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Study of the representation of even numbers and the proof of the Goldbach's binary conjecture

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Abstract
The paper analyzes the representation of even number as a sum of two odd summands. To describe even numbers and their representations, arithmetic progressions of a special kind are used, which allow us to divide (split) the set of even numbers into classes, in each of which the representation depends on a small number of generators. On this basis the proof of Goldbach's binary conjecture is given. The connection of this conjecture with other problems of number theory is considered.
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I. Introduction

Goldbach's binary conjecture is the statement: every even number $2n > 2$ admits a representation as a sum of two primes. Experimental studies and calculations confirm its validity for large numbers [1 – 3]. The purpose of this article is to study the connection between the Goldbach's conjecture and the problem of representing natural numbers and to prove the Goldbach's binary conjecture. The article is a development of the author's previous work on this problem [4]. The representation of an arbitrary even number $2n$ as a sum of two odd terms consists of the set of pairs $\{g_i, (2n - g_i)\}$. The number n is the center of the representation. The numbers g_i are to the left of the center. We will call them representation generators of even number, or shortly representation generators, because they completely define the structure of representation and the number of pairs. The numbers $(2n - g_i)$ are to the right of the center. The sum of the numbers in each pair is $2n$. The pair $[g_i, (2n - g_i)]$ will be called the pair of conjugate numbers. If both numbers in a pair are prime, then we call it a pair of prime conjugate numbers. To avoid duplication, we will assume that always $g_i \le (2n - g_i)$. If n is an even number, then the representation of the even number $2n$ can be written as

$$2n = \left[1 + (2n-1)\right] = \left[3 + (2n-3)\right] = ... = \left[(n-3) + (n+3)\right] = \left[(n-1) + (n+1)\right]. \tag{1}$$

If n is an odd number, then the representation has the form

$$2n = \left[1 + (2n-1)\right] = \left[3 + (2n-3)\right] = ... = \left[(n-2) + (n+2)\right] = \left[n + n\right]. \tag{2}$$

If the number in a pair is composite, then its smallest prime divisor we call the generator of this composite number, and the composite number we call the number formed by this generator. If the numbers g_i and $(2n - g_i)$ are composite, then they are formed using the generators of the number $2n$. The number of generators forming composite numbers participating in the representation is less (or much less) than the number of conjugate pairs in the representation $2n$, as well as the number of prime representation generators. To describe even numbers and their representations, we use arithmetic progressions of a special kind, which allow us to divide the set of even numbers into classes, in each of which the representation depends on a small number of representation generators. All odd numbers participating in the representation belong to these progressions. Representations of even numbers of one class form a connected system. We will prove that a system of equations describing the representation of an arbitrary even number from a given class cannot have a solution that does not include a pair of prime conjugate numbers. This conclusion is valid for even numbers of an arbitrary class, which is equivalent to the validity of the Goldbach's binary conjecture.

II. Choice of representation

Analysis of the tables of prime numbers shows that the distance between adjacent primes, as a rule, does not exceed 30. For example, exceeding the distance by 2, 4 and 6 as compared to 30 is observed once among the first 10000 primes. We can assume that an interval of 30 or a multiple of 30 is characteristic for the distribution of prime numbers. This is one of the arguments we take into account to choose a representation and an interval for analyze of the distribution of even numbers. It is easily verified that the triplet ($x = 3k$, $y = 9k^2 - 1$, $z = 9k^2 + 1$), where k is an even number, i.e. k = 2, 4, 6, etc., is a solution of the quadratic Fermat equation $x^2 + y^2 = z^2$ (see [5, 6]). The expression for y is the product of two factors $y = (3k - 1) \cdot (3k + 1)$. Odd numbers in the sequences $(3k - 1)$ and $(3k + 1)$ follow each other with step of 6 units. Numbers in these sequences differ by 2 for the same value of k. If we combine sequences and rank the numbers in ascending order, they alternate with step 2 – 4 – 2 – 4 etc., namely, we have pairs 5 – 7 (k = 2), 11 – 13 (k = 4), 17 – 19 (k = 6), etc. The first number in each pair corresponds to $(3k - 1)$ and the second number corresponds to $(3k + 1)$. Let us prove two assertions. Lemma 1: "Any prime number (except for numbers 2 and 3) belongs to one of the sequences $(3k - 1)$ or $(3k + 1)$". Lemma 2: "Any even number (except for numbers 2, 4, 6 and 8) can be represented as a sum of two odd numbers belonging to sequences $(3k - 1)$ or $(3k + 1)$". To prove the first assertion, we note that the prime number is odd and it is not divisible by 3. Consider odd numbers $2n + 3$, where n = 1, 2, 3, etc. We put $2n + 3 = 3k - 1$, then $n = (3k - 4)/2$. From this it follows that n can take values 1, 4, 7, etc. with step 3. Put $2n + 3 = 3k + 1$, then $n = (3k - 2)/2$. From this it follows that n takes values 2, 5, 8, etc. with step 3. It can be seen that the sequences $(3k - 1)$ and $(3k + 1)$ do not include the numbers n = 3, 6, 9, etc. with step 3. However, these numbers are divided by 3; therefore they are not prime numbers (we excluded number 3), which proves the first assertion. To prove the second assertion, we consider even numbers $2n$, where n = 5, 6, 7, etc. Divide the set of even numbers into 6 subsets (series). In the first series, we put $2n = (3k - 1) + (3k - 1) = 6k - 2$, then $n = 3k - 1$. From this it follows that n can take values n = 5, 11, 17, etc. with step 6. In the second series, we put $2n = (3k - 1) + (3k + 1) = 6k$, then $n = 3k$. From this it follows that n can take values n = 6, 12, 18, etc. with step 6. In the third series, we put $2n = (3k + 1) + (3k + 1) = 6k + 2$, then $n = 3k + 1$. From this it follows that n can take values n = 7, 13, 19, etc. with step 6. In the fourth series, we put $2n = (3k - 1) + (3(k + 2) - 1) = 6k + 4$, then $n = 3k + 2$. From this it follows that n can take values n = 8, 14, 20, etc. with step 6. In the fifth series, we put $2n = (3k - 1) + (3(k + 2) + 1) = 6k + 6$, then $n = 3k + 3$. From this it follows that n can take values n = 9, 15, 21, etc. with step 6. In the sixth series, we put $2n = (3k + 1) + (3(k + 2) + 1) = 6k + 8$, then $n = 3k + 4$. From this it follows that n can take values n = 10, 16, 22, etc. with step 6. We see that the six considered series cover all even numbers, which proves the second assertion. It also follows from the above analysis that prime numbers n belong only to the first or third series. If we put $k + 2 = k^1$, then series 4, 5, 6 are described in the same way as series 1, 2, 3, respectively; the difference is the change in the parity of the center. We divide even numbers belonging to one series into 5 groups, in each of which the numbers have the same ends, namely, the numbers with the end 0 form the first group, with the end 2 – the second, with the end 4 – the third, with the end 6 – the fourth, with end 8 – the fifth. The sequence of ends is different for different series. We divide the numbers in each group into two subgroups; in the first subgroup, the center of representation of even number $2n$ is even, and in the second subgroup, the center is odd. Every even number is completely determined by indicating three numbers (m_1, m_2, m_3), where m_1 is the series number, m_2 is the group number, m_3 is the number of the subgroup. We will say that even numbers corresponding to each combination (m_1, m_2, m_3) belong to one class. If m_1 and m_2 are given, then this defines m_3; if m_2 and m_3 are given, then this defines m_1. Therefore, some of the combinations are excluded. In total, there are 30 permissible combinations or classes. In each subgroup, the distance between adjacent numbers is 60. Arbitrary even number can be written in the form $2n = n_0 + 60(t - 1)$, where n_0 is initial number, t is the ordinal number of even number; t = 1, 2, 3, etc. We will also use the notation $2n = n_0 + 60t$, then t = 0, 1, 2, 3, etc. For even numbers ending in 0, n_0 can take values 10, 20, 30, 40, 50 and 60 depending on class of even number. In particular, for the class (2, 1, 1), which includes even numbers of the form $60 + 60(t - 1)$, n_0 is 60; for the class (5, 1, 2) including numbers of the form $30 + 60(t - 1)$, n_0 is 30, etc. For even numbers ending in 2, n_0 can take values 12, 22, 32, 42, 52 and 62 depending on class. For even numbers ending in 4, n_0 can take values 14, 24, 34, 44, 54 and 64 depending on class. For even numbers ending in 6, n_0 can take values 16, 26, 36, 46, 56 and 66 depending on class. For even numbers ending in 8, n_0 can take values 18, 28, 38, 48, 58 and 68 depending on class. We use sequences $(3k - 1)$ and $(3k + 1)$ to represent even numbers as the sum of two odd summands. The advantages of such a representation are obvious. Prime numbers appear regularly in sequences consisting of the solutions of the quadratic Fermat equation. In the representation of an arbitrary even number $2n$, the first pair $(1, 2n - 1)$ is absent, since the number 1 does not belong to the sequences $(3k - 1)$ or $(3k + 1)$. At the same time, the number 1 is not a prime number. In these sequences, there is no number 3, which has the shortest repetition period, and no odd numbers that are divisible by 3. In the general case, an arbitrary prime number g generates (forms) two sequences (arithmetic progressions) of composite numbers (g, $6g$) and ($5g$, $6g$), where g and $5g$ are the initial values, and $6g$ is the period. We call the period $6g$ a small period. Depending on the value of the number g, one of the progressions belongs to the sequence $(3k + 1)$, and the other belongs to the sequence $(3k - 1)$. The value $5 \cdot 6g = 30g$ corresponds to the complete cycle of changing of ends in the series and we will call it a large half-period. After each large half-period, the number, position and ends of the composite numbers are repeated. The value $60g$ we will call a large period. After each large period, the entire set (spectrum) of combinations of composite numbers participating in the representation of even numbers of a given class is repeated. The interval between adjacent numbers of the first and second progression can take the value $2g$ or $4g$. Within a large half-period, the sum of the intervals is $28g$. We have $4g + 2g + 4g + 2g + 4g + 2g + 4g + 2g + 4g = 28g$. If we exclude numbers divisible by 5, then the equality takes the form $6g + 4g + 2g + 4g + 2g + 4g + 6g = 28g$. The adjacent numbers, separated by an interval of $6g$, belong to the same progression. The alternation (sequence) of intervals is repeated in each large half-period. In addition, for generators with the same endings, the endings of the composite numbers formed by them are repeated with the same sequence. In the future, we exclude from the representation generators 3 and 5, as well as numbers that are multiples of 3 or 5. If necessary, numbers 3 and 5 are taken into account additionally (see below).

III. Properties of representation of numbers from different classes

It is easy to verify the validity of two statements. Lemma 3: "Any odd number more than 31 can be obtained from the numbers 7, 11, 13, 17, 19, 23, 29, 31 by adding the number 30 or a multiple of 30". Lemma 4: "Any odd number more than 61 can be obtained from the numbers 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61 by adding the number 60, or a multiple of 60". We will call these numbers the initial numbers. Using Lemma 3, we define arithmetic progressions corresponding to odd numbers with different endings. Any odd number ending in 7 belongs to one of the arithmetic progressions $(7 + 30l_7)$ or $(17 + 30l_{17})$. Any odd number ending in 1 belongs to one of the arithmetic progressions $(11 + 30l_{11})$ or $(31 + 30l_{31})$; any odd number ending in 3 belongs to one of the arithmetic progressions $(13 + 30l_{13})$ or $(23 + 30l_{23})$; any odd number ending in 9 belongs to one of the arithmetic progressions $(19 + 30l_{19})$ or $(29 + 30l_{29})$. We write the representation of even numbers with different endings using Lemma 3. The representation of even numbers with different endings is obtained by combining numbers from the corresponding arithmetic progressions. The representation for even numbers ending in 0 has the form $\{[(7 + 30l_7)$ or $(17 + 30l_{17})] + [(13 + 30l_{13})$ or $(23 + 30l_{23})]\}$ or $\{[(11 + 30l_{11})$ or $(31 + 30l_{31})] + [(19 + 30l_{19})$ or $(29 + 30l_{29})]\}$. Hereinafter, the choice of pairs depends on the class to which an even number belongs. If the even number is not divisible by 3, then the following pairs are additionally taken into account $\{3 + [(7 + 30l_7)$ or $(17 + 30l_{17})]\}$; the choice of one or another pair depends on the class to which an even number belongs. The representation for even numbers ending in 2 has the form $\{[(11 + 30l_{11})$ or $(31 + 30l_{31})] + [(11 + 30l_{11})$ or $(31 + 30l_{31})]\}$ or $\{[(13 + 30l_{13})$ or $(23 + 30l_{23})] + [(19 + 30l_{19})$ or $(29 + 30l_{29})]\}$ or $\{5 + [(7 + 30l_7)$ or $(17 + 30l_{17})]\}$. If the even number is not divisible by 3, then the following pairs are additionally taken into account $\{3 + [(19 + 30l_{19})$ or $(29 + 30l_{29})]\}$. The representation for even numbers ending in 4 has the form $\{[(7 + 30l_7)$ or $(17 + 30l_{17})] + [(7 + 30l_7)$ or $(17 + 30l_{17})]\}$ or $\{[(11 + 30l_{11})$ or $(31 + 30l_{31})] + [(13 + 30l_{13})$ or $(23 + 30l_{23})]\}$ or $\{5 + [(19 + 30l_{19})$ or $(29 + 30l_{29})]\}$. If the even number is not divisible by 3, then the following pairs are additionally taken into account $\{3 + [(11 + 30l_{11})$ or $(31 + 30l_{31})]\}$. The representation for even numbers ending in 6 has the form $\{[(7 + 30l_7)$ or $(17 + 30l_{17})] + [(19 + 30l_{19})$ or $(29 + 30l_{29})]\}$ or $\{[(13 + 30l_{13})$ or $(23 + 30l_{23})] + [(13 + 30l_{13})$ or $(23 + 30l_{23})]\}$ or $\{5 + [(11 + 30l_{11})$ or $(31 + 30l_{31})]\}$. If the even number is not divisible by 3, then the following pairs are additionally taken into account $\{3 + [(13 + 30l_{13})$ or $(23 + 30l_{23})]\}$. The representation for even numbers ending in 8 has the form $\{[(7 + 30l_7)$ or $(17 + 30l_{17})] + [(11 + 30l_{11})$ or $(31 + 30l_{31})]\}$ or $\{[(19 + 30l_{19})$ or $(29 + 30l_{29})] + [(19 + 30l_{19})$ or $(29 + 30l_{29})]\}$ or $\{5 + [(13 + 30l_{13})$ or $(23 + 30l_{23})]\}$. The numbers l_7, l_{17}, l_{11}, l_{31}, etc. can take values 0, 1, 2, 3, etc. The index shows their connection with the corresponding initial numbers. Lemma 3 allows us to establish a connection between arithmetic progressions, as well as between representations of even numbers from classes that differ only in the parity of the center. We define arithmetic progressions corresponding to odd numbers with different endings using Lemma 4. Any odd number ending in 7 belongs to one of the arithmetic progressions $(7 + 60l_7)$ or $(17 + 60l_{17})$ or $(37 + 60l_{37})$ or $(47 + 60l_{47})$. Any odd number ending in 1 belongs to one of the arithmetic progressions $(11 + 60l_{11})$ or $(31 + 60l_{31})$ or $(41 + 60l_{41})$ or $(61 + 60l_{61})$. Any odd number ending in 3 belongs to one of the arithmetic progressions $(13 + 60l_{13})$ or $(23 + 60l_{23})$ or $(43 + 60l_{43})$ or $(53 + 60l_{53})$; any odd number ending in 9 belongs to one of the arithmetic progressions $(19 + 60l_{19})$ or $(29 + 60l_{29})$ or $(49 + 60l_{49})$ or $(59 + 60l_{59})$. The representation of even numbers with different endings is obtained in the same way as in the case of Lemma 3 by combining numbers from the corresponding arithmetic progressions. The representation for even numbers ending in 0 has the form $\{[(7 + 60l_7)$ or $(17 + 60l_{17})$ or $(37 + 60l_{37})$ or $(47 + 60l_{47})] + [(13 + 60l_{13})$ or $(23 + 60l_{23})$ or $(43 + 60l_{43})$ or $(53 + 60l_{53})]\}$ or $\{[(11 + 60l_{11})$ or $(31 + 60l_{31})$ or $(41 + 60l_{41})$ or $(61 + 60l_{61})] + [(19 + 60l_{19})$ or $(29 + 60l_{29})$ or $(49 + 60l_{49})$ or $(59 + 60l_{59})]\}$. Hereinafter, the choice of combinations of pairs depends on the class to which an even number belongs (see below). If the even number is not divisible by 3, then the following pairs are additionally taken into account $\{3 + [(7 + 60l_7)$ or $(17 + 60l_{17})$ or $(37 + 60l_{37})$ or $(47 + 60l_{47})]\}$. The representation of even numbers ending in 2 has the form $\{[(11 + 60l_{11})$ or $(31 + 60l_{31})$ or $(41 + 60l_{41})$ or $(61 + 60l_{61})] + [(11 + 60l_{11})$ or $(31 + 60l_{31})$ or $(41 + 60l_{41})$ or $(61 + 60l_{61})]\}$ or $\{[(13 + 60l_{13})$ or $(23 + 60l_{23})$ or $(43 + 60l_{43})$ or $(53 + 60l_{53})] + [(19 + 60l_{19})$ or $(29 + 60l_{29})$ or $(49 + 60l_{49})$ or $(59 + 60l_{59})]\}$ or $\{5 + [(7 + 60l_7)$ or $(17 + 60l_{17})$ or $(37 + 60l_{37})$ or $(47 + 60l_{47})]\}$. If the even number is not divisible by 3, then the following pairs are additionally taken into account $\{3 + [(19 + 60l_{19})$ or $(29 + 60l_{29})$ or $(49 + 60l_{49})$ or $(59 + 60l_{59})]\}$. The representation for even numbers ending in 4 has the form {[(7 + 60l_7) or (17 + 60l_{17}) or (37 + 60l_{37}) or (47 + 60l_{47})] + [(7 + 60l_7) or (17 + 60l_{17}) or (37 + 60l_{37}) or (47 + 60l_{47})]} or {[(11 + 60l_{11}) or (31 + 60l_{31}) or (41 + 60l_{41}) or (61 + 60l_{61})] + [(13 + 60l_{13}) or (23 + 60l_{23}) or (43 + 60l_{43}) or (53 + 60l_{53})]} or {5 + [(19 + 60l_{19}) or (29 + 60l_{29}) or (49 + 60l_{49}) or (59 + 60l_{59})]}. If the even number is not divisible by 3, then the following pairs are additionally taken into account {3 + [(11 + 60l_{11}) or (31 + 60l_{31}) or (41 + 60l_{41}) or (61 + 60l_{61})]}. The representation for even numbers ending in 6 has the form {[(7 + 60l_7) or (17 + 60l_{17}) or (37 + 60l_{37}) or (47 + 60l_{47})] + [(19 + 60l_{19}) or (29 + 60l_{29}) or (49 + 60l_{49}) or (59 + 60l_{59})]} or {[(13 + 60l_{13}) or (23 + 60l_{23}) or (43 + 60l_{43}) or (53 + 60l_{53})] + [(13 + 60l_{13}) or (23 + 60l_{23}) or (43 + 60l_{43}) or (53 + 60l_{53})]} or {5 + [(11 + 60l_{11}) or (31 + 60l_{31}) or (41 + 60l_{41}) or (61 + 60l_{61})]}. If the even number is not divisible by 3, then the following pairs are additionally taken into account {3 + [(13 + 60l_{13}) or (23 + 60l_{23}) or (43 + 60l_{43}) or (53 + 60l_{53})]}. The representation for even numbers ending in 8 has the form {[(7 + 60l_7) or (17 + 60l_{17}) or (37 + 60l_{37}) or (47 + 60l_{47})] + [(11 + 60l_{11}) or (31 + 60l_{31}) or (41 + 60l_{41}) or (61 + 60l_{61})]} or {[(19 + 60l_{19}) or (29 + 60l_{29}) or (49 + 60l_{49}) or (59 + 60l_{59})] + [(19 + 60l_{19}) or (29 + 60l_{29}) or (49 + 60l_{49}) or (59 + 60l_{59})]} or {5 + [(13 + 60l_{13}) or (23 + 60l_{23}) or (43 + 60l_{43}) or (53 + 60l_{53})]}. We write the arithmetic progressions corresponding to Lemma 4 in explicit form. Hereinafter, primes are shown in bold. For odd numbers ending in 7, we have four arithmetic progressions **7**, **67**, **127**, 187, 247, **307**, **367**, 427, **487**, **547**, **607**, 667, **727**, **787**, 847, etc.; **17**, 77, **137**, **197**, **257**, **317**, **377**, 437, 497, **557**, **617**, **677**, 737, **797**, **857**, etc.; **37**, **97**, **157**, 217, **277**, **337**, **397**, **457**, 517, **577**, 637, 697, **757**, 817, **877**, etc.; **47**, **107**, **167**, **227**, 287, **347**, 407, **467**, 527, **587**, **647**, 707, 767, **827**, **887**, etc. For odd numbers ending in 1, we have four arithmetic progressions **11**, **71**, **131**, **191**, **251**, **311**, **371**, **431**, **491**, 551, **611**, **671**, 731, 791, 851, **911**, **971**, etc.; **31**, 91, **151**, **211**, **271**, **331**, 391, 451, 511, **571**, **631**, **691**, **751**, **811**, 871, 931, **991**, etc.; **41**, **101**, 161, 221, **281**, 341, **401**, **461**, **521**, 581, 641, **701**, **761**, **821**, **881**, etc.; **61**, 121, **181**, **241**, 301, 361, **421**, 481, **541**, **601**, **661**, 721, 781, 841, 901, 961, **1021**, etc. For odd numbers ending in 3, we have four arithmetic progressions **13**, **73**, 133, **193**, 253, **313**, **373**, **433**, 493, 553, **613**, **673**, **733**, 793, etc.; **23**, **83**, 143, 203, **263**, 323, **383**, **443**, **503**, **563**, 623, **683**, **743**, 803, etc.; **43**, **103**, **163**, **223**, **283**, 343, 403, **463**, **523**, 583, **643**, 703, 763, **823**, etc.; **53**, **113**, **173**, **233**, **293**, **353**, 413, 473, 533, **593**, **653**, 713, **773**, 833, etc. For odd numbers ending in 9, we have four arithmetic progressions **19**, **79**, **139**, **199**, 259, 319, **379**, **439**, **499**, 559, **619**, 679, **739**, 799, **859**, **919**, etc.; **29**, **89**, **149**, 209, **269**, 329, **389**, **449**, **509**, **569**, 629, 689, 749, **809**, 869, **929**, etc.; 49, **109**, 169, **229**, 289, **349**, **409**, 469, 529, 589, 649, **709**, 769, **829**, 889, 949, etc.; **59**, 119, **179**, **239**, 299, **359**, **419**, **479**, 539, **599**, **659**, **719**, 779, **839**, 899, 959, etc. Progressions corresponding to a certain class of even numbers we call basic progressions. Any prime number, except 3 and 5, participating in the representation of even numbers of a given class, belongs to one of the basic progressions. For example, the representation of numbers of the form 60 + 60(t – 1) from the class (2, 1, 1) is obtained by combining numbers from 16 arithmetic progressions: (7 + 60l_7) and (53 + 60l_{53}), (17 + 60l_{17}) and (43 + 60l_{43}), (23 + 60l_{23}) and (37 + 60l_{37}), (13 + 60l_{13}) and (47 + 60l_{47}); (11 + 60l_{11}) and (49 + 60l_{49}), (29 + 60l_{29}) and (31 + 60l_{31}), (19 + 60l_{19}) and (41 + 60l_{41}), (59 , 60l_{59}) and (61 + 60l_{61}). The relation $t = l_i + l_k + 1$ is satisfied; l_i , l_k take values 0, 1, 2, etc., where l_i we denote l_7, l_{17}, l_{23}, l_{13}, l_{11}, l_{29}, l_{19}, l_{59}, and l_k, respectively, l_{53}, l_{43}, l_{37}, l_{47}, l_{49}, l_{31}, l_{41}, l_{61}. Determine the number of pairs participating in the representation of even numbers from different classes. To avoid duplication, we assume that the smaller number in a pair is always to the left of the center, and the larger number is to the right of the center. The representation of an arbitrary even number of the form $2n$ = 60 + 60(t – 1) from the class (2, 1, 1) consists of 8t – 1 pairs (combinations) of numbers from the corresponding arithmetic progressions. The representation of an arbitrary even number of the form $2n$ = 30 + 60(t – 1) from the class (5, 1, 2) consists of 8t – 5 pairs (combinations). For even numbers from the classes (2, 1, 1) and (5, 1, 2), if t increases by 1, then the number of pairs in the representation increases by 8. The representation of an arbitrary even number of the form $2n$ = 50 + 60(t – 1) from the class (3, 1, 2) consists of 4t – 1 pairs (combinations). If we take into account an additional pair (3, $2n$ – 3), then the number of pairs will be 4t. For even numbers of the form $2n$ = 20 + 60(t – 1) from the class (6, 1, 1), the number of pairs is 4t – 3; if we take into account the pair (3, $2n$ – 3), the number of pairs is 4t – 2. For even numbers from the classes (3, 1, 2) and (6, 1, 1), if t increases by 1, then the number of pairs in the representation increases by 4. For even numbers of the form $2n$ = 10 + 60(t – 1) from the class (1, 1, 2), the number of pairs is 4t – 4; if we take into account the pair (3, $2n$ – 3), the number of pairs is 4t – 3. For even numbers of the form $2n$ = 40 + 60(t – 1) from the class (4, 1, 1), the number of pairs is 4t – 2; if we take into account the pair (3, $2n$ – 3), the number of pairs is 4t – 1. For even numbers from the classes (1, 1, 2) and (4, 1, 1), if t increases by 1, then the number of pairs in the representation increases by 4. For even numbers of the form $2n$ = 12 + 60(t – 1) from the class (2, 2, 1), the number of pairs is 6t – 6; if we take into account the pair (5, $2n$ – 5), the number of pairs is 6t – 5. For even numbers of the form $2n$ = 42 + 60(t – 1) from the class (5, 2, 2), the number of pairs is 6t – 3; if we take into account the pair (5, $2n$ – 5), the number of pairs is 6t – 2. For even numbers from the classes (2, 2, 1) and (5, 2, 2), if t increases by 1, then the number of pairs in the representation increases by 6. For even numbers of the form $2n$ = 22 + 60(t – 1) from the class (1, 2, 2), the number of pairs is 3t – 2; if we take into account two pairs (3, $2n$ – 3) and (5, $2n$ – 5), the number of pairs is 3t. For even numbers of the form $2n$ = 52 + 60(t – 1) from the class (4, 2, 1), the number of pairs is 3t – 1; if we take into account two pairs (3, $2n-3$) and (5, $2n-5$), the number of pairs is $3t+1$. For even numbers from the classes (1, 2, 2) and (4, 2, 1), if t increases by 1, then the number of pairs in the representation increases by 3. For even numbers of the form $2n = 62 + 60(t-1)$ from the class (3, 2, 2), the number of pairs is $3t$; if we take into account the pair (3, $2n-3$), the number of pairs is $3t+1$. For even numbers of the form $2n = 32 + 60(t-1)$ from the class (6, 2, 1), the number of pairs is $3t-2$; if we take into account the pair (3, $2n-3$), the number of pairs is $3t-1$. For even numbers from the classes (3, 2, 2) and (6, 2, 1), if t increases by 1, then the number of pairs in the representation increases by 3. For even numbers of the form $2n = 14 + 60(t-1)$ from the class (3, 3, 2), the number of pairs is $3t-2$; if we take into account the pair (3, $2n-3$), the number of pairs is $3t-1$. For even numbers of the form $2n = 44 + 60(t-1)$ from the class (6, 3, 1), the number of pairs is $3t-1$; if we take into account the pair (3, $2n-3$), the number of pairs is $3t$. For even numbers from the classes (3, 3, 2) and (6, 3, 1), if t increases by 1, then the number of pairs in the representation increases by 3. For even numbers of the form $2n = 24 + 60(t-1)$ from the class (2, 3, 1), the number of pairs is $6t-4$; if we take into account the pair (5, $2n-5$), the number of pairs is $6t-3$. For even numbers of the form $2n = 54 + 60(t-1)$ from the class (5, 3, 2), the number of pairs is $6t-1$; if we take into account the pair (5, $2n-5$), the number of pairs is $6t$. For even numbers from the classes (2, 3, 1) and (5, 3, 2), if t increases by 1, then the number of pairs in the representation increases by 6. For even numbers of the form $2n = 34 + 60(t-1)$ from the class (1, 3, 2), the number of pairs is $3t-1$; if we take into account two pairs (3, $2n-3$) and (5, $2n-5$), the number of pairs is $3t+1$. For even numbers of the form $2n = 64 + 60(t-1)$ from the class (4, 3, 1), the number of pairs is $3t$; if we take into account two pairs (3, $2n-3$) and (5, $2n-5$), the number of pairs is $3t+2$. For even numbers from the classes (1, 3, 2) and (4, 3, 1), if t increases by 1, then the number of pairs in the representation increases by 3. For even numbers of the form $2n = 46 + 60(t-1)$ from the class (1, 4, 2), the number of pairs is $3t-1$; if we take into account two pairs (3, $2n-3$) and (5, $2n-5$), the number of pairs is $3t+1$. For even numbers of the form $2n = 16 + 60(t-1)$ from the class (4, 4, 1), the number of pairs is $3t-3$; if we take into account two pairs (3, $2n-3$) and (5, $2n-5$), the number of pairs is $3t-1$. For even numbers from the classes (1, 4, 2) and (4, 4, 1), if t increases by 1, then the number of pairs in the representation increases by 3. For even numbers of the form $2n = 26 + 60(t-1)$ from the class (3, 4, 2), the number of pairs is $3t-1$; if we take into account the pair (3, $2n-3$), the number of pairs is $3t$. For even numbers of the form $2n = 56 + 60(t-1)$ from the class (6, 4, 1), the number of pairs is $3t$; if we take into account the pair (3, $2n-3$), the number of pairs is $3t+1$. For even numbers from the classes (3, 4, 2) and (6, 4, 1), if t increases by 1, then the number of pairs in the representation increases by 3. For even numbers of the form $2n = 36 + 60(t-1)$ from the class (2, 4, 1), the number of pairs is $6t-3$; if we take into account the pair (5, $2n-5$), the number of pairs is $6t-2$. For even numbers of the form $2n = 66 + 60(t-1)$ from the class (5, 4, 2), the number of pairs is $6t$; if we take into account the pair (5, $2n-5$), the number of pairs is $6t+1$. For even numbers from the classes (2, 4, 1) and (5, 4, 2), if t increases by 1, then the number of pairs in the representation increases by 6. For even numbers of the form $2n = 48 + 60(t-1)$ from the class (2, 5, 1), the number of pairs is $6t-2$; if we take into account the pair (5, $2n-5$), the number of pairs is $6t-1$. For even numbers of the form $2n = 18 + 60(t-1)$ from the class (5, 5, 2), the number of pairs is $6t-5$; if we take into account the pair (5, $2n-5$), the number of pairs is $6t-4$. For even numbers from the classes (2, 5, 1) and (5, 5, 2), if t increases by 1, then the number of pairs in the representation increases by 6. For even numbers of the form $2n = 58 + 60(t-1)$ from the class (1, 5, 2), the number of pairs is $3t$; if we take into account the pair (5, $2n-5$), the number of pairs is $3t+1$. For even numbers of the form $2n = 28 + 60(t-1)$ from the class (4, 5, 1), the number of pairs is $3t-2$; if we take into account the pair (5, $2n-5$), the number of pairs is $3t-1$. For even numbers from the classes (1, 5, 2) and (4, 5, 1), if t increases by 1, then the number of pairs in the representation increases by 3. For even numbers of the form $2n = 38 + 60(t-1)$ from the class (3, 5, 2), the number of pairs is $3t-1$. For even numbers of the form $2n = 68 + 60(t-1)$ from the class (6, 5, 1), the number of pairs is $3t$. For even numbers from the classes (3, 5, 2) and (6, 5, 1), if t increases by 1, then the number of pairs in the representation increases by 3. Thus, when t increases by 1, which corresponds to the increase of the even number by 60, the number of pairs in the representation increases by 3, 4, 6, or 8, depending on the class to which the even number belongs. Thus, all odd numbers (composite and prime) participating in the representation of a given even number belong to one of the arithmetic progressions determined by the class to which the even number belongs. At the same time, any composite number belongs to one of the arithmetic progressions formed by generators, the number of which depends on the order of magnitude of an even number. It should be borne in mind that the generators participating in the formation of composite numbers located on the right of the center of the representation do not necessarily coincide with the prime representation generators (prime numbers on the left of the center), which depends on the class of even numbers (see below). Generators of composite numbers in the representation of even number $2n$ are prime numbers (remind that numbers 1, 2, 3, and 5, as well as numbers that are divisible by 3 or 5 are excluded), that do not exceed $[(2n)^{1/2}]$, where $[(2n)^{1/2}]$ is the integer part of the number $(2n)^{1/2}$. The number of generators is a small part of the total number of pairs in the representation of the number $2n$. For the number of generators, we have the following upper estimate $n_a < [[(2n)^{1/2}]]/4$. Therefore, in addition to generators, there are other primes on the left of the center outside the boundary value $[(2n)^{1/2}]$. Analysis of even numbers from different classes confirms that the number of generators is less than the total number of prime representation generators (primes to the left of the center in the representation of $2n$). For large t, with the increase of the parameter t by 1, which corresponds to the increase of the even number by 60, i.e. from $2n$ to $2n + 60$, the number of generators remains the same as for $2n$ or can increase by 1 due to the "accumulated effect", as a rule, when the number $2n + 60$ coincides with the large period $60g$ for certain generator g. This new generator in the representation of $2n + 60$ forms a pair (g, g^2) or $(2n + 60 - g, g^2)$. The value of the numbers in this pair depends on the class of even numbers and the order of magnitude of the numbers. For example, the pair $(7, 7^2)$ first appears in the representation of 56; a pair $(11, 11^2)$ – in the representation of the number 132; a pair $(13, 13^2)$ – in the representation of the number 182; a pair $(17, 17^2)$ – in the representation of the number 306, etc., a pair $(59, 59^2)$ – in the representation of the number 3540, a pair $(61, 61^2)$ – in the representation of the number 3782, etc. Analysis of even numbers from different classes allows us to establish the validity of Lemma 5: "Any even number $2n > 154$ is not divisible by all its generators; moreover, any even number $2n > 168$ is less than the product of its generators". Since the number of prime representation generators is more than the number of generators, Lemma 5 is undoubtedly valid for the prime representation generators. Analysis of the block structure of the representation of even numbers for different classes (see below) shows that Corollary 1 is valid: "There is no even number $2n$ satisfying one of the conditions $2n \geq n_0$ or $2n \geq n_0 + 60$, that is divisible by all its prime representation generators (primes to the left of the center of the representation); moreover, any even number satisfying one of these conditions is less than the product of its prime representation generators". The first part of Corollary 1 is valid under the condition $2n \geq n_0 + 60$, if $n_0 = 10, 12, 22, 14, 16$, and under the condition $2n \geq n_0$ for all other n_0 (see the distribution of even numbers by classes). The second part of Corollary 1 is valid under the condition $2n \geq n_0 + 60$, if $n_0 = 10, 20, 12, 22, 32, 14, 16, 18, 28$, and under the condition $2n \geq n_0$ for all other n_0. Taking into account additional generators 3 and 5, the estimates of Lemma 5 and Corollary 1 are shifted towards lower values. The first part of Lemma 5 is valid for $2n > 30$, and the second – for $2n > 120$. Corollary 1 is valid in this case under the condition $2n \geq n_0$; the exception is the values $n_0 = 12$ and $n_0 = 16$, for which the second part of Corollary 1 is fulfilled under the condition $2n \geq n_0 + 60$.

IV. Block structure of representations of even numbers

Consider the structure of representation for even numbers of a certain class. We use Lemma 4, since such a representation is more convenient than using Lemma 3. Divide the representation of an arbitrary even number $2n$ from this class into blocks. The first block can include pairs $(7, 2n - 7)$, $(11, 2n - 11)$, $(13, 2n - 13)$, etc., $(61, 2n - 61)$. If some generators are absent in the representation of numbers of this class, then the corresponding pairs are excluded. The second block is obtained from the first block by increasing the first number in each pair by 60, at the same time, the second number in each pair is decreasing by 60. So the second block can include pairs $(67, 2n - 7 - 60)$, $(71, 2n - 11 - 60)$, etc., $(121, 2n - 61 - 60)$. The third block is obtained from the first one by increasing the first number in each pair by 120, at the same time, the second number in each pair is decreasing by 120. So the third block can include pairs $(127, 2n - 7 - 120)$, $(131, 2n - 11 - 120)$, etc., $(181, 2n - 61 - 120)$. In all pairs in which the first numbers differ by 60 or a multiple of 60, the second numbers belong to the same arithmetic progression. Subsequent blocks are obtained similarly. The structure of the blocks depends on the class of even numbers, while the last block may be incomplete. The representation of even numbers of one class has vertical-horizontal symmetry. Thus, with a sequential increase of the number $2n$ by 60, the numbers to the right of the center in each pair of the first block in the representation of $2n$ coincide with the numbers to the right of the center of the second block in the representation of $2n + 60$, with the numbers to the right of the center of the third block in the representation of $2n + 120$, etc. Simultaneously, the numbers to the right of the center in each pair of the first block in the representation of $2n + 60$ coincide with the numbers to the right of the center of the second block in the representation of $2n + 120$, etc. With a sequential decrease of the number $2n$ by 60, the numbers to the right of the center in each pair of the second block in the representation of $2n$ coincide with the numbers to the right of the center of the first block in the representation of $2n - 60$. Simultaneously, the numbers to the right of the center in each pair of the third block in the representation of $2n$ coincide with the numbers to the right of the center of the second block in the representation of $2n - 60$, with the numbers to the right of the center of the first block in the representation of $2n - 120$. Similar relations take place for other blocks in the representation of even numbers that differ by 60 or a multiple of 60. Lemma 6 is valid: "The numbers on the right of the center in the representation of $2n(n_0, 60t)$ located in block $k + l$ coincide with the corresponding numbers on the right of the center in the representation of $2n(n_0, 60t) - 60l$ located in block k". Here k and l can take values 1, 2, 3, etc. The value of $k + l$ does not exceed the number of blocks in the representation of $2n$. Hereinafter, we accept that the numeration of blocks increases towards the center of the representation of the number $2n$. We determine the range of variation of quantities that influence the use of Lemma 6. The number of complete blocks in the representation of an arbitrary even number $2n = n_0 + 60(t - 1)$ is determined by the relation $N = \min\{[(n - g_{11})/60] + 1, [(n - g_{1m})/60] + 1\}$, where g_{11} and

g_{1m} are respectively the smallest and largest generators of the first block in the representation of even numbers of this class; $[\cdot]$ – integer part of a number in square brackets. If the equality $\{[(n - g_{11})/60] + 1 = [(n - g_{1m})/60] + 1$ holds, then all blocks are complete; if $\{[(n - g_{11})/60] + 1 > [(n - g_{1m})/60] + 1$, then the last block is incomplete. We put $t_0 = (2n - n_0)/60$. The numbers to the right of the center in the last (incomplete) block of the representation of the number $2n$ coincide with the numbers to the right of the center in the first block of the representation of the number $2n_{th}$. The numbers to the right of the center in the penultimate (complete) block of the representation of the number $2n$ coincide with the numbers to the right of the center in the first block of the representation of the number $2n_{th} + 60$. If t_0 is an even number, then $2n_{th} = n_0 + 60t_0/2$; if t_0 is an odd number, then $2n_{th} = n_0 + 60 + 60(t_0 - 1)/2$. We designate $2n_{th}$ – the threshold number, which is the closest even number to the number $2n/2$ from the considered class of even numbers. Calculations assume that $n_0 \geq 30$, so that not to consider exceptions. If $n_0 < 30$, then the value of n_0 should be increased by 60, for example, if $n_0 = 10$, then we take $n_0 = 10 + 60 = 70$, which corresponds to the displacement of the beginning of the counting for numbers of this class. We write explicitly the representation generators (numbers to the left of the center) of the first block for even numbers from different classes. In the representation of even numbers from classes that differ only in the parity of the center, the representation generators in the first block are the same. For classes (1, 1, 2) and (4, 1, 1) including even numbers of the form $10 + 60(t-1)$ and $40 + 60(t-1)$, respectively, the representation generators in the first block are 11, 17, 23. 29, 41, 47, 53, 59; so there are 8 numbers. For classes (3, 1, 2) and (6, 1, 1) including even numbers of the form $50 + 60(t-1)$ and $20 + 60(t-1)$, respectively, the representation generators in the first block are 7, 13, 19, 31, 37, 43, 49, 61; so there are 8 numbers. For classes (2, 1, 1) and (5, 1, 2) including even numbers of the form $60 + 60(t-1)$ and $30 + 60(t-1)$, respectively, the representation generators in the first block are 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61; so there are 16 numbers. For classes (2, 2, 1) and (5, 2, 2) including even numbers of the form $12 + 60(t-1)$ and $42 + 60(t-1)$, respectively, the representation generators in the first block are 11, 13, 19, 23, 29, 31, 41, 43, 49, 53, 59, 61; so there are 12 numbers. For classes (1, 2, 2) and (4, 2, 1) including even numbers of the form $22 + 60(t-1)$ and $52 + 60(t-1)$, respectively, the representation generators in the first block are 11, 23, 29, 41, 53, 59; so there are 6 numbers. For classes (3, 2, 2) and (6, 2, 1) including even numbers of the form $62 + 60(t-1)$ and $32 + 60(t-1)$, respectively, the representation generators in the first block are 13, 19, 31, 43, 49, 61; so there are 6 numbers. For classes (3, 3, 2) and (6, 3, 1) including even numbers of the form $14 + 60(t-1)$ and $44 + 60(t-1)$, respectively, the representation generators in the first block are 7, 13, 31, 37, 43, 61; so there are 6 numbers. For classes (2, 3, 1) and (5, 3, 2) including even numbers of the form $24 + 60(t-1)$ and $54 + 60(t-1)$, respectively, the representation generators in the first block are 7, 11, 13, 17, 23, 31, 37, 41, 43, 47, 53, 61; so there are 12 numbers. For classes (1, 3, 2) and (4, 3, 1) including even numbers of the form $34 + 60(t-1)$ and $64 + 60(t-1)$, respectively, the representation generators in the first block are 11, 17, 23, 41, 47, 53; so there are 6 numbers. For classes (1, 4, 2) and (4, 4, 1) including even numbers of the form $46 + 60(t-1)$ and $16 + 60(t-1)$, respectively, the representation generators in the first block are 17, 23, 29, 47, 53, 59; so there are 6 numbers. For classes (3, 4, 2) and (6, 4, 1) including even numbers of the form $26 + 60(t-1)$ and $56 + 60(t-1)$, respectively, the representation generators in the first block are 7, 13, 19, 37, 43, 49; so there are 6 numbers. For classes (2, 4, 1) and (5, 4, 2) including even numbers of the form $36 + 60(t-1)$ and $66 + 60(t-1)$, respectively, the representation generators in the first block are 7, 13, 17, 19, 23, 29, 37, 43, 47, 49, 53, 59; so there are 12 numbers. For classes (2, 5, 1) and (5, 5, 2) including even numbers of the form $48 + 60(t-1)$ and $18 + 60(t-1)$, respectively, the representation generators in the first block are 7, 11, 17, 19, 29, 31, 37, 41, 47, 49, 59, 61; so there are 12 numbers. For classes (1, 5, 2) and (4, 5, 1) including even numbers of the form $58 + 60(t-1)$ and $28 + 60(t-1)$, respectively, the representation generators in the first block are 11, 17, 29, 41, 47, 59; so there are 6 numbers. For classes (3, 5, 2) and (6, 5, 1) including even numbers of the form $38 + 60(t-1)$ and $68 + 60(t-1)$, respectively, the representation generators are 7, 19, 31, 37, 49, 61; so there are 6 numbers. Thus, classes (2, 1, 1) and (5, 1, 2) each have 16 representation generators (numbers to the left of center) in the first block, that are the same for both classes. Pairs of classes (1, 1, 2) and (4, 1, 1), as well as (3, 1, 2) and (6, 1, 1) have 8 generators, that are the same for the classes of each pair. The pairs of classes (2, 2, 1) and (5, 2, 2), as well as (2, 3, 1) and (5, 3, 2), (2, 4, 1) and (5, 4, 2), (2, 5, 1) and (5, 5, 2) have 12 generators that are the same for the classes of each pair. The pairs of classes (1, 2, 2) and (4, 2, 1), (3, 2, 2) and (6, 2, 1), (3, 3, 2) and (6, 3, 1), (1, 3, 2) and (4, 3, 1), (1, 4, 2) and (4, 4, 1), (3, 4, 2) and (6, 4, 1), (1, 5, 2) and (4, 5, 1), (3, 5, 2) and (6, 5, 1), have 6 generators that are the same for the classes of each pair. The composition and number of representation generators in the first block completely determine the structure of other blocks in representation of even numbers. Therefore, the representation of even numbers in each pair of classes with the same number of representation generators in the first block has the same block structure of representation, which is different from the structure in other pairs of classes. Pairs of classes with the same number and composition of representation generators in the first block have the same distribution of endings for representation generators (numbers to the left of the center of the representation) in all blocks. If the number of generators in the first block is 16, then in the first and subsequent complete blocks there are 4 generators with the end 7, 4 generators with the end 1, 4 generators with the end 3 and 4 generators with the end 9 (taking into account the number 49). If the number of generators is 12, then the distribution depends on the ends of the even numbers. For even numbers with the end 2, in the first and subsequent complete blocks, there are 4 generators with the end 1, 4 generators with the end 3 and 4 generators with the end 9, but there are not generators with the end 7. For even numbers with the end 4, in the first and subsequent complete blocks, there are 4 generators with the end 7, 4 generators with the end 1 and 4 generators with the end 3, but there are not generators with the end 9. For even numbers with the end 6, in the first and subsequent complete blocks, there are 4 generators with the end 7, 4 generators with the end 3 and 4 generators with the end 9, but there are not generators with the end 1. For even numbers with the end 8, in the first and subsequent complete blocks, there are 4 generators with the end 7, 4 generators with the end 1 and 4 generators with the end 9, but there are not generators with the end 3. If the number of generators is 8, then in the first and subsequent complete blocks there are 2 generators with the end 7, 2 generators with the end 1, 2 generators with the end 3 and 2 generators with the end 9. If the number of generators is 6, then the distribution depends on the ends of the even numbers. For even numbers with the end 2, in the first and subsequent complete blocks, there are 2 generators with the end 1, 2 generators with the end 3 and 2 generators with the end 9, but there are not generators with the end 7. For even numbers with the end 4, in the first and subsequent complete blocks, there are 2 generators with the end 7, 2 generators with the end 1 and 2 generators with the end 3, but there are not generators with the end 9. For even numbers with the end 6, in the first and subsequent complete blocks, there are 2 generators with the end 7, 2 generators with the end 3 and 2 generators with the end 9, but there are not generators with the end 1. For even numbers with the end 8, in the first and subsequent complete blocks, there are 2 generators with the end 7, 2 generators with the end 1 and 2 generators with the end 9, but there are not generators with the end 3. The numbers on the right of the center, which form pairs with numbers on the left of the center, belong to arithmetic progressions defined according to Lemma 4. For even numbers of a given class, all representation generators (numbers on the left of the center) obtained from the same generator of the first block by Lemma 4 form conjugate pairs with numbers on the right of the center belonging to the same arithmetic progression. In the general case, for even numbers of the form $2n = n_0 + 60(t-1)$ from an arbitrary class, an arbitrary representation generator (number to the left of the center) g_i forms a pair with a number (prime or composite) belonging to the arithmetic progression with the initial term $n_0 - g_i$,, if $n_0 > g$, or progressions with an initial term $n_0 + 60 - g_i$, if $n_0 < g_i$. Thus, all odd numbers participating in the representation of even numbers of a given class belong to one of the arithmetic progressions. The period (difference) of each progression is 60, and the first member coincides with one of representation generators in the first block. We call these progressions basic for this class. The number of basic progressions is equal to the number of representation generators in the first block. Prime representation generators (prime numbers to the left of center) in the representation of an even number $2n$ do not necessarily coincide with generators that participate in the formation of composite numbers to the right of center. A coincidence is possible in classes with the maximum number of representation generators in the first block. For other classes such a coincidence may not take place, since the generators participating in the formation of composite numbers to the right of the center can also be other prime numbers from the first and subsequent blocks for classes with a maximum (largest) number of generators equal to 16. For example, for classes (1, 5, 2) and (4, 5, 1), the numbers on the left of the center in the first block are 11, 17, 29, 41, 47, 59 (see above), but the generators that participate in the formation of composite numbers on the right of the center can also be 7, 13, 19, 23, 31, 37, 43, 53, 61, which, of course, depends on the order of magnitude of an even number. However, this does not lead to an increase of the number of composite numbers in the representation of even numbers from classes with the number of representation generators in the first block less than 16. From the above analysis, it follows that if the number of representation generators in the first block is less than 16, then the representation of even numbers of this class will not have prime and composite numbers to the left of the center, corresponding to the generators of the first block with missing endings. Moreover, the representation of numbers of this class will not have prime and composite numbers to the right of the center of the representation from arithmetic progressions corresponding to the missing generators. Therefore, in classes with the number of representation generators in the first block less than 16, the number of composite numbers to the left and to right of the center of representation in all blocks is no more than in classes with 16 generators. We give in an explicit form the basic arithmetic progressions for all classes of numbers. We write them in the form (g_{i1}, g_{k1}). The first number in brackets g_{i1} corresponds to the first member of the progression that includes the representation generators (numbers to the left of the center) in the representation of the even number. The number g_{i1} is equal to one of the generators of the first block. The second number in brackets g_{k1} corresponds to the first member of the progression that includes numbers to the right of the center in the representation of the even number. The number g_{k1} is also equal to one of the generators of the first block and depends on g_{i1}. In the basic progressions, $g_{i1} \leq g_{k1}$. The pair (g_{i1}, g_{k1}) is a pair of conjugate numbers in the representation of the number n_0 or $n_0 + 60$. The sum of the first members of the progressions ($g_{i1} + g_{k1}$) is equal to the smallest even number from the considered class. If moreover generators 3 and/or 5 are taken into account, the progression relates only to the member on the right of the center, since multiples of 3 or 5 are excluded. If $n_0 > g_{i1}$ and $g_{i1} \leq n_0/2$, then $g_{k1} = n_0 - g_{i1}$; if $n_0 < g_{i1}$ and $g_{i1} \leq n_0/2 + 30$, then $g_{k1} = n_0 + 60 - g_{i1}$. If $n_0 > g_{i1}$ and $g_{i1} > n_0/2$, then the pair (g_{i1}, g_{k1}) has the form $(g_{i1}, n_0 + 60 - g_{i1})$. If $n_0 < g_{i1}$ and $g_{i1} > n_0/2 + 30$, then the pair (g_{i1}, g_{k1}) has the form $(g_{i1}, n_0 + 120 - g_{i1})$. In the last two cases, the pairs are included in the representation of the even numbers $n_0 + 60$ and $n_0 + 120$, respectively, but the numbers g_{i1} and g_{k1} from these pairs may not be the first members of the basic progressions. For classes (1, 1, 2) and (4, 1, 1), the number of basic progressions is 8. For the class (1, 1, 2), they have the form (11, 59), (17, 53), (23, 47), (29, 41). Additional generator 3 corresponds to the progression (3, 7). For the class (4, 1, 1), basic progressions have the form (11, 29), (17, 23), (41, 59), (47, 53). Additional generator 3 corresponds to the progression (3, 37). For classes (3, 1, 2) and (6, 1, 1), the number of basic progressions is 8. For the class (3, 1, 2), basic progressions have the form (7, 43), (13, 37), (19, 31), (49, 61). Additional generator 3 corresponds to the progression (3, 47). For the class (6, 1, 1), the basic progressions have the form (7, 13), (19, 61), (31, 49), (37, 43). Additional generator 3 corresponds to the progression (3, 17). For classes (2, 1, 1) and (5, 1, 2), the number of basic progressions is 16. For the class (2, 1, 1), they have the form (7, 53), (11, 49), (13, 47), (17, 43), (19, 41), (23, 37), (29, 31), (59, 61). For the class (5, 1, 2), the basic progressions have the form (7, 23), (11, 19), (13, 17), (29, 61), (31, 59), (37, 53), (41, 49), (43, 47). For classes (2, 2, 1) and (5, 2, 2), the number of basic progressions is 12. For the class (2, 2, 1), they have the form (11, 61), (13, 59), (19, 53), (23, 49), (29, 43), (31, 41). Additional generator 5 corresponds to the progression (5, 7). For the class (5, 2, 2), basic progressions have the form (11, 31), (13, 29), (19, 23), (41, 61), (43, 59), (49, 53). Additional generator 5 corresponds to the progression (5, 37). For classes (1, 2, 2) and (4, 2, 1), the number of basic progressions is equal to 6. For the class (1, 2, 2), basic progressions have the form (11, 11), (23, 59), (29, 53), (41, 41). Additional generators 3 and 5 correspond to the progression (3, 19) and (5, 17), respectively. For the class (4, 2, 1), the basic progressions have the form (11, 41), (23, 29), (53, 59). Additional generators 3 and 5 correspond to the progression (3, 49) and (5, 47), respectively. For classes (3, 2, 2) and (6, 2, 1), the number of basic progressions is equal to 6. For the class (3, 2, 2) they have the form (13, 49), (19, 43), (31, 31), (61, 61). Additional generator 3 corresponds to the progression (3, 59). For the class (6, 2, 1), the basic progressions have the form (13, 19), (31, 61), (43, 49). Additional generator 3 corresponds to the progression (3, 29). For classes (3, 3, 2) and (6, 3, 1), the number of basic progressions is equal to 6, For the class (3, 3, 2), they have the form (7, 7), (13, 61), (31, 43), (37, 37). Additional generator 3 corresponds to the progression (3, 11). For the class (6, 3, 1), basic progressions have the form (7, 37), (13, 31), (43, 61). Additional generator 3 corresponds to the progression (3, 41). For classes (2, 3, 1) and (5, 3, 2), the number of basic progressions is 12. For the class (2, 3, 1), they have the form (7, 17), (11, 13), (23, 61), (31, 53), (37, 47), (41, 43). Additional generator 5 corresponds to the progression (5, 19). For the class (5, 3, 2), basic progressions have the form (7, 47), (11, 43), (13, 41), (17, 37), (23, 31), (53, 61). Additional generator 5 corresponds to the progression (5, 49). For classes (1, 3, 2) and (4, 3, 1), the number of basic progressions is equal to 6. For the class (1, 3, 2), they have the form (11, 23), (17, 17), (41, 53), (47, 47). Additional generators 3 and 5 correspond to the progression (3, 31) and (5, 29), respectively. For the class (4, 3, 1), basic progressions have the form (11, 53), (17, 47), (23, 41). Additional generators 3 and 5 correspond to the progression (3, 61) and (5, 59), respectively. For classes (1, 4, 2) and (4, 4, 1), the number of basic progressions is equal to 6. For the class (1, 4, 2), they have the form (17, 29), (23, 23), (47, 59), (53, 53). Additional generators 3 and 5 correspond to the progression (3, 43) and (5, 41), respectively. For the class (4, 4, 1), basic progressions have the form (17, 59), (23, 53), (29, 47). Additional generators 3 and 5 correspond to the progression (3, 13) and (5, 11), respectively. For classes (3, 4, 2) and (6, 4, 1), the number of basic progressions is equal to 6. For the class (3, 4, 2), they have the form (7, 19), (13, 13), (37, 49), (43, 43). Additional generator 3 corresponds to the progression (3, 23). For the class (6, 4, 1), basic progressions have the form (7, 49), (13, 43), (19, 37). Additional generator 3 corresponds to the progression (3, 53). For classes (2, 4, 1) and (5, 4, 2), the number of basic progressions is 12. For the class (2, 4, 1), they have the form (7, 29), (13, 23), (17, 19), (37, 59), (43, 53), (47, 49). Additional generator 5 corresponds to the progression (5, 31). For the class (5, 4, 2), basic progressions have the form (7, 59), (13, 53), (17, 49), (19, 47), (23, 43), (29, 37). Additional generator 5 corresponds to the progression (5, 61). For classes (2, 5, 1) and (5, 5, 2), the number of basic progressions is 12. For the class (2, 5, 1), they have the form (7, 41), (11, 37), (17, 31), (19, 29), (47, 61), (49, 59). Additional generator 5 corresponds to the progression (5, 43). For the class (5, 5, 2), basic progressions have the form (7, 11), (17, 61), (19, 59), (29, 49), (31, 47), (37, 41). Additional generator 5 corresponds to the progression (5, 13). For classes (1, 5, 2) and (4, 5, 1), the number of basic progressions is equal to 6. For the class (1, 5, 2) they have the form (11, 47), (17, 41), (29, 29), (59, 59). Additional generator 5 corresponds to the progression (5, 53). For the class (4, 5, 1), basic progressions have the form (11, 17), (29, 59), (41, 47). Additional generator 5 corresponds to the progression (5, 23). For classes (3, 5, 2) and (6, 5, 1), the number of basic progressions is equal to 6. For the class (3, 5, 2) they have the form (7, 31), (19, 19), (37, 61), (49, 49). For the class (6, 5, 1), basic progressions have the form (7, 61), (19, 49), (31, 37). For classes of even numbers that differ only in the parity of the center, the first members of basic arithmetic progressions are either equal or differ by 30, according to Lemma 3. Thus, the representation of even numbers of an arbitrary class is completely determined by specifying several arithmetic progressions, the number of which is equal to the number of representation generators in the first block.

V. Proof of the Goldbach's binary conjecture

Consider the process of proof for numbers of an arbitrary class $2n = n_0 + 60(t-1)$. Designate g_{i1} are the representation generators (numbers to the left of the center) from the first block in the representation of even numbers of this class. The class of even numbers (see above) determines the number of generators and their composition. Each generator g_{i1} forms arithmetic progression of the form $g_i = g_{i1} + 60(t_{gi}-1)$, where $t_{gi} = 1$ for the first block, $t_{gi} = 2$ for the second block, and so on. For the classes of even numbers, the representation of which contains in the first block additional generators $g_{i1} = 3$ and/or $g_{i1} = 5$, we assume $t_{gi} = 1$ for these generators. According to Lemma 4, all numbers to the left of the center in the representation of $2n$ belong to one of these progressions. The last block may be incomplete, and only a part of the generators of the first block is involved in its formation. The numbers to the right of the center in the representation of even numbers of this class, that form pairs of conjugate numbers with the generators g_{i1} of the first block and with numbers to the left of the center of the other blocks obtained from g_{i1}, according to Lemma 4, belong to arithmetic progressions of the form $\delta_i = \Delta_i + 60(t-1) - 60(t_{gi}-1)$ with the first term $\Delta_i = n_0 - g_{i1}$, if $n_0 > g_{i1}$ or $\Delta_i = n_0 + 60 - g_{i1}$, if $n_0 < g_{i1}$. For generator 3, the progression has the form $n_0 - 3 + 60(t-1)$, and for generator 5, the progression has the form $n_0 - 5 + 60(t-1)$. For even numbers in the representation of which numbers $n_0 - 3 + 60(t-1)$ or $n_0 - 5 + 60(t-1)$ are primes, Goldbach's conjecture is obviously valid, and such numbers are excluded from consideration. For the rest of the even numbers, in the representation of which $n_0 - 3 + 60(t-1)$ and $n_0 - 5 + 60(t-1)$ are composite, we will not take into account the pairs $(3, n_0 - 3 + 60(t-1))$ and $(5, n_0 - 5 + 60(t-1))$, since this does not affect the generality of the proof. The representation of the number $2n$ of this class consists of the set of pairs $\{g_i, \delta_i\}$. We write the representation of the number $2n$ of this class in the form of the system of equations

$$g_i + \delta_i = 2n \tag{3}$$

where g_i are representation generators (numbers to the left of center of representation) and δ_i are numbers to the right of the center, forming pairs of conjugate numbers with the corresponding g_i. We write δ_i in the form $\delta_i = a_j'' g_j''$, then (3) takes the form

$$g_i + a_j'' g_j'' = 2n \tag{4}$$

where a_j'' are unknown coefficients (natural numbers), g_j'' are the generators of the number $2n$ participating in the formation of composite numbers to the right of the center of the representation, and then the corresponding $a_j'' \neq 1$, or g_j'' are primes that are not generators, and then the corresponding $a_j'' = 1$. The system of equations (4) has a block structure corresponding to the representation of the number $2n$. Transformations that do not change the number of coefficients a_j'' equal to 1, i.e. the number of pairs of prime conjugate numbers in the representation of the number $2n$, we will call permissible for the given system of equations. Systems of equations with the same number of coefficients a_j'' equal to 1 are equivalent. We will say that these systems have solution of the same type. In particular, the systems (4) corresponding to the representation of even numbers of the same class without pairs of prime conjugate numbers in all blocks, i.e. not having coefficients a_j'' equal to 1 are equivalent. Equivalent systems can be obtained from each other using permissible transformations. In particular, permissible transformations include: 1) for a given number $2n$, the exclusion of equations on the left side of which one or both terms are composite numbers; 2) the change of number $2n$ on the right side of all equations by 60 or a multiple of 60, if this does not change the number of coefficients a_j'' equal to 1 (in this case, of course, the left sides of the equations also change); 3) the combination of the first and second transformations. When comparing equivalent systems, the number of equations in (4) does not matter. Transformations 1) or 2) change the number of equations; transformation 3) allows us to keep the number of equations if needed. According to Lemma 6, the representation of the number $2n$ has vertical-horizontal symmetry. Therefore, a system similar to (4) simultaneously corresponds to the first block in the representation of even numbers $2n$, $2n - 60$, $2n - 120$, etc. up to $2n_{th}$. In this case, the second members δ_i in pairs of the representation of these even numbers are the same as in the corresponding blocks in representation of the number $2n$, and the representation generators g_i correspond to the first block and are the same for all even numbers from the considered class. We keep on the left-hand side of (4) only g_i, that are prime numbers, thus we exclude from the representation pairs in which the first number is composite (an exclusion is made only after obtaining a system of equations for a given even number). This transformation is permissible and does not affect the generality of the proof. If Goldbach's conjecture is not valid for the number $2n$, then all a_j'' are different from 1, and if this conjecture is valid for $2n$, then some a_j'' should equal 1, and the corresponding g_j'' are prime numbers. When we successively increase even number by 60 from $2n_{th}$ to $2n$, some of the numbers to the right of the center are replaced by new ones, and some of the numbers move to the left of the center of representation, so the number of primes to the left of the center increases. In particular, in the representation of an even number $n = 2n/2$, the set of numbers to the left and to right of the center coincides with the set of numbers to the left of the center in the representation of $2n$. The system of equations (4) is redundant, since the number of prime representation generators g_i (prime numbers to the left of the center) exceeds the number of generators g_j''. We will prove that for an arbitrary even number $2n$, system (4) cannot have a solution in which all coefficients a_j'' differ from 1, i.e., all numbers to the right of the center are formed by generators g_j'' and are composite. The proof is by induction. It is directly verified that Goldbach's conjecture is valid for even numbers $4 \le 2n \le 168$. Put $2n_1 = n_0 + 60t_1$ and $2n_2 = 2n_1 + 60$. Suppose that Goldbach's conjecture is valid for even numbers $2n(n_0, t) \le 2n(n_0, t_1)$, where the even numbers $2n$ and $2n_1$ belong to the same class. Then system (4) for any number $2n \le 2n_1$ has a solution, and some a_j'' are equal to 1. We prove that the conjecture is valid for the number $2n_2(n_0, t_2)$, where $t_2 = t_1 + 1$. Compare the representations of the numbers $2n_1$ and $2n_2$ using the results of the previous analysis. The number of representation generators (numbers to the left of the center) in the first block, depending on the class, is 16, 8, 12 or 6 (taking into account the number 49, which we then exclude from the system of equations (4)). In blocks of the representation of even numbers with the number of generators less than 16, there will be no primes and composite numbers to the left and to right of the center, which are included in the arithmetic progressions corresponding to the missing generators. Therefore, the number of composite numbers to the left and to right of the center of the representation in all blocks is no more than in the classes of even numbers with the maximum number of generators equal to 16. Depending on the class of even numbers, 16, 8, 12 or 6 new numbers appear to the right of the center in the representation $2n_2$ (we do not take into account the numbers 3 and 5) and at the same time 8, 4, 6 or 3 numbers located to the right of the center in the representation of $2n_1$ move to the left from the center in the representation of $2n_2$. So the number of pairs in the representation of $2n_2$ is increased by 8, 4, 6, or 3, respectively, depending on the class of considered even numbers. Suppose that the hypothesis is not valid for $2n_2$, then in system (4) all a_j'' should differ from 1. This is possible if one of the assumptions is satisfied: 1) in all blocks of the representation of $2n_2$ there are no primes to the right of the center, 2) there are such numbers, but prime representation generators (primes to the left of the center) form conjugate pairs only with composite numbers. We will show that both of these assumptions lead to a contradiction. We accept that the number of generators for $2n_1$ and $2n_2$ is the same. Otherwise, we can always exclude the pair formed by the new generator g from the representation of $2n_2$. The process of proof is the same for all classes. Consider the first assumption. We cannot use Bertrand's postulate, since pairs in which the first number is composite are excluded. If there are no prime numbers to the right of the center of the representation $2n_2$, then the representation of $2n_2$ contains no pairs of prime conjugate numbers and, consequently, the system of equations (4) corresponding to the representation of $2n_2$ has no coefficients a_j'' equal to 1. Hence, the system of equations (4) for the number $2n_2$ is equivalent to the system for the number $2n$, which is divisible by all its prime representation generators. Since the numbers $2n_2$ and $2n$ belong to the same class, they differ by 60 or a multiple of 60, and one can be obtained from another using permissible transformations. In this case, transformation 3) allows us to keep the number of equations of the system. But according to Corollary 1 of Lemma 5, there is no such number $2n$. Therefore, the first assumption is incorrect. It also follows from the first assumption that in the first block of representation of any number $2n$ from the interval $n_{th} + 60 \le 2n \le 2n_2$, all numbers to the right of the center are composite, i.e. the corresponding basic arithmetic progressions do not contain primes in this interval. Since the first assumption is incorrect, then in the first block of the representation of some even numbers from the interval $n_{th} + 60 \le 2n \le 2n_2$ to the right of the center there are always primes, and hence pairs of conjugate prime numbers, since all numbers to the left of the center are prime. Hence, it follows that the number of primes participating in the representation should increase as the even number $2n$ increases. Note that even numbers with no prime numbers to the right of the center in the first block exist. For example, it is take place for the number $2n_2 = 1923610$, which belongs to the class (1, 1, 2). However, in this case, as follows from the table of prime numbers, in the first block of representation of numbers 1923610

± 60 there are pairs of prime conjugate numbers. The largest numbers for given class that have this property are the numbers $2n_2$, which are divisible by all prime representation generators of the first block. In this case, $2n_2$ has the form $2n_2 = AG_1 + 60AG_1(t - 1)$, where G_1 is the product of prime representation generators of the first block, A is a coefficient (even number), the value of which depends on the considered class of even numbers. The smallest possible value of the coefficient A is 2. The number AG_1 belongs to the considered class, therefore it has the form $AG_1 = n_0 + 60(t_A - 1)$, which allows us to determine the value of the coefficient A. In particular, for the class (1, 1, 2), $AG_1 = 10 \times 751\,583\,152\,441$, here $A = 10$. The systems of equations (4) for such even numbers have a solution of the same type if in the first block of systems (4) all a_j'' are different from 1, and the remaining blocks have the same number of coefficients a_j'' equal to 1. In a more general case, primes to the right of the center may be absent in the first l blocks of the number $2n_2$, where l can take values 1, 2, 3, etc. This is equivalent to the fact that primes to the right of the center are missing in the first blocks of $l + 1$ even numbers $2n_2$, $2n_2 - 60$, $2n_2 - 120$, etc. up to $2n_2 - 60(l - 1)$, where $2n_2 - 60(l - 1)$ is less than $2n_{\text{th}}$. The value of l is limited by the condition that the number of prime representation generators located to the left of the center in l blocks should be less than the number of generators of $2n_2$. The value of l also depends on the class of even numbers; the number of prime representation generators located to the left of the center in l blocks should be undoubtedly less than the total number of prime representation generators (prime numbers to the left of the center) participating in the representation of $2n_2$. The systems of equations (4) for such even numbers have a solution of the same type if in the first l blocks of systems (4) all a_j'' are different from 1, and the remaining blocks have the same number of coefficients a_j'' equal to 1. The largest numbers for given class that have this property are the numbers $2n_2$, that are divisible by all prime representation generators of the first l blocks. In this case, $2n_2$ has the form $2n_2 = AG_l + 60AG_l(t - 1)$, where G_l is the product of prime representation generators of the first l blocks, A is a coefficient (even number), the value of which depends on the considered class of even numbers. The smallest possible value of the coefficient A is 2. The number AG_l belongs to the considered class, therefore it has the form $AG_l = n_0 + 60(t_A - 1)$, which allows us to determine the value of the coefficient A. From Lemma 6 it follows that in this case, the number $2n_1$ is not divisible by any representation generator from the first l blocks. In the system of equations (4), in the first blocks of representation of even numbers $2n_2$, $2n_1$, $2n_1 - 60$, etc. up to $2n_2 - 60(l - 1)$ all a_j'' differ from 1. In other words, in the basic arithmetic progressions, to which all numbers to the right of the center belong in the first blocks of the representation of the corresponding even numbers, there are no prime numbers. But of course, there are primes in the first blocks of representation of some even numbers from $2n_2 - 60l$ to $2n_{\text{th}}$, since the first assumption is wrong (see above). Now consider the second assumption. Suppose there are primes to the right of the center in the representation of $2n_2$, but the prime representation generators (prime numbers to the left of the center) form conjugate pairs only with composite numbers. The system of equations (4) for all such even numbers has a solution of the same type: in all blocks of equations (4), all a_j'' differ from 1. Consequently, the system of equations (4) for the number $2n_2$ is equivalent to the system for the number $2n$, which is divisible by all its prime representation generators. Since the numbers $2n_2$ and $2n$ belong to the same class, they differ by 60 or a multiple of 60, and one can be obtained from another using permissible transformations. In this case, transformation 3) allows us to keep the number of equations of the system. For such numbers $2n$, the system of equations (4) has the obvious solution: $g_j'' = g_i$, $2n = a_i g_i$, $1 + a_j'' = a_i$ which corresponds to the case when $2n$ is divisible by all its prime representation generators. But according to Corollary 1 of Lemma 5, such a number does not exist. Therefore, our second assumption is incorrect and the hypothesis is valid for the number $2n_2$, and hence for all numbers of this class. Since the class of even numbers was chosen arbitrarily, the binary hypothesis is valid for all even numbers. For numbers of different classes, the number of representation generators in the first block can vary, and hence the number of basic arithmetic progressions, as well as the number of conjugate pairs in the representation. For classes with the same generators, but different parity of the center, in the representation of even numbers with an odd center, an additional pair of conjugate prime numbers may appear if the center is a prime number. These differences should be taken into account when analyzing the representations of even numbers, but they are not essential for our proof of the Goldbach's binary conjecture.

VI. Connection of the binary hypothesis with the Landau problem

The second Landau problem (conjecture) consists in proving the statement that there are an infinite number of primes the distance between which is 2 (the so-called twin prime numbers). Obviously, there are always such pairs on the left of the center in the representation of an arbitrary even number. However, we cannot know with certainty whether new pairs of such numbers will appear on the right of the center when an even number increases. We are going to prove this statement, assuming that the binary hypothesis is valid. We accept that two odd (prime) numbers, the distance between which is 2, form a pair of conjugate numbers, so one number is located to the left of the center and the other to the right of the center in the representation of even number. This pair of numbers is adjacent to the center of the representation. They can also both be to the right of the center and be located in adjacent conjugate pairs. If we will increase an even number, then the second case is reduced to the first. Consider the first case, which simplifies the proof. In this case, a pair of prime conjugate numbers, the distance between which is 2, can have only the following endings (1, 3), (7, 9) or (9, 1). It follows from the previous analysis that three classes of even numbers satisfy these conditions. Class (2, 3, 1) includes even numbers of the form $2n_{13} = 24 + 60(t - 1)$, in the representation of which the pair of twin numbers has endings (1, 3). Class (2, 4, 1) includes even numbers of the form $2n_{79} = 36 + 60(t - 1)$, in the representation of which the pair of twin numbers has endings (7, 9). Class (2, 1, 1) includes even numbers of the form $2n_{91} = 60 + 60(t - 1)$, in the representation of which the pair of twin numbers has endings (9, 1). The first ending corresponds to the number on the left of the center, and the second one corresponds to the number on the right of the center. In the class of numbers (2, 1, 1), pairs of numbers, the distance between which is 2, belong to the basic arithmetic progressions $[29 + 30l_1, 31 + 30l_2]$. In class (2, 3, 1), such pairs belong to the basic arithmetic progressions $[11 + 30l_3, 13 + 30l_4]$. In class (2, 4, 1), such pairs belong to the basic arithmetic progressions $[17 + 30l_5, 19 + 30l_6]$. The first progression in brackets corresponds to the numbers on the left of the center in the representation of even number, and the second progression corresponds to the numbers on the right of the center. Parameters $l_1, l_2, \ldots, l_6$ can take values 0, 1, 2, etc. depending on the order of magnitude of an even number, i.e. on the value of the parameter t. It follows from Lemmas 1 and 3 that all twin primes (except 3 and 5) belong to these progressions. We will consider the arithmetic progressions of each class as independent. In each of them, one or both numbers can be prime numbers, or both numbers can be composite. To prove the validity of Landau's conjecture, it is sufficient that, with increasing t, pairs of twin prime numbers appear in progressions belonging to at least one class. Consider the course of the proof for the class of numbers (2, 1, 1). There are infinitely many prime numbers in the basic arithmetic progressions corresponding to this class. We will sequentially increase the parameters l_1 and l_2, which corresponds to an increase of the parameter t. Since Goldbach's conjecture is valid for all even numbers from this class, after a certain number of steps, a pair of prime conjugate numbers with endings (9, 1) will appear in the representation of the corresponding even number. Such pairs will appear further as l_1 and l_2 increase. Otherwise, in the representation of even numbers from the class (2, 1, 1), pairs of prime conjugate numbers could not take the endings (9, 1), which is absurd (illogical), since there is no reason to prefer one pair of endings to the other. This conclusion is valid, mutatis mutandis, for even numbers from other classes with other pairs of admissible endings. Thus, in each of the classes considered above, there are infinitely many pairs of twin prime numbers. This conclusion is undoubtedly valid if the three classes of even numbers (2, 1, 1), (2, 3, 1) and (2, 4, 1) are considered as complementary to each other.

VII. Conclusion

The considered proof of Goldbach's conjecture is based on the invariance of the system of equations (4), which describes the representation of even numbers of one class, to transformations that do not change the number of prime conjugate pairs. Representations of even numbers of one class depend on several basic arithmetic progressions and form a connected system. The system of equations (4), which describes the representation of even numbers of the same class, cannot have a solution in which there are no pairs of prime conjugate numbers. This conclusion is valid for numbers of an arbitrary class. From the validity of the Goldbach’s conjecture, it follows the validity of other assertions, in particular, the Legendre conjecture and the Landau conjecture.

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