

## Real Price Index and Alternative Methods of Weighting

Amit Kumar

Lecturer (Economics)

Govt. Intermediate College, Farsadi, Pauri Garhwal, Uttarakhand, India

---

### Abstract

We currently have a variety of price indexes, the most popular of which are the Paasche's price index, the Laspeyres' price index and the Fisher's price index. But there is no such price index yet that satisfies all the four tests of adequacy. The most successful price index of all time is Fisher's ideal price index, which satisfies three of the four tests of adequacy —unit test, factor reversal test, and time reversal test. But it fails to complete circular test. We present a new price index which will be called real price index. This price index satisfies all the four tests of adequacy - unit test, factor reversal test, time reversal test and Circular test. Real price index is a generalized measure of relative changes in the prices of goods and services during a given time period, effectively incorporating the effects of relative changes in the demand-quantities of these goods and services over this fixed time period. A direct and equal effect of the changes in both prices and quantities of the commodities is obvious on this price index. We compare fisher's price index with real price index to find out why fisher's price index fails to satisfy the circular test and to prove that fisher's price index is not always free of all kinds of bias. Real price index consists of 2 parts: - a summative part i.e. the expense-ratio ( $\Sigma P_1 Q_1 / \Sigma P_0 Q_0$ ) and the second, geometric part due to which we need a new weighting method - the proportional cum exponential weighting method. This weighting method appropriately affects both the parts of the real price index. At last, we can say that Fisher's Price Index is only an ideal price index while real price Index is a perfect price index.

**Keywords:** Real Price Index, Perfect Price Index, Fisher's Price Index, Geometric and Summative Parts of Real Price Index, Proportional cum Exponential Weighting Method, Commodities, Adequacy Tests.

---

Date of Submission: 28-04-2021

Date of Acceptance: 12-05-2021

---

### I. Introduction

There are various price indexes developed by the various genius economists, Statisticians and Mathematicians. Among them, the most discussed price indexes are the Paasche price index, the Laspeyres price index and the Fisher's price index. But there is no such price index yet that satisfies all the four tests of adequacy. The four tests of adequacy are unit test, time reversal test, factor reversal test and circular test. The most successful price index of all time is Fisher's ideal price index which satisfies three of the four tests of adequacy - unit test, factor reversal test, and time reversal test. But it fails to complete circular test. We describe here a new price index that satisfies all the four tests of adequacy. We shall call it real price index. This article explains reasons why Fisher's price index fails to meet the Circular test. This article proves that Fisher's Price Index is not always free of bias and explains reasons for it.

### II. Real Price Index

Real price index is a generalized measure of relative changes in the prices of goods and services during a given time period, effectively incorporating the effects of relative changes in the demand-quantities of these goods and services over this fixed time period. It gives equal importance to both the prices of commodities and their quantities demanded. Thus there are two factors that play an important role in determining real price index:- First, the relative changes in the prices of the commodities and second, Changes in the structure of expenditure on the commodities due to the changes in the relative importance and preferences of the commodities for the people who purchase them.

Since the changes in the quantities of the commodities affect real price index in the same way as the changes in their prices do. So it is very important that such commodities should be included in the basket of commodities for the determination of real price index which are very essential to the society, because :-

- There is relatively more stability in the demand for these commodities, due to which the changes in their prices significantly affect the purchasing power of the people of the society;
- Being very essential for the society, such commodities keep their place in the basket of commodities selected for real price index for a long time. Thus the selected commodities, that serve as the basis for

determination of real price index, remains more homogenous and therefore the conclusions based on comparative analysis of real price indices related to different time periods, will be more accurate and practical. Real price index highlights the biggest human bias in the formulation of a price index formula : “Trying to keep separate the effects of changes in quantities of the commodities from the effects of changes in their prices” and removes this human bias by including obviously the effects of changes in the quantities of commodities.

**1. Method to calculate real price index**

Real price index is a square root of the product of the ratio between the total expense in current year and that in base year ( $\Sigma P_1 Q_1 / \Sigma P_0 Q_0$ ) and the ratio between the geometric mean of price ratios and the geometric mean of quantity ratios  $[G.M.(P_1/P_0)]/[G.M.(Q_1/Q_0)]$ . It is multiplied by 100 to express it in the form of percentage.

Formula for calculating real price index can be written in many ways. The formula is as follows :-

$$\text{Real Price Index (RP}_{01}) = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0) \div G.M.(Q_1/Q_0)]\}} \times 100$$

----- Or -----

$$\text{Real Price Index (RP}_{01}) = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0)] \times [G.M.(Q_0/Q_1)]\}} \times 100$$

----- Or -----

$$\text{Real Price Index (RP}_{01}) = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1 Q_0) / G.M.(P_0 Q_1)]\}} \times 100$$

----- Or -----

$$\text{Real Price Index (RP}_{01}) = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times G.M.(P_1 Q_0 / P_0 Q_1)\}} \times 100$$

We can calculate real price index with the help of logarithm and antilogarithm. Formula with the use of logarithm and antilogarithm is as follows. It can be written in many ways.

$$\text{Real Price Index (RP}_{01}) = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times \text{Antilog} [1/N(\log(P_1/P_0)_1 + \log(P_1/P_0)_2 + \log(P_1/P_0)_3 + \dots + \log(P_1/P_0)_N)] \div \text{Antilog} [1/N(\log(Q_1/Q_0)_1 + \log(Q_1/Q_0)_2 + \log(Q_1/Q_0)_3 + \dots + \log(Q_1/Q_0)_N)]\}} \times 100$$

..... Or .....

$$\text{Real Price Index (RP}_{01}) = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times \text{Antilog} [1/N(\log(P_1/P_0)_1 + \log(P_1/P_0)_2 + \log(P_1/P_0)_3 + \dots + \log(P_1/P_0)_N)] \times \text{Antilog} [1/N(\log(Q_0/Q_1)_1 + \log(Q_0/Q_1)_2 + \log(Q_0/Q_1)_3 + \dots + \log(Q_0/Q_1)_N)]\}} \times 100$$

..... Or .....

$$\text{Real Price Index (RP}_{01}) = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times \text{Antilog} [1/N(\log(P_1 Q_0)_1 + \log(P_1 Q_0)_2 + \log(P_1 Q_0)_3 + \dots + \log(P_1 Q_0)_N)] \div \text{Antilog} [1/N(\log(P_0 Q_1)_1 + \log(P_0 Q_1)_2 + \log(P_0 Q_1)_3 + \dots + \log(P_0 Q_1)_N)]\}} \times 100$$

..... Or .....

$$\text{Real Price Index (RP}_{01}) = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times \text{Antilog} [1/N(\log(P_1 Q_0 / P_0 Q_1)_1 + \log(P_1 Q_0 / P_0 Q_1)_2 + \log(P_1 Q_0 / P_0 Q_1)_3 + \dots + \log(P_1 Q_0 / P_0 Q_1)_N)]\}} \times 100$$

Explanation with the help of an example

Example:1:

commodities	P <sub>0</sub>	Q <sub>0</sub>	P <sub>1</sub>	Q <sub>1</sub>	P <sub>0</sub> Q <sub>0</sub>	P <sub>1</sub> Q <sub>1</sub>	(P <sub>1</sub> /P <sub>0</sub> )	(Q <sub>1</sub> /Q <sub>0</sub> )
A	05	12	06	06	60	36	6/5	1/2
B	06	10	08	03	60	24	4/3	3/10
C	08	06	10	05	48	50	5/4	5/6
D	02	09	06	10	18	60	03	10/9
E	09	04	05	02	36	10	5/9	1/2
N= 05,					$\Sigma P_0 Q_0$ = 222,	$\Sigma P_1 Q_1$ = 180,	G.M. (P <sub>1</sub> /P <sub>0</sub> ) = 1.27226,	G.M. (Q <sub>1</sub> /Q <sub>0</sub> ) = 0.5866,

$$G.M.(P_1/P_0) = (6/5 \times 4/3 \times 5/4 \times 3 \times 5/9)^{(1/N)}$$

$$G.M.(P_1/P_0) = (10/3)^{(1/5)}$$

$$G.M.(P_1/P_0) = 1.2722596365$$

And

$$G.M.(Q_1/Q_0) = (1/2 \times 3/10 \times 5/6 \times 10/9 \times 1/2)^{(1/N)}$$

$$G.M.(Q_1/Q_0) = (5/72)^{(1/5)}$$

$$G.M.(Q_1/Q_0) = 0.5865803382$$

Now,

$$\text{Real Price Index (RP}_{01}) = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0) \div G.M.(Q_1/Q_0)]\}} \times 100$$

$$\text{Real Price Index (RP}_{01}) = \sqrt{\{(180/222) \times (1.2722596365 \div 0.5865803382)\}} \times 100$$

$$\text{Real Price Index (RP}_{01}) = \sqrt{(0.810810811 \times 2.1689435422)} \times 100$$

$$\text{Real Price Index (RP}_{01}) = \sqrt{(1.7586028721)} \times 100$$

$$\text{Real Price Index (RP}_{01}) = 1.3261232492 \times 100$$

$$\text{Real Price Index (R}_{01}) = 132.61$$

**2. Real price index and tests of adequacy**

There are four available tests of adequacy to choose a suitable price index. These tests are called tests of consistency also. Now, On the basis of these tests, we will examine the validity of real price index.

**2.1. Unit test**

*For this it is necessary that the method of index construction should be independent of the problem of units. According to this test prices and quantities are presented in any unit.<sup>[1]</sup>*

**2.1.1. Unit test and real price index**

$$RP_{01} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0)] \times [G.M.(Q_0/Q_1)]\}}$$

It is clear from the formula that the real price index is independent of the problem of units.

Hence, the Real Price Index satisfies the unit test.

**2.2. Time reversal test**

*It is clear from this test that if the current year index is based on the base year and the base year index is based on the current year, then both should be inverse of each other and the product of both should be one (1).<sup>[1]</sup>*

Symbolically, the presentation of the test is as follows: -

$$P_{01} \times P_{10} = 1 \text{ (Excluding the factor 100 from each index)}$$

Where  $P_{01}$  shows the price index for the current year (1) based on the base year (0) and  $P_{10}$  shows the price index for the current year (0) based on the base year (1).

**2.2.1. Time reversal test and real price index**

$$RP_{01} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0)] \times [G.M.(Q_0/Q_1)]\}}$$

And

$$RP_{10} = \sqrt{\{(\Sigma P_0 Q_0 / \Sigma P_1 Q_1) \times [G.M.(P_0/P_1)] \times [G.M.(Q_1/Q_0)]\}}$$

Now, by multiplying  $RP_{01}$  by  $RP_{10}$ , we get :-

$$RP_{01} \times RP_{10} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0)] \times [G.M.(Q_0/Q_1)]\}} \times \sqrt{\{(\Sigma P_0 Q_0 / \Sigma P_1 Q_1) \times [G.M.(P_0/P_1)] \times [G.M.(Q_1/Q_0)]\}}$$

$$RP_{01} \times RP_{10} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times (\Sigma P_0 Q_0 / \Sigma P_1 Q_1) \times [G.M.(P_1/P_0)] \times [G.M.(P_0/P_1)] \times [G.M.(Q_0/Q_1)] \times [G.M.(Q_1/Q_0)]\}}$$

$$RP_{01} \times RP_{10} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_1 Q_1 \times \Sigma P_0 Q_0 / \Sigma P_0 Q_0) \times 01 \times 01\}}$$

$$RP_{01} \times RP_{10} = 01$$

Hence, the real price index satisfies the time reversal test.

**2.3. Factor reversal test**

*It says that the product of price index and quantity index should be equal to the expense ratio.<sup>[1]</sup>*

If  $P_{01}$  is a price index with base year (0) for current year (1) and  $Q_{01}$  is a quantity index with base year (0) for current year (1). The product of  $P_{01}$  and  $Q_{01}$  must be equal to the expense ratio  $(\Sigma P_1 Q_1 / \Sigma P_0 Q_0)$ .

Symbolically the presentation of the test is as follows:-

$$P_{01} \times Q_{01} = \Sigma P_1 Q_1 / \Sigma P_0 Q_0 \text{ (Excluding the factor 100 from each index)}$$

**2.3.1. Factor reversal test and real price index**

$$RP_{01} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0)] \times [G.M.(Q_0/Q_1)]\}}$$

And

$$RQ_{01} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(Q_1/Q_0)] \times [G.M.(P_0/P_1)]\}}$$

Now, by multiplying  $RP_{01}$  by  $RQ_{01}$ , we get :-

$$RP_{01} \times RQ_{01} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0)] \times [G.M.(Q_0/Q_1)]\}} \times$$

$$\sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(Q_1/Q_0)] \times [G.M.(P_0/P_1)]\}}$$

$$RP_{01} \times RQ_{01} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0)] \times [G.M.(Q_0/Q_1)] \times (\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(Q_1/Q_0)] \times [G.M.(P_0/P_1)]\}}$$

$$RP_{01} \times RQ_{01} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times (\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0)] \times [G.M.(P_0/P_1)] \times [G.M.(Q_0/Q_1)] \times [G.M.(Q_1/Q_0)]\}}$$

$$RP_{01} \times RQ_{01} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times (\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times 01 \times 01\}}$$

$$RP_{01} \times RQ_{01} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0)^2\}}$$

$$RP_{01} \times RQ_{01} = \Sigma P_1 Q_1 / \Sigma P_0 Q_0$$

Hence, the real price index satisfies the factor reversal test.

**2.4. Circular test**

*This test is an extended form of time reversal test. According to this test the indices are prepared in the form of cycles and the product of them should be one (1).<sup>[1]</sup>*

If an index is calculated for the year (1) on the base year (0) and another index is calculated for the year (2) on the base year (1), and still another index is calculated for the year (0) on the base year (2), then the product of these three indices must be equal to one.

Symbolically the presentation of the test is as follows:

$$P_{01} \times P_{12} \times P_{23} \dots \dots \times P_{N0} = 1 \text{ (Excluding the factor 100 from each index)}$$

**2.4.1. Circular test and real price index**

$$RP_{01} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0)] \times [G.M.(Q_0/Q_1)]\}}$$

And

$$RP_{12} = \sqrt{\{(\Sigma P_2 Q_2 / \Sigma P_1 Q_1) \times [G.M.(P_2/P_1)] \times [G.M.(Q_1/Q_2)]\}}$$

And

$$RP_{20} = \sqrt{\{(\Sigma P_0 Q_0 / \Sigma P_2 Q_2) \times [G.M.(P_0/P_2)] \times [G.M.(Q_2/Q_0)]\}}$$

Now, by multiplying  $RP_{01}$  by  $RP_{12}$  and  $RP_{20}$ , we get :-

$$RP_{01} \times RP_{12} \times RP_{20} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0)] \times [G.M.(Q_0/Q_1)]\}} \times \sqrt{\{(\Sigma P_2 Q_2 / \Sigma P_1 Q_1) \times [G.M.(P_2/P_1)] \times [G.M.(Q_1/Q_2)]\}} \times \sqrt{\{(\Sigma P_0 Q_0 / \Sigma P_2 Q_2) \times [G.M.(P_0/P_2)] \times [G.M.(Q_2/Q_0)]\}}$$

$$RP_{01} \times RP_{12} \times RP_{20} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0)] \times [G.M.(Q_0/Q_1)] \times (\Sigma P_2 Q_2 / \Sigma P_1 Q_1) \times [G.M.(P_2/P_1)] \times [G.M.(Q_1/Q_2)] \times (\Sigma P_0 Q_0 / \Sigma P_2 Q_2) \times [G.M.(P_0/P_2)] \times [G.M.(Q_2/Q_0)]\}}$$

$$RP_{01} \times RP_{12} \times RP_{20} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times (\Sigma P_2 Q_2 / \Sigma P_1 Q_1) \times (\Sigma P_0 Q_0 / \Sigma P_2 Q_2) \times [G.M.(P_1/P_0)] \times [G.M.(Q_0/Q_1)] \times [G.M.(P_2/P_1)] \times [G.M.(Q_1/Q_2)] \times [G.M.(P_0/P_2)] \times [G.M.(Q_2/Q_0)]\}}$$

$$RP_{01} \times RP_{12} \times RP_{20} = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_1 Q_1) \times (\Sigma P_2 Q_2 / \Sigma P_2 Q_2) \times (\Sigma P_0 Q_0 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0)] \times [G.M.(Q_0/Q_1)] \times [G.M.(P_2/P_1)] \times [G.M.(Q_1/Q_2)] \times [G.M.(P_0/P_2)] \times [G.M.(Q_2/Q_0)]\}}$$

$$RP_{01} \times RP_{12} \times RP_{20} = \sqrt{\{01 \times 01 \times 01 \times [G.M.(P_1/P_0)] \times [G.M.(P_2/P_1)] \times [G.M.(P_0/P_2)] \times [G.M.(Q_0/Q_1)] \times [G.M.(Q_1/Q_2)] \times [G.M.(Q_2/Q_0)]\}}$$

$$RP_{01} \times RP_{12} \times RP_{20} = \sqrt{\{[G.M.(P_1/P_0)] \times [G.M.(P_2/P_1)] \times [G.M.(P_0/P_2)] \times [G.M.(Q_0/Q_1)] \times [G.M.(Q_1/Q_2)] \times [G.M.(Q_2/Q_0)]\}}$$

$$RP_{01} \times RP_{12} \times RP_{20} = \sqrt{(01 \times 01)}$$

$$RP_{01} \times RP_{12} \times RP_{20} = 01$$

Hence, real price index satisfies the circular test.

Thus it is clear that real price index is perfect in itself as it satisfies all the four adequacy tests. So, the real price index can also be called perfect price index.

**3. Real price index and Fisher's price index : A comparative study**

Now we will do a comparative study of Fisher's price index and real price index.

3.1. Fisher's price index formula is :-  $\sqrt{[(\Sigma P_1 Q_1 / \Sigma P_0 Q_1) \times (\Sigma P_1 Q_0 / \Sigma P_0 Q_0)]}$  and real price index formula is  $\sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1/P_0) \div G.M.(Q_1/Q_0)]\}}$ . It is clear from view of both the formulae that real price index uses the geometric mean method more intensively than Fisher's price index does.

3.2. we can write Fisher's price index formula as :-  $\sqrt{[(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times (\Sigma P_1 Q_0 / \Sigma P_0 Q_1)]}$ . Now we can see clearly that Fisher's price index has 2 summative parts, one of which is the expense-ratio and the other is the ratio of the sums of irrelevant products, while real price index has 2 parts :- one summative part i.e. the expense-ratio  $(\Sigma P_1 Q_1 / \Sigma P_0 Q_0)$  and the other, geometric part  $[G.M.(P_1/P_0) \div G.M.(Q_1/Q_0)]$ . Thus we can clearly see that both parts of the real price index are relevant.

3.3. Fisher's price index fails to meet the circular test, while real price index satisfies all the four tests of adequacy.

The real price index is always free of all kinds of biases while Fisher's price index is not always free of biases. We clarify it on the basis of the two following conditions and with the help of some examples.

The two conditions given below are satisfied by the only price index which satisfies the four tests of adequacy and so real price index satisfies the two conditions. But Fisher's price index satisfies 1<sup>st</sup> condition but is unable to satisfy 2<sup>nd</sup> condition. The two conditions are as follows :-

Condition :1: When the quantity ratio  $(Q_1/Q_0)$  is the same for each commodity and the price ratio  $(P_1/P_0)$  for each commodity also is the same, the price index  $(P_{01})$  should be always equal to the price ratio  $(P_1/P_0)$  because the same quantity ratio  $(Q_1/Q_0)$  for each commodity naturally and logically refers to similarity in preferences and relative importance of commodities for the people of the society in both the years. In such a situation, the impact of changes in the prices of those commodities on the purchasing power of the people of the society is same between the base year and the current year. See example : 2.

Example:2:

(When the quantity ratio  $(Q_1/Q_0)$  is the same for each commodity and the price ratio  $(P_1/P_0)$  for each commodity also is the same.)

commodities	P <sub>0</sub>	Q <sub>0</sub>	P <sub>1</sub>	Q <sub>1</sub>	P <sub>0</sub> Q <sub>0</sub>	P <sub>1</sub> Q <sub>1</sub>	P <sub>0</sub> Q <sub>1</sub>	P <sub>1</sub> Q <sub>0</sub>
A	05	02	15	04	10	60	20	30
B	06	04	18	08	24	144	48	72
C	09	05	27	10	45	270	90	135
D	02	11	06	22	22	132	44	66
E	12	10	36	20	120	720	240	360
F	14	20	42	40	280	1680	560	840

G	03	25	09	50	75	450	150	225
H	08	09	24	18	72	432	144	216
I	07	06	21	12	42	252	84	126
J	10	02	30	04	20	120	40	60

$$N = 10, \quad \Sigma P_0 Q_0 = 710, \quad \Sigma P_1 Q_1 = 4260, \quad \Sigma P_0 Q_1 = 1420, \quad \Sigma P_1 Q_0 = 2130$$

$$G.M.(P_0 Q_1) = (20 \times 48 \times 90 \times 44 \times 240 \times 560 \times 150 \times 144 \times 84 \times 40)^{(1/N)}$$

$$G.M.(P_0 Q_1) = (37081621463040000000)^{(1/10)}$$

$$G.M.(P_0 Q_1) = 90.5557165939$$

And

$$G.M.(P_1 Q_0) = (30 \times 72 \times 135 \times 66 \times 360 \times 840 \times 225 \times 216 \times 126 \times 60)^{(1/N)}$$

$$G.M.(P_1 Q_0) = (2138313150167040000000)^{(1/10)}$$

$$G.M.(P_1 Q_0) = 135.8335748909$$

NOW

$$\text{Real price index } (RP_{01}) = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1 Q_0) / G.M.(P_0 Q_1)]\}}$$

$$\text{Real price index } (RP_{01}) = \sqrt{\{(4260/710) \times (135.8335748909 / 90.5557165939)\}}$$

$$\text{Real price index } (RP_{01}) = \sqrt{(06 \times 1.5)} = \sqrt{(09)}$$

$$\text{Real price index } (RP_{01}) = 03$$

And

$$\text{Fisher's price index } (F_{01}) = \sqrt{[(\Sigma P_1 Q_1 / \Sigma P_0 Q_1) \times (\Sigma P_1 Q_0 / \Sigma P_0 Q_0)]}$$

$$\text{Fisher's price index } (F_{01}) = \sqrt{[(4260/1420) \times (2130/710)]}$$

$$\text{Fisher's price index } (F_{01}) = \sqrt{(3 \times 3)} = \sqrt{(09)}$$

$$\text{Fisher's price index } (F_{01}) = 03$$

In Example: 2,

$$\text{Fisher's price index } (F_{01}) = \text{real price index } (RP_{01}) = \text{price ratio } (P_1/P_0) = 03.$$

Both the Fisher price index and real price index meet condition 1.

So here, both the price indexes have no bias.

Condition : 2 : When the quantity ratio ( $Q_1/Q_0$ ) is not the same for each commodity but the price ratio ( $P_1/P_0$ ) for each commodity is the same, the price index ( $P_{01}$ ) is not always equal to the price ratio ( $P_1/P_0$ ) because It depends on how the expenditure on commodities by people changes. The variations in the quantity ratios ( $Q_1/Q_0$ ) naturally and logically refers to a variation in preferences and relative importance of commodities for the society between the base year and the current year. In such a situation, the impact of changes in the prices of those commodities on the purchasing power of the people of the society is not the same between the two years. See example 3 and 4.

Example:3:

(When the quantity ratio ( $Q_1/Q_0$ ) is not the same for each commodity but the price ratio ( $P_1/P_0$ ) for each commodity is the same.

Commodities	P <sub>0</sub>	Q <sub>0</sub>	P <sub>1</sub>	Q <sub>1</sub>	P <sub>0</sub> Q <sub>0</sub>	P <sub>1</sub> Q <sub>1</sub>	P <sub>0</sub> Q <sub>1</sub>	P <sub>1</sub> Q <sub>0</sub>
A	02	48	01	12	96	12	24	48
B	04	12	02	15	48	30	60	24
C	08	15	04	03	120	12	24	60
D	06	12	03	14	72	42	84	36
E	08	21	04	06	168	24	48	84
F	06	16	03	06	96	18	36	48
G	01	48	0.5	48	48	24	48	24

$$N=07, \quad \Sigma P_0 Q_0 = 648, \quad \Sigma P_1 Q_1 = 162, \quad \Sigma P_0 Q_1 = 324, \quad \Sigma P_1 Q_0 = 324$$

$$G.M.(P_0 Q_1) = (24 \times 60 \times 24 \times 84 \times 48 \times 36 \times 48)^{(1/N)}$$

$$G.M.(P_0 Q_1) = (240789749760)^{(1/7)}$$

$$G.M.(P_0 Q_1) = 42.2618286703$$

and

$$G.M.(P_1 Q_0) = (48 \times 24 \times 60 \times 36 \times 84 \times 48 \times 24)^{(1/N)}$$

$$G.M.(P_1 Q_0) = (240789749760)^{(1/7)}$$

$$G.M.(P_1 Q_0) = 42.2618286703$$

Now,

$$\text{Real price index } (RP_{01}) = \sqrt{\{(\Sigma P_1 Q_1 / \Sigma P_0 Q_0) \times [G.M.(P_1 Q_0) / G.M.(P_0 Q_1)]\}}$$

$$\text{Real price index } (RP_{01}) = \sqrt{\{(162/648) \times (42.2618286703 / 42.2618286703)\}}$$

$$\text{Real price index } (RP_{01}) = \sqrt{\{(1/4) \times (01)\}} = \sqrt{(1/4)}$$

$$\text{Real price index } (RP_{01}) = 1/2$$

and

$$\text{Fisher's price index } (F_{01}) = \sqrt{[(\sum P_1 Q_1 / \sum P_0 Q_1) \times (\sum P_1 Q_0 / \sum P_0 Q_0)]}$$

$$\text{Fisher's price index } (F_{01}) = \sqrt{[(162/324) \times (324/648)]} = \sqrt{(1/4)}$$

$$\text{Fisher's price index } (F_{01}) = 1/2$$

in Example:3

$$\text{Fisher's price index } (F_{01}) = \text{Real Price Index } (RP_{01}) = \text{Price Ratio } (P_1/P_0) = 1/2.$$

Hence, there is no bias in Fisher's price index.

Example:4:

(Again, when the quantity ratio ( $Q_1/Q_0$ ) is not the same for each commodity but the price ( $P_1/P_0$ ) for each commodity is the same.)

Commodities	P <sub>0</sub>	Q <sub>0</sub>	P <sub>1</sub>	Q <sub>1</sub>	P <sub>0</sub> Q <sub>0</sub>	P <sub>1</sub> Q <sub>1</sub>	P <sub>0</sub> Q <sub>1</sub>	P <sub>1</sub> Q <sub>0</sub>
A	02	10	04	10	20	40	20	40
B	04	05	08	20	20	160	80	40
C	05	18	10	22	90	220	110	180
D	07	20	14	20	140	280	140	280
E	09	32	18	28	288	504	252	576
F	06	15	12	18	90	216	108	180
G	10	16	20	12	160	240	120	320
H	12	15	24	20	180	480	240	360
I	03	30	06	35	90	210	105	180
J	08	25	16	50	200	800	400	400
N= 10,					$\sum P_0 Q_0$ = 1278,	$\sum P_1 Q_1$ = 3150,	$\sum P_0 Q_1$ = 1575,	$\sum P_1 Q_0$ = 2556

$$\text{G.M.}(P_0 Q_1) = (20 \times 80 \times 110 \times 140 \times 252 \times 108 \times 120 \times 240 \times 105 \times 400)^{(1/N)}$$

$$\text{G.M.}(P_0 Q_1) = (81116046950400000000)^{(1/10)}$$

$$\text{G.M.}(P_0 Q_1) = 123.2851244189$$

and

$$\text{G.M.}(P_1 Q_0) = (40 \times 40 \times 180 \times 280 \times 576 \times 180 \times 320 \times 360 \times 180 \times 400)^{(1/N)}$$

$$\text{G.M.}(P_1 Q_0) = (69347447930880000000000)^{(1/10)}$$

$$\text{G.M.}(P_1 Q_0) = 192.3548089473$$

Now,

$$\text{Real price index } (RP_{01}) = \sqrt{\{(\sum P_1 Q_1 / \sum P_0 Q_0) \times [\text{G.M.}(P_1 Q_0) / \text{G.M.}(P_0 Q_1)]\}}$$

$$\text{Real price index } (RP_{01}) = \sqrt{\{(3150/1278) \times (192.3548089473/123.2851244189)\}}$$

$$\text{Real price index } (RP_{01}) = \sqrt{\{(2.4647887324) \times (1.5602434588)\}}$$

$$\text{Real price index } (RP_{01}) = \sqrt{(3.8456704971)}$$

$$\text{Real price index } (RP_{01}) = 1.9610381172$$

And

$$\text{Fisher's price index } (F_{01}) = \sqrt{[(\sum P_1 Q_1 / \sum P_0 Q_1) \times (\sum P_1 Q_0 / \sum P_0 Q_0)]}$$

$$\text{Fisher's Price Index } (F_{01}) = \sqrt{[(3150/1575) \times (2556/1278)]}$$

$$\text{Fisher's price index } (F_{01}) = \sqrt{(2 \times 2)}$$

$$\text{Fisher's price index } (F_{01}) = 02$$

In example:4,

$$\text{Real price index } (RP_{01}) = 1.9610381172 < \text{Fisher's price index } (F_{01}) = \text{price ratio } (P_1/P_0) = 02.$$

It is clear in this example that Fisher's price index is free from the influence of the demand-quantities of commodities selected for price index determination, which makes Fisher's Price Index ( $F_{01}$ ) equal to the Price Ratio ( $P_1/P_0$ )(i.e. 02).

Hence, Fisher's price index here is biased.

Whereas the effect of the demand quantities of selected commodities for price index determination on the real price index is obvious due to which real price index ( $RP_{01}$ ) is less than the price ratio ( $P_1/P_0$ ).

On the basis of the above two conditions, it is clear that Fisher's price index is not always free of bias whereas real price index always free of bias. Thus we can say that if Fisher's price index is different from real price index, it will suffer from bias. See example 5 and 6.

Example: 5

(When the quantity ratio ( $Q_1/Q_0$ ) is not the same for each commodity and the price ratio ( $P_1/P_0$ ) for each commodity is also not the same.)

commodities	P <sub>0</sub>	Q <sub>0</sub>	P <sub>1</sub>	Q <sub>1</sub>	P <sub>0</sub> Q <sub>0</sub>	P <sub>1</sub> Q <sub>1</sub>	P <sub>0</sub> Q <sub>1</sub>	P <sub>1</sub> Q <sub>0</sub>
A	06	07	04	10	42	40	60	28
B	07	19	02	06	133	12	42	38
C	03	11	04	19	33	76	57	44

D	11	04	10	06	44	60	66	80
E	09	05	12	09	45	108	81	60
F	05	08	05	18	40	90	90	40
G	08	09	06	15	72	90	120	54
H	12	07	08	07	84	56	84	56
N= 08,					$\Sigma P_0Q_0$ = 493,	$\Sigma P_1Q_1$ = 532,	$\Sigma P_0Q_1$ = 600,	$\Sigma P_1Q_0$ = 400

G.M.(P<sub>0</sub>Q<sub>1</sub>) = (60 × 42 × 57 × 66 × 81 × 90 × 120 × 84)<sup>(1/8)</sup>

G.M.(P<sub>0</sub>Q<sub>1</sub>) = (696638371968000)<sup>(1/8)</sup>

G.M.(P<sub>0</sub>Q<sub>1</sub>) = 71.6763570252

and

G.M.(P<sub>1</sub>Q<sub>0</sub>) = (28 × 38 × 44 × 80 × 60 × 40 × 54 × 56)<sup>(1/8)</sup>

G.M.(P<sub>1</sub>Q<sub>0</sub>) = (27181744128000)<sup>(1/8)</sup>

G.M.(P<sub>1</sub>Q<sub>0</sub>) = 47.7842380168

Now

Real price index (RP<sub>01</sub>) = √{(ΣP<sub>1</sub>Q<sub>1</sub>/ΣP<sub>0</sub>Q<sub>0</sub>) × [G.M.(P<sub>1</sub>Q<sub>0</sub>)/G.M.(P<sub>0</sub>Q<sub>1</sub>)]}

Real price index (RP<sub>01</sub>) = √{(532/493) × (47.7842380168/71.6763570252)}

Real price index (RP<sub>01</sub>) = √{(1.0791075051) × (0.66666666667)} = √(0.7194050034)

Real price index (RP<sub>01</sub>) = 0.8481774598

and

Fisher's price index (F<sub>01</sub>) = √[(ΣP<sub>1</sub>Q<sub>1</sub>/ΣP<sub>0</sub>Q<sub>1</sub>) × (ΣP<sub>1</sub>Q<sub>0</sub>/ΣP<sub>0</sub>Q<sub>0</sub>)]

Fisher's price index (F<sub>01</sub>) = √[(532/600) × (400/493)]

Fisher's price index (F<sub>01</sub>) = √[(0.88666666667) × (0.8113590264)] = √(0.7194050034)

Fisher's price index (F<sub>01</sub>) = 0.8481774598

In example : 5,

Fisher's price index (F<sub>01</sub>) = real price index (RP<sub>01</sub>) = 0.8481774598.

Hence, there is no bias in Fisher's price index.

Example:6:

(Again, when the quantity ratio (Q<sub>1</sub>/Q<sub>0</sub>) is not the same for each commodity and the price ratio (P<sub>1</sub>/P<sub>0</sub>) for each commodity is also not the same.)

Commodities	P <sub>0</sub>	Q <sub>0</sub>	P <sub>1</sub>	Q <sub>1</sub>	P <sub>0</sub> Q <sub>0</sub>	P <sub>1</sub> Q <sub>1</sub>	P <sub>0</sub> Q <sub>1</sub>	P <sub>1</sub> Q <sub>0</sub>
A	10	05	02	10	50	20	100	10
B	10	10	04	05	100	20	50	40
C	06	30	10	35	180	350	210	300
D	08	20	12	25	160	300	200	240
E	12	40	15	35	480	525	420	600
F	20	30	25	25	600	625	500	750
G	25	20	18	18	500	324	450	360
H	15	15	16	20	225	320	300	240
I	20	12	30	16	240	480	320	360
J	02	25	08	25	50	200	50	200

N= 10,					$\Sigma P_0Q_0$ = 2585,	$\Sigma P_1Q_1$ = 3164,	$\Sigma P_0Q_1$ = 2600,	$\Sigma P_1Q_0$ = 3100
--------	--	--	--	--	----------------------------	----------------------------	----------------------------	---------------------------

G.M.(P<sub>0</sub>Q<sub>1</sub>) = (100 × 50 × 210 × 200 × 420 × 500 × 450 × 300 × 320 × 50)<sup>(1/10)</sup>

G.M.(P<sub>0</sub>Q<sub>1</sub>) = (95256000000000000000)<sup>(1/10)</sup>

G.M.(P<sub>0</sub>Q<sub>1</sub>) = 198.5588432417

and

G.M.(P<sub>1</sub>Q<sub>0</sub>) = (10 × 40 × 300 × 240 × 600 × 750 × 360 × 240 × 360 × 200)<sup>(1/10)</sup>

G.M.(P<sub>1</sub>Q<sub>0</sub>) = (8062156800000000000000)<sup>(1/10)</sup>

G.M.(P<sub>1</sub>Q<sub>0</sub>) = 195.2743154584

Now,

Real price index (RP<sub>01</sub>) = √{(ΣP<sub>1</sub>Q<sub>1</sub>/ΣP<sub>0</sub>Q<sub>0</sub>) × [G.M.(P<sub>1</sub>Q<sub>0</sub>)/G.M.(P<sub>0</sub>Q<sub>1</sub>)]}

Real price index (RP<sub>01</sub>) = √{(3164/2585) × (195.2743154584/198.5588432417)}

Real price index (RP<sub>01</sub>) = √{(1.2239845261) × (0.9834581642)}

Real price index (RP<sub>01</sub>) = √(1.2037375751)

Real price index (RP<sub>01</sub>) = 1.0971497505

And

Fisher's price index (F<sub>01</sub>) = √[(ΣP<sub>1</sub>Q<sub>1</sub>/ΣP<sub>0</sub>Q<sub>1</sub>) × (ΣP<sub>1</sub>Q<sub>0</sub>/ΣP<sub>0</sub>Q<sub>0</sub>)]

Fisher's price index (F<sub>01</sub>) = √[(3164/2600) × (3100/2585)]

Fisher's price index  $(F_{01}) = \sqrt{\{(1.2169230769) \times (1.1992263056)\}} = \sqrt{1.4593661657}$   
 Fisher's price index  $(F_{01}) = 1.2080422864$

In example:6,

Fisher's Price Index  $(F_{01}) = 1.2080422864 > \text{Real Price Index (RP}_{01}) = 1.0971497505$ .

Hence, Fisher's price index here is biased.

The comparative study of Fisher's price index and real price index makes it clear that Fisher's price index is not always free of biases. The main reason for this is that like other price indices, it also does not always give equal importance to the prices of commodities and their quantities.

**4. Calculation of real price index and methods of weighting**

With the help of the methods of weighting, the weights assigned to the selected commodities for calculating the price index are not determined, but they are related to the process of applying the weights given to the selected commodities in calculating the price index. The weight given to a selected commodity for the price index depends on how much essential that particular commodity is to the people of the society.

Here, we will study alternative methods of weighting especially in the context of real price index. But first we will explain why the currently prevalent weighting method i.e. proportional weighting method is inappropriate for real price index and after that we will describe alternative methods of weighting and try to find out which of the alternative methods of weighting is the best one for real price index.

**4.1. 'The currently prevalent method of weighting : the proportional weighting method' and calculation of real price index**

According to this method, the prescribed weights are used as ratios. This method can be understood by the following example :-

Example:7:

Wei- ghts (In%)	Comm- odities.	P <sub>0</sub>	Q <sub>0</sub>	P <sub>1</sub>	Q <sub>1</sub>	P <sub>0</sub> Q <sub>0</sub>	P <sub>1</sub> Q <sub>1</sub>	P <sub>1</sub> Q <sub>0</sub>	P <sub>0</sub> Q <sub>1</sub>	P <sub>0</sub> Q <sub>0</sub> with weights as ratios 'P <sub>0</sub> Q <sub>0</sub> '	P <sub>1</sub> Q <sub>1</sub> with weights as ratios 'P <sub>1</sub> Q <sub>1</sub> '	P <sub>1</sub> Q <sub>0</sub> with weights as ratios 'P <sub>1</sub> Q <sub>0</sub> '	P <sub>0</sub> Q <sub>1</sub> with weights as ratios 'P <sub>0</sub> Q <sub>1</sub> '
15	A	05	03	06	10	15	60	18	50	15×15%=02.25	09.0	18×15%=02.7	07.50
10	B	06	03	08	15	18	120	24	90	18×10%=01.80	12.0	24×10%=02.4	09.00
25	C	08	06	10	12	48	120	60	96	48×25%=12.00	30.0	60×25%=15.0	24.00
35	D	02	20	06	14	40	84	120	28	40×35%=14.00	29.4	120×35%=42.0	09.80
15	E	09	20	05	06	180	30	100	54	180×15%=27.00	04.5	100×15%=15.0	08.10
N= 05, ('P <sub>0</sub> Q <sub>1</sub> ')										$\Sigma$ 'P <sub>0</sub> Q <sub>0</sub> '	$\Sigma$ 'P <sub>1</sub> Q <sub>1</sub> '	G.M. ('P <sub>1</sub> Q <sub>0</sub> ')	G.M. ('P <sub>0</sub> Q <sub>1</sub> ')
										= 57.05,	= 84.9	=09.0657,	=10.5159

Real price index  $(RP_{01}) = \sqrt{\{(\Sigma 'P_1Q_1' / \Sigma 'P_0Q_0') \times [G.M. ('P_1Q_0') / G.M. ('P_0Q_1')] \}} \times 100$

Real price index  $(RP_{01}) = \sqrt{\{(84.90/57.05) \times (09.0657/10.5159)\}} \times 100$

Real price index  $(RP_{01}) = \sqrt{(1.4881682734 \times 0.8620970144)} \times 100$

Real price index  $(RP_{01}) = \sqrt{(1.2829454254)} \times 100$

Real price index  $(RP_{01}) = 1.1326718083 \times 100$

Real price index  $(RP_{01}) = 113.2672$

**4.1.1. Demerit of the proportional weighting method**

The weights given according to the proportional weighting method do not affect the geometric part  $[G.M.(P_1Q_0)/G.M.(P_0Q_1)]$  of real price index. We can see this fact clearly in the above example, where  $[G.M.(P_1Q_0)/G.M.(P_0Q_1)] = 49.953192444/57.943817934 = 0.8620970144 = [G.M.('P_1Q_0')/G.M.('P_0Q_1')]$

Therefore, it is not a good idea to use the proportional weighting method in calculating real price index.

**4.2. 'First alternative method of weighting : the exponential weighting method' and calculation of real price index**

According to this method, the prescribed weights are used as exponents and not as ratios. The use of this method can be understood with the help of the following example :-

Example: 8:

Wei- ghts P <sub>0</sub> Q <sub>1</sub> with weights as exponents	Comm- odities	P <sub>0</sub> Q <sub>0</sub> with weights as exponents	P <sub>1</sub> Q <sub>1</sub> with weights as exponents	P <sub>1</sub> Q <sub>0</sub> with weights as exponents.
---	---------------	---	---	--

(In%) 'P <sub>0</sub> Q <sub>1</sub> '		P <sub>0</sub>	Q <sub>0</sub>	P <sub>1</sub>	Q <sub>1</sub>	P <sub>0</sub> Q <sub>0</sub>	P <sub>1</sub> Q <sub>1</sub>	P <sub>1</sub> Q <sub>0</sub>	P <sub>0</sub> Q <sub>1</sub>	'P <sub>0</sub> Q <sub>0</sub> '	'P <sub>1</sub> Q <sub>1</sub> '	'P <sub>1</sub> Q <sub>0</sub> '
15.7982	A	05	03	06	10	15	60	18	50	(15) <sup>(0.15)</sup> =1.5011	1.8481.	(18) <sup>(0.15)</sup> =1.5427
1.5683	B	06	03	08	15	18	120	24	90	(18) <sup>(0.10)</sup> =1.3351	1.6141	(24) <sup>(0.10)</sup> =1.3741
3.1302	C	08	06	10	12	48	120	60	96	(48) <sup>(0.25)</sup> =2.6321	3.3098	(60) <sup>(0.25)</sup> =2.7832
3.21	D	02	20	06	14	40	84	120	28	(40) <sup>(0.35)</sup> =3.6368.	4.7152	(120) <sup>(0.35)</sup> = 5.3421
1.8191	E	09	20	05	06	180	30	100	54	(180) <sup>(0.15)</sup> =2.1792.	1.6656	(100) <sup>(0.15)</sup> =1.9953
N=05, G.M. <sub>d</sub> ('P <sub>0</sub> Q <sub>1</sub> ')**										Σ'P <sub>0</sub> Q <sub>0</sub>	Σ'P <sub>1</sub> Q <sub>1</sub> '	G.M. <sub>d</sub> ('P <sub>1</sub> Q <sub>0</sub> ')*
										=11.2843,	= 13.1528,	=62.8873,
										=51.5468		

\* G.M.<sub>d</sub> ('P<sub>1</sub>Q<sub>0</sub>') = Product of (P<sub>1</sub>Q<sub>0</sub>) s with different exponents.

\*\* G.M.<sub>d</sub> ('P<sub>0</sub>Q<sub>1</sub>') = Product of (P<sub>0</sub>Q<sub>1</sub>) s with different exponents

Real price index (RP<sub>01</sub>) = √{(Σ'P<sub>1</sub>Q<sub>1</sub>'/Σ'P<sub>0</sub>Q<sub>0</sub>') × [G.M.<sub>d</sub>('P<sub>1</sub>Q<sub>0</sub>')/G.M.<sub>d</sub>('P<sub>0</sub>Q<sub>1</sub>')]} × 100

Real price index (RP<sub>01</sub>) = √{(13.1528/11.2843) × (62.8873/51.5468)} × 100

Real price index (RP<sub>01</sub>) = √{(1.1655840415 × 1.2200041665)} × 100

Real price index (RP<sub>01</sub>) = √{(1.422017387)} × 100

Real price index (RP<sub>01</sub>) = 1.1924837051 × 100

Real price index (RP<sub>01</sub>) = 119.2484

#### 4.2.1. Demerit of the exponential weighting method

The weights given according to the exponential weighting method improperly affect the expense-ratio (ΣP<sub>1</sub>Q<sub>1</sub>/ΣP<sub>0</sub>Q<sub>0</sub>) i.e. the summative part of real price index.

#### 4.3. 'Second alternative method of weighting : the proportional cum exponential weighting method' and calculation of real price index

According to the proportional cum exponential weighting method, the assigned weights are applied on the expense ratio (ΣP<sub>1</sub>Q<sub>1</sub>/ΣP<sub>0</sub>Q<sub>0</sub>) i.e. the summative part of the real price index with the proportional weighting method and on the geometric part [GM(P<sub>1</sub>Q<sub>0</sub>)/GM(P<sub>0</sub>Q<sub>1</sub>)] with the exponential weighting method.

The use of this method can be understood with the help of the following example: -

Example:9:

Wei- ghts	Comm- odities								P <sub>0</sub> Q <sub>0</sub> with weights as ratios	P <sub>1</sub> Q <sub>1</sub> with weights as ratios	P <sub>1</sub> Q <sub>0</sub> with weights as exponents	P <sub>0</sub> Q <sub>1</sub> with weights as exponents	
(In%)	P <sub>0</sub>	Q <sub>0</sub>	P <sub>1</sub>	Q <sub>1</sub>	P <sub>0</sub> Q <sub>0</sub>	P <sub>1</sub> Q <sub>1</sub>	P <sub>1</sub> Q <sub>0</sub>	P <sub>0</sub> Q <sub>1</sub>	'P <sub>0</sub> Q <sub>0</sub> '	'P <sub>1</sub> Q <sub>1</sub> '	'P <sub>1</sub> Q <sub>0</sub> '	'P <sub>0</sub> Q <sub>1</sub> '	
15	A	05	03	06	10	15	60	18	50	15×15%=02.25	09.0	(18) <sup>(0.15)</sup> =1.5427	1.7982
10.	B	06	03	08	15	18	120	24	90	18×10%=01.80	12.0	(24) <sup>(0.10)</sup> =1.3741	1.5683
25	C	08	06	10	12	48	120	60	96	48×25%=12.00	30.0	(60) <sup>(0.25)</sup> =2.7832	3.1302
35	D	02	20	06	14	40	84	120	28	40×35%=14.00	29.4	(120) <sup>(0.35)</sup> =5.3421	3.21
15	E	09	20	05	06	180	30	100	54	180×15%=27.00	04.5	(100) <sup>(0.15)</sup> =1.9953	1.8191
N= 05,										Σ'P <sub>0</sub> Q <sub>0</sub> '	Σ'P <sub>1</sub> Q <sub>1</sub> '	G.M. <sub>d</sub> ('P <sub>1</sub> Q <sub>0</sub> ')	G.M. <sub>d</sub> ('P <sub>0</sub> Q <sub>1</sub> ')
										=57.05,	= 84.90,	=62.8873,	=51.5468

Real price index (RP<sub>01</sub>) = √{(Σ'P<sub>1</sub>Q<sub>1</sub>'/Σ'P<sub>0</sub>Q<sub>0</sub>') × [G.M.<sub>d</sub>('P<sub>1</sub>Q<sub>0</sub>')/G.M.<sub>d</sub>('P<sub>0</sub>Q<sub>1</sub>')]} × 100

Real price index (RP<sub>01</sub>) = √{(84.90/57.05) × (62.8873/51.5468)} × 100

Real price index (RP<sub>01</sub>) = √{(1.4881682734 × 1.2200041665)} × 100

Real price index (RP<sub>01</sub>) = √{(1.8155714941)} × 100

Real price index (RP<sub>01</sub>) = 1.3474314432 × 100

Real price index (RP<sub>01</sub>) = 134.7431

#### 4.3.1. Justification of the use of proportional cum exponential weighting method in context of real price index

First we consider the summative part of real price index, that is, the expense ratio (ΣP<sub>1</sub>Q<sub>1</sub>/ΣP<sub>0</sub>Q<sub>0</sub>) : -

Suppose, the total number of commodities included in the basket of commodities selected for determining the real price index is N. If we do not use weights in the price index, it means that we give equal weights to all commodities which will be 1/N for each commodity. If equal weight 1/N is given for each commodity included in the basket, it means that an equal proportion (1/N or 100%) of the expenditure on each commodity in a year is

included in the total expenditure of the concerned year (base year or current year). Thus we see that the expense ratio ( $\Sigma P_1 Q_1 / \Sigma P_0 Q_0$ ) remains unchanged.

Hence, the weighting should be done in such a way that the proportion of expenditure incurred on each commodity included in the basket, which is included in the total expenditure ( $\Sigma P_1 Q_1$  or  $\Sigma P_0 Q_0$ ) of the relevant year, is affected by the weighting.

When the weights assigned to commodities are applied with the proportional weighting method on the summative portion of real price index, i.e., the expense-ratio ( $\Sigma P_1 Q_1 / \Sigma P_0 Q_0$ ), then a different ratio of expenditure on each commodity, corresponding to the weights assigned to them, Consists in the total expenditure of the respective year.

Now we talk about the second part of real price index, that is, the geometric part [ $G.M.(P_1/P_0) / G.M.(Q_1/Q_0)$ ] :- As we have already assumed that the total number of commodities included in the basket of commodities selected for determining real price index is N, and we do not use weights in the price index calculation, it means that we take all the commodities with equal weights Which will be  $1/N$  for each commodity.

In this case the weights which are in the form of exponents, are the same ( $1/N$ ) for all commodities, even for the geometric part of real price index. It can be understood as follows: -

Suppose, the N commodities included in the basket of commodities are : - commodity-1, commodity-2, commodity-3, commodity-4, ..... commodity-N.

Then, the geometric part of real price index can be written as follows: -

$$(A_1 \times A_2 \times A_3 \times A_4 \times \dots \times A_N)^{(1/N)}$$

OR

$$[A_1^{(1/N)} \times A_2^{(1/N)} \times A_3^{(1/N)} \times A_4^{(1/N)} \times \dots \times A_N^{(1/N)}]$$

And

$$\text{Antilog } [1/N(\log A_1 + \log A_2 + \log A_3 + \log A_4 \dots + \log A_N)]$$

OR

$$\text{Antilog } (1/N \log A_1 + 1/N \log A_2 + 1/N \log A_3 + 1/N \log A_4 \dots + 1/N \log A_N)$$

Where,

$$A_1 = (P_1 Q_0) / (P_0 Q_1) \text{ for commodity-1} \quad A_2 = (P_1 Q_0) / (P_0 Q_1) \text{ for commodity-2,}$$

$$A_3 = (P_1 Q_0) / (P_0 Q_1) \text{ for commodity-3,} \quad A_4 = (P_1 Q_0) / (P_0 Q_1) \text{ for commodity-4;}$$

$$A_N = (P_1 Q_0) / (P_0 Q_1) \text{ for commodity-N,} \quad (1/N) = \text{Weight as exponent;}$$

Hence, the weighting should be done in such a way that the exponent ( $1/N$ ) of  $[(P_1 Q_0) / (P_0 Q_1)]$  for each commodity included in the basket is affected by the weighting.

When the weights assigned to the commodities are applied with the exponential weighting method on the geometric portion of real price index, i.e. [ $GM (P_1 Q_0) / GM(P_0 Q_1)$ ], the exponent ( $1/N$ ) of  $[(P_1 Q_0) / (P_0 Q_1)]$  for each commodity, corresponding to the weights assigned to them, is influenced by the weighting.

In conclusion, it can be said that the proportional cum exponential weighting method appropriately affects both the parts of real price index - the summative portion i.e. the expense-ratio ( $\Sigma P_1 Q_1 / \Sigma P_0 Q_0$ ) and the geometric part [ $GM(P_1 Q_0) / GM(P_0 Q_1)$ ].

### 5. *Characteristics of real price index*

On the basis of above study, main characteristics of real price index are as follows :-

#### 5.1. *Merits of real price index*

5.1.1. It satisfies all the four adequacy tests.

5.1.2. The real price index uses the geometric mean method more intensively. *The geometric mean is the most useful when the changes tend to make large fluctuations. It is also well known that Geometric mean is the most reliable mean of ratios.*<sup>[2]</sup>

5.1.3. It gives equal importance to the prices of commodities and their quantities demanded; i.e. it gives equal importance to the changes in the prices and changes in the relative importance (preferences) of the commodities for the people who purchase them.

5.1.4. It is free from all kinds of biases.

#### 5.2. *Demerit of real price index*

5.2.1. We need a new method of weighting for calculating real price index.

5.2.2. It becomes more complicated to calculate as the number of commodities increases.

### III. Conclusion

Real price index satisfies all the four tests of adequacy. It is also clear from comparative study of Fisher's price index and real price index that Fisher's price index is not always free of bias, whereas real price index is always free of bias. The main reason for this is that real price index always gives equal importance to the prices of commodities and their quantities whereas Fisher's price index always does not give equal importance to the prices of commodities and their quantities.

Real price index highlights the biggest human bias in the formulation of a price index formula : “Trying to keep separate the effects of changes in quantities of the commodities from the effects of changes in their prices” and removes this human bias by including obviously the effects of changes in the quantities of commodities.

Finally, we can say that real price index is a perfect price index. This index consists of 2 parts: - a summative part i.e. the expense-ratio ( $\Sigma P_1 Q_1 / \Sigma P_0 Q_0$ ) and the second, geometric part due to which we need a new weighting method - the proportional cum exponential weighting method. This weighting method appropriately affects both the parts of real price index. So the proportional cum exponential weighting method is the best one for real price index.

But it is also important to note that a price index formula measures the change in the general price level on the basis of the given conditions - demands and prices of commodities. It is a mathematical method that gives equal importance to all commodities. Whereas in reality the changes in the prices of all commodities do not affect human interests and purchasing power equally because all commodities are not equally important (necessary) to the people of society. Therefore, it is not enough to just create a price index formula that satisfies all the four tests of adequacy, but also human discretion is required in every step of the price index determination process - from the selection of commodities to assigning and applying weights. As the selection of commodities and the weights for selected commodities are decided on a subjective basis, due to which the probability of the price index being affected by human biases increases significantly. Therefore, the need of taking special care, caution and honesty in this work is unavoidable.

### **References**

- [1]. Dr. V. K. Nigam, Dr. Ankita Gupta, Dr. Anamika Chaudhary, Dr. Monika Malhotra, Dr. S. V. Rawat, Pro. A. P. Sen ( Hindi Edition April 2016), Quantative Methods, Directorate Study and Publications, Uttarakhand Open University, Haldwani, Nainital, Uttarakhand, India
- [2]. Dr. V. C. Sinha, Dr. B. N. Gupta (Revised Hindi Edition 2012) Economics, S.B. P.D. Publications, 3/20B, Agra-Mathura Bypass Road, Near Tulsi Cinema, Agra, Uttar Pradesh, India

### **Acknowledgement**

“I would like to pay my special regards to my wife Mrs. Komal and my brother Mr. Raj Kumar to encourage me for this research work.”

### **Conflicts of Interests**

None

Amit Kumar. "Real Price Index and Alternative Methods of Weighting." *IOSR Journal of Mathematics (IOSR-JM)*, 17(3), (2021): pp. 09-19.