e., Alternating Direction Implicit (ADI), implicit and explicit approaches (Thiault 1985).

Explicit Methods

When the method is applied on Eqn21 for any point (i, j) in Cartesian coordinates when a simple forward difference for the time term is utilized can be expressed as;

$$\frac{\emptyset_{i,j}^{n+1}-\emptyset_{i,j}^{n}}{\Delta\tau}=\delta_X^2\emptyset_{i,j}^n+\delta_Y^2\emptyset_{i,j}^n+f_{i,j}^n \qquad 25$$

Where $\emptyset_{i,j}^n$ and $\emptyset_{i,j}^{n+1}$denote the estimate of dependent parameter$\emptyset$at node (i, j) at nth and (n+1)th time steps, correspondingly. By taking the numerical spatial derivatives of dependent parameter in the preceding time step, nth inEqn25 the unknown estimate of dependent parameter at point (i, j),$\emptyset_{i,j}^{n+1}$can be found. Since values of the dependent parameter at all points of the computational area at nth time step are identified, it's easy to determine the unknown $\emptyset_{i,j}^{n+1}$in Eqn25.

Implicit Method

Applying implicit techniquewholly on eqn21 for any point (i, j) in Cartesian coordinate, when a simple forward difference for time term is utilized, can be expressed as;

$$\frac{\emptyset_{i,j}^{n+1}-\emptyset_{i,j}^{n}}{\Delta\tau}=\delta_X^2\emptyset_{i,j}^{n+1}+\delta_Y^2\emptyset_{i,j}^{n+1}+f_{i,j}^{n+1} \qquad 26$$

In implicit technique, the spatial derivative of dependent parameter at the same time step determines the dependent parametervalue at a new time step for a point (i, j),$\emptyset_{i,j}^{n+1}$. Therefore, m nodal equations must be acquired and solutions found so as the value of dependent parametercan be determinedfor a new time step in the computational area with m points. Thus, parabolic differential equationsolution with implicit techniquemight be more complexcompared to explicit technique.

ADI method

The ADI method splits the finite difference equation into two, one having the x – direction and the other the y – direction. Every time step is divided into two sub-steps of equal duration $1/2\,\Delta t$ and approximating the spatial derivatives in a partially implicit manner while working sequentially and alternating in the x – and y – direction

Use of ADI technique on Eqn21when simple forward difference for time term is utilized for any point (i, j) in Cartesian coordinatemay be expressed in 2stages as;

$$\frac{\emptyset_{i,j}^{n+1/2}-\emptyset_{i,j}^{n}}{\Delta\tau/2}=\delta_X^2\emptyset_{i,j}^{n+1/2}+\delta_Y^2\emptyset_{i,j}^n+f_{i,j}^n \qquad 27$$

$$\frac{\emptyset_{i,j}^{n+1}-\emptyset_{i,j}^{n+1/2}}{\Delta\tau/2}=\delta_X^2\emptyset_{i,j}^{n+1/2}+\delta_Y^2\emptyset_{i,j}^{n+1}+f_{i,j}^{n+1/2} \qquad 28$$

Where Eqn27 is explicit for y - direction and implicit for x-direction and Eqn28 is explicit for x - direction and implicit for y-direction.

Eqns 27 and 28 can be organized as;

$$\left(1-\frac{\Delta\tau}{2}\delta_X^2\right)\emptyset_{i,j}^{n+1/2}=\left(1+\frac{\Delta\tau}{2}\delta_Y^2\right)\emptyset_{i,j}^n+\frac{\Delta\tau}{2}f_{i,j}^n \qquad 29$$

$$\left(1-\frac{\Delta\tau}{2}\delta_Y^2\right)\emptyset_{i,j}^{n+1}=\left(1+\frac{\Delta\tau}{2}\delta_X^2\right)\emptyset_{i,j}^n+\frac{\Delta\tau}{2}f_{i,j}^{n+1/2} \qquad 30$$

As it can be seen, the most importantbenefit of ADI techniqueis that result of the equations can be found after two stages with regard to a fully implicit technique.

V. Discussionof Results

Isotherms

In figure 2, 3, 4 and 5 the maximum temperature is 117K, 56.7K, 42.6K and 34.2K respectively. The high temperatures are evident on the left side wall. In all cases two round motion in opposite directions (anticlockwise and clockwise direction). There is rises up of hot less dense particles which losses its heat with distance as shown by change in color. In between the two isothermal walls there is mixing of air particles which is a region of thermal equilibrium and is a relatively warm region. In fig 4 and 5, temperature uniformity is achieved. In conclusion, it is evident that maximum temperature decreases with increase in aspect ratio.

Fig. 2 Isotherm of aspect ratio 2

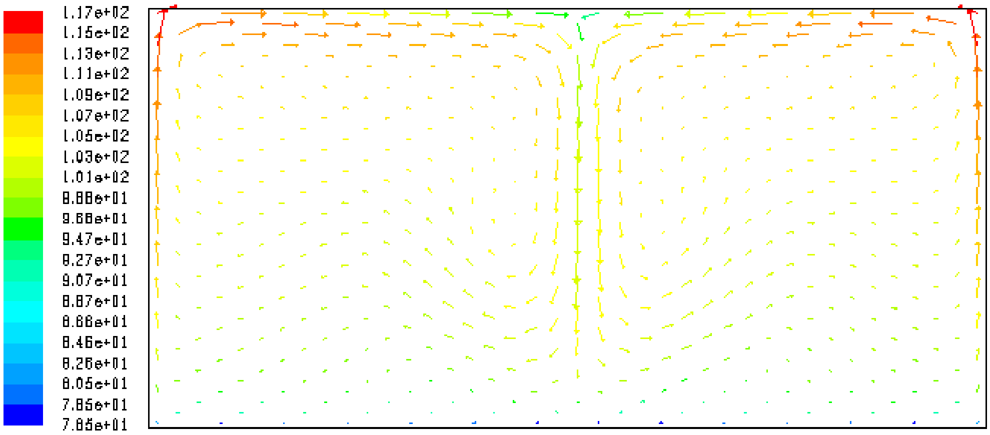


Fig. 3 Isotherm of aspect ratio 4

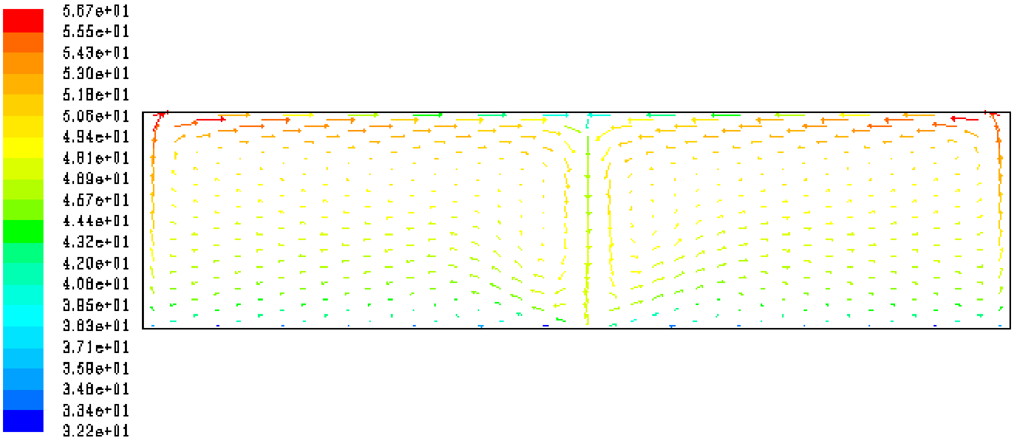


Fig. 4 Isotherm of aspect ratio 6

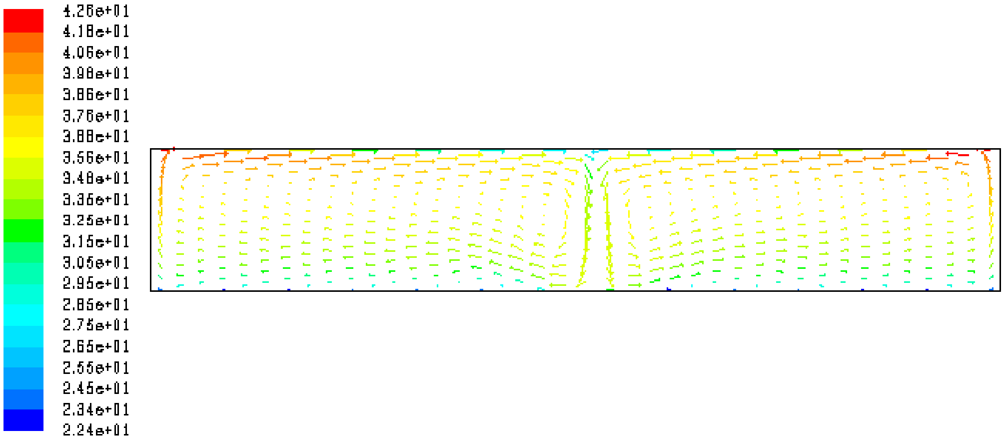
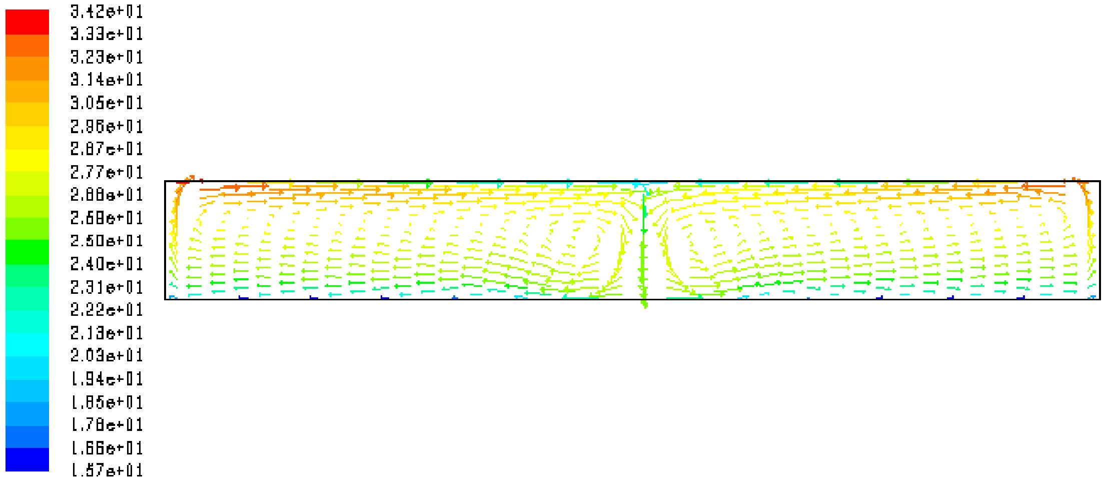


Fig. 5 Isotherm of aspect ratio 8



Contours of Velocity Magnitudes

In figures 6, 7, 8 and 9, the highest velocity of air particles is 0.456m/s, 0.352m/s, 0.308m/s and 0.303 m/s respectively.Vortices are more for aspect ratio of 2 which become parallel as aspect ratio increases. At this point it is evident that as aspect ratio increases the flow becomes less turbulent.

Fig. 6 Contours of velocity magnitude of aspect ratio 2

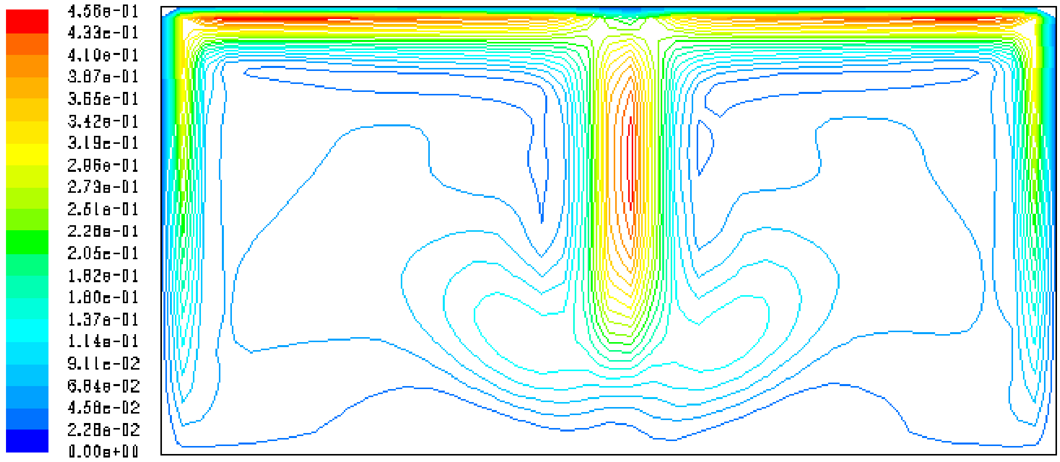


Fig. 7 Contours of velocity magnitude of aspect ratio 4

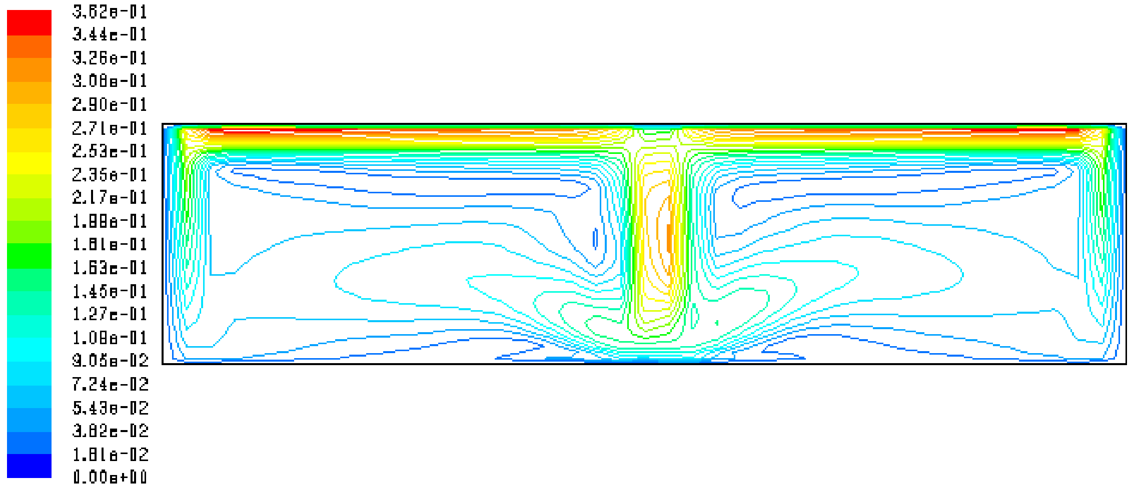


Fig. 8 Contours of velocity magnitude of aspect ratio 6

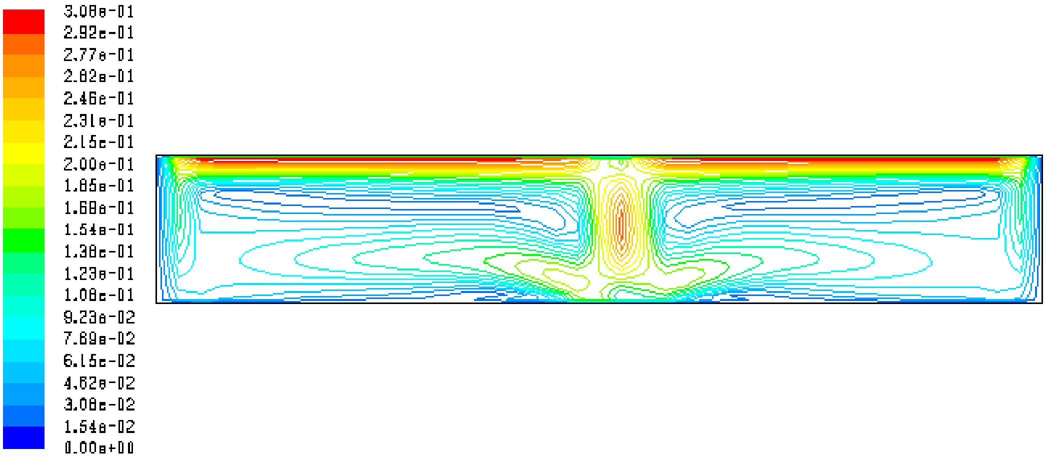
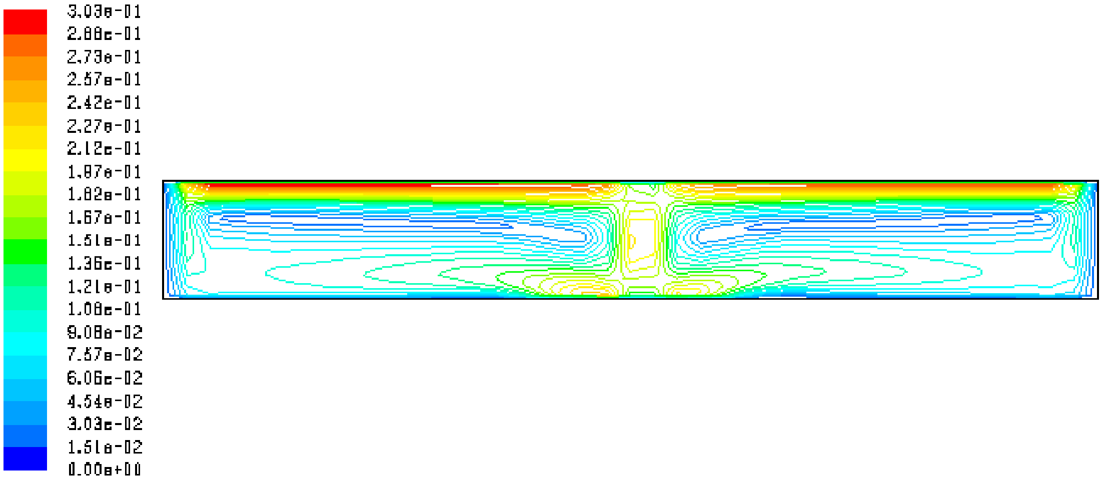


Fig. 9 Contours of velocity magnitude of aspect ratio 8



Streamline Distribution

The lowestspeed of an element indicated here for aspect ratio of 2 is 0.158Kg/s followed by that of aspect ratio 4 which is 0.185Kg/s. This value increases as aspect ratio increases as depicted by that of aspect ratio 6 which is 0.246Kg/s and the highest speed which is 0.278 Kg/s as shown by that of aspect ratio 8. In fig 10, the vortices are big in size and they assume a circular path which deforms as distance increases from their centers. Infig 11, radius of circle reduces which as well decreases as the aspect ratio increases to 8 as seen in fig 12. In fig 13 the two centre cell deforms and takes an oval shape. The vortices become parallel as aspect ratio increases.

Fig. 10 Contours of streamlines of aspect ratio 2

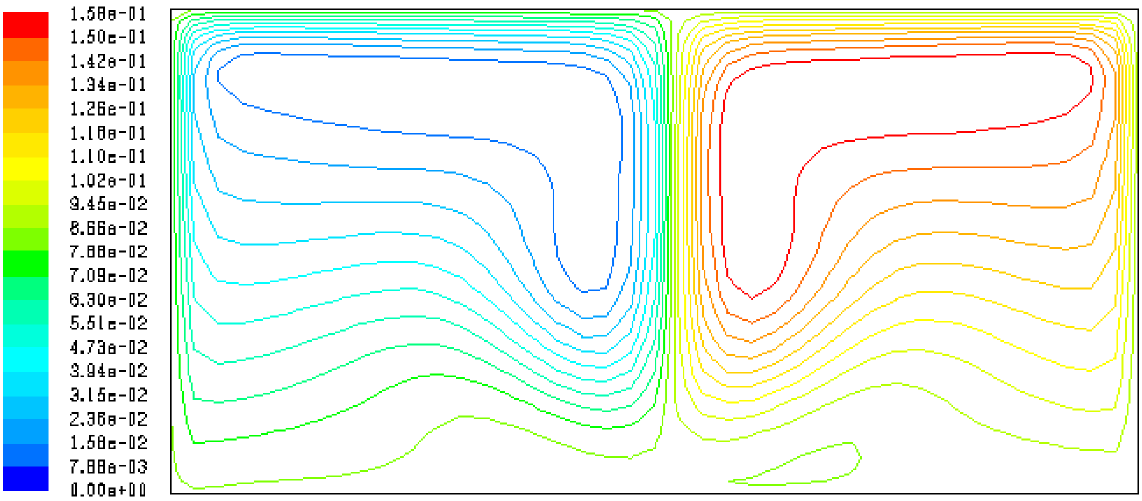


Fig. 11 Contours of streamlines of aspect ratio 4

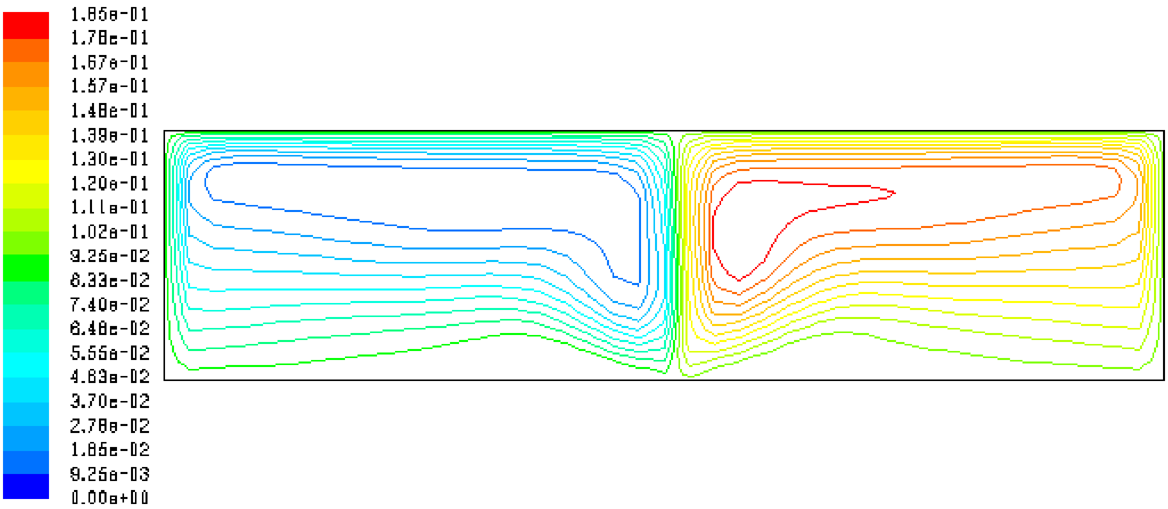


Fig. 12 Contours of streamlines of aspect ratio 6

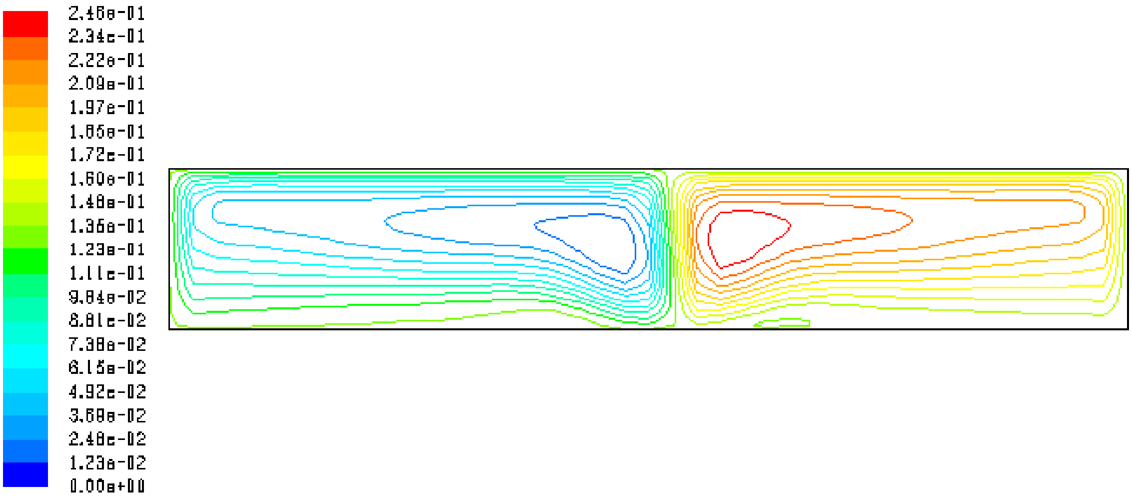
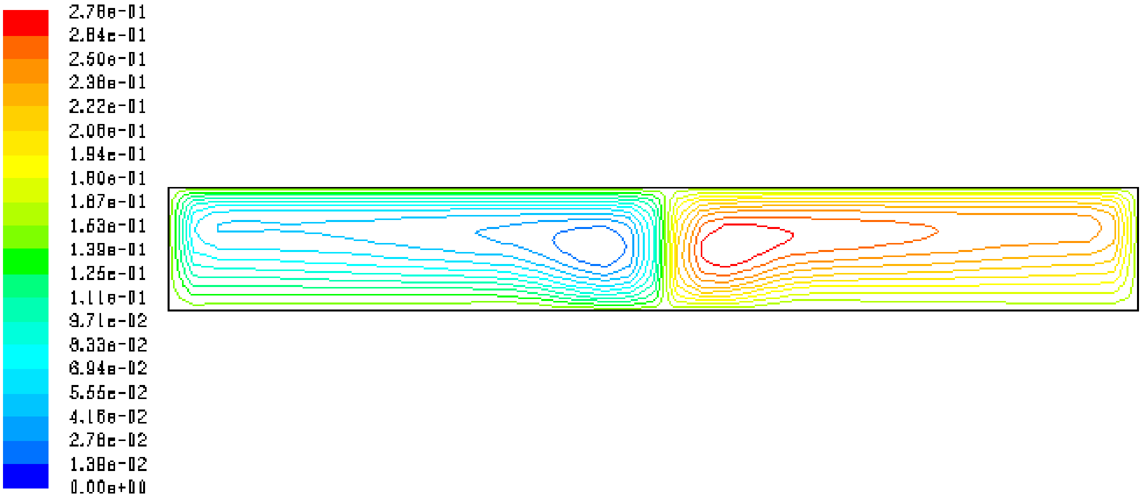


Fig. 13 Contours of streamlines of aspect ratio 8



VI. Conclusion

Streamlines, isotherms and velocity magnitudes for aspect ratio 2, 4, 6, and 8 were generated and showed that the increase in aspect ratio decreased the turbulence.The results showed that increased aspect ratio decreased speed and vortices became more parallel thus decreasing turbulence. So, the aspect ratio has animportantinfluence in temperature field and fluid stream in horizontal enclosures heated from the side.

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