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Two Storage Facilities Supply Chain Inventory Model for Deteriorating Items with Multiple Buyers Single Vendor under Time and Price Dependent Demand

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Abstract:- *A supply chain inventory model for deteriorating items under price and time dependent demand is formulated for obtaining optimal policy between buyer and vendor. For obtaining optimal cycle time and prices for a system with multiple buyers and single vendor under two level storage for buyers as profit maximization is developed. Joint profits for buyers and vendor have also computed. Numerical example is presented with its sensitivity analysis to demonstrate the utility of the model.*

Key Words:- *Different Deterioration, Multiple Buyers, Optimal strategy, Price dependent demand, Single Vendor, Supply chain, Time dependent demand, Time varying holding cost, Two storage facility*

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I. Introduction:

For getting economic advantage like reduce possibility of shortages, reduce possibility of unit purchase cost, reduce costs of ordering, etc. for buyers and vendor both, additional amount of items are to be purchased and simultaneously also provide quantity discounts. For storing additional units additional storage facility known as rented warehouse is taken which has better storage facility than own warehouse and having higher storage cost than own warehouse. An inventory model under two facilities location was developed by Hartley [8]. Later, Sarma [18] extended Harley's [8] model by taking effect of deterioration in rented as well as own warehouses. A time dependent demand rate under two storage facility locations deterministic inventory model was developed by Kar et al. [13]. A two storage facilities location inventory model under time dependent demand was formulated by Banerjee and Agarwal [1]. A three parameters Weibull distribution deterioration rate was taken into consideration. Shortages were also taken into consideration. Yu et al. [26] gave two warehouses deteriorating items inventory model with decreasing rental over time. A two facility location inventory model was constructed by Tyagi and Singh [23] under the assumption of linear demand and variable holding cost. One retailer one wholesaler collaborative supply chain stock model under partially backlogged shortages was formulated by Ghiami et al. [4], where the demand of retailer depends on inventory level. Retailer has limited warehouse capacity therefore retailer has taken rented warehouse. A non-instantaneous deteriorating items inventory model under two storage facility locations and price dependent demand was formulated by Jaggi and Tiwari [10]. Storage cost was taken as time dependent. Backorders were also taken into consideration. Sheikh and Patel [20] obtained two facilities location inventory model under varying deterioration. Demand was taken as power function of time. An unsteady deteriorating items inventory model under two warehouses was formulated by Patel [16]. Time dependent storage cost, and time and price dependent demand was considered. Inflation factor was also considered with permissible delay.

In any supply chain system, inventory plays an important role. For efficient running of the supply chain's operation, there must be association between buyer and vendor about inventory movement. Buyer and vendor apply a strategy of business, generally, in classical methods of economic order quantity (EOQ) or Economic production quantity (EPQ). But for today's competitive situation concept of integrated economic lot size (IELS) or supply chain strategy is used for business requirements. Under various assumptions for demand pattern, researchers have developed vendor buyers supply chain inventory models. Lu [15] introduced the optimal policy when the delivered quantity sent to the buyer is identical and the stock was replenished for every time and developed collaborative inventory model for single vendor and multiple buyers. Yang and Wee [25] investigated joint inventory model when multiple buyers and single vendor considered under assumption that after receiving inventory by vendor, buyers demanded units are deteriorated. Multiple buyers and single vendor integrated inventory model constructed by Woo et al. [24] assumed reduction efforts for ordering cost is to be updated through the relevant information taken by IT and results in highest co-ordination and mechanization between associated business parties. Chan and Kingsman [2] constructed collaborative integrated inventory model. In this supply chain model single vendor deals multiple buyers with co-ordinated delivery and manufacturing cycles. This co-ordinated management reached its goals through schedule of the definite released days of product to the buyers and co-ordination with the manufacture goods produced by vendor, where the buyers were allowed to decide their own batch amount and ordering cycles. Karabati and Sayin [14] construct collaborative inventory model to develop supply chain between single suppler and multiple buyers where suppler offers quantity discount, buyer expected positive return from the coordination. Supply chain inventory model derived by Shah et al. [19] considered multiple buyers and single vendor, demand is a function of increasing and quadratic time dependent with invariable deterioration unit. A service level constraints and controllable lead time integrated production stock model under one wholesaler and multi-retailers was developed by Jha and Shanker [11]. Supply chain inventory model was derived by Glock and Kim [7] for single supplier and multiple retailers, where supplier transport complete manufactured products to the retailers while the supplier waits until the complete produced lot has been ended and consignments can be manufactured by batch. Giri and Roy [6] derived two levels supply chain inventory model by assuming the collaboration between multiple buyers and single manufacturer when lead-time demand was normally distributed and demand is price dependent. Ghiami and Williams [5] delivered two levels production inventory models with multiple buyers and one manufacturer when deteriorating items has fixed production rate and the order quantities are dispatched by the manufacturer to the consumers for definite period and the surplus inventory supplies for successive deliveries. Srinivas and Reddy [21] determined the integrated inventory model with consignment strategy when multiple buyers and single vendor included in supply chain inventory. Gani and Dharik [3] developed multiple buyers and single vendor supply chain inventory model based on a shipment stock and vendor followed inventory policy suggested by management where demand and manufacture rate of the vendor are considered trapezoidal fuzzy numbers. EOQ and EPQ inventory model derived by Jonrinaldi et al. [12] under supply chain strategy considered multiple buyers and single vendor with multiple items and determine the vendor's optimal lot size and buyers' number of orders for shipment policy by considering discrete and continuous demand. For multiple products when single supplier and multiple buyers consist in inventory model and derived integrated model by Powar and Nandurkar [17] under supply chain policy determined the optimal joint reorder point, shipments and order quantity for each buyer subject to decrease the co-ordinated cost of buyers and vendor. Tarhini, et al. [22] considered warehouse capacity as a constraint in the developed model to ensure that replenishment policy did not exceed it. Islam, et al. [9] obtained a three-tier green supply chain model for an agricultural product where byproducts are used for some purposes. They have also derived the solution procedure.

One vendor multiple buyers combined time and price dependent demand two warehouses inventory model for varying deterioration for buyers and changing storage cost for buyers and vendor both is considered in this paper. Under the assumption that vendor has better preservation technology, so preservation technology cost is included for vendor and therefore there is no deterioration cost for vendor.

II. Assumptions And Notations

NOTATIONS:

For obtaining model, list of notations used are:

$D(t)$: $a_i + b_i t - \rho_i p_i$, where $a_i > 0$, $0 < b_i < 1$, $p_i > 0$, $\rho_i > 0$.

$HC_{bi}(OW)$: OW of i^{th} buyer has time varying holding cost

$HC_{bi}(RW)$: RW of i^{th} buyer has time varying holding cost

$I_{0bi}(t)$: OW stock size of i^{th} buyer

$I_{rbi}(t)$: RW stock size of i^{th} buyer

$I_v(t)$: Inventory size of vendor at time t

A_{bi} : Ordering cost of i^{th} buyer's per order

A_v : Ordering cost of vendor per order

c_b : Unit cost of purchasing of buyer

θ_i : i^{th} buyer's deterioration rate during $t_1 \le t \le t_2$, $0 < \theta_i < 1$

$\theta_i t$: i^{th} buyer's deterioration rate during , $t_2 \le t \le \frac{T}{n_i}$, $0 < \theta_i < 1$

x_{ib1} : Fixed holding cost in OW of i^{th} buyer

y_{ib1} : i^{th} Buyer's varying holding cost in OW

x_{ib2} : RW fixed holding cost of i^{th} buyer

y_{ib2} : Varying holding cost in RW of i^{th} buyer

x_v : Fixed holding cost of vendor

y_v : Varying holding cost of vendor

p_i : Selling price of i^{th} buyer's per unit (decision variables)

m : Preservation technology cost for vendor (fixed)
n_i : Number of time orders placed by i^{th} buyer during cycle time.
t_r : When level of inventory of buyer in RW becomes nil (a decision variable)
W_i : Capacity of own warehouse of i^{th} buyer
TP_{bi} : Total profit of i^{th} buyer
TP_v : Total profit of vendor
TP : Vendor buyers' total profit

$t_1=v_1*(\frac{T}{n_i})$, $t_2=v_2*(\frac{T}{n_i})$, where $T_b = T/n_i$

T : Cycle time of vendor.

ASSUMPTIONS:
For developing the model, assumptions considered are:

- Item's demand depends on time and price.
- Multiple buyers' and one vendor are considered.
- Stock out is not permitted.
- Lead time is zero.
- During the cycle time, no repairing or replacement of deteriorated units and deterioration is dependent on time for buyer's inventory.
- For buyers and vendor both, time varying holding cost is considered.
- W_i units fixed capacity in OW of i^{th} buyer and unlimited capacity in RW of i^{th} buyer are considered.
- First RW goods of ith buyer are consumed and then goods of ith buyer in OW are consumed.
- Unit inventory cost in i^{th} buyer OW is less than unit inventory cost in RW in i^{th} buyer warehouse.

III. Mathematical Model And Analysis:

Level of inventory of i^{th} buyer's at time t be given by $I_{bi}(t)$ $(0 \le t \le T/n_i)$ is shown below:
Buyers' Inventory

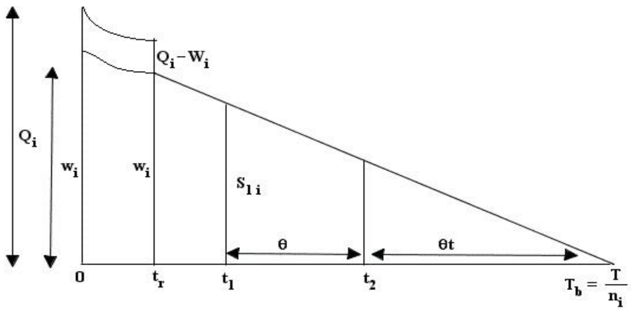


Figure 1

Two situations are discussed. In the first situation there is no collaboration between vendor and buyers, while in the second there is collaboration of buyers and vendor. Considering time and price dependent demand, inventory size is given for buyers and vendor.
Change in inventory sizes are given by following differential equations in RW and OW for buyers and vendor:

$$\frac{dI_{rbi}(t)}{dt} = -(a_i + b_i t - \rho_i p_i), \qquad 0 \le t \le t_r \qquad (1)$$

$$\frac{dI_{0bi}(t)}{dt} = 0, \qquad 0 \le t \le t_r \qquad (2)$$

$$\frac{dI_{0bi}(t)}{dt} = -(a_i + b_i t - \rho_i p_i), \qquad t_r \le t \le t_1 \qquad (3)$$

$$\frac{dI_{0bi}(t)}{dt} + \theta_i I_{0bi}(t) = -(a_i + b_i t - \rho_i p_i), \qquad t_1 \le t \le t_2 \qquad (4)$$

$$\frac{dI_{0bi}(t)}{dt} + \theta_i t I_{0bi}(t) = -(a_i + b_i t - \rho_i p_i), \qquad t_2 \le t \le \frac{T}{n_i} \qquad (5)$$

$$\frac{dI_v(t)}{dt} = -\sum_{i=1}^{N}(a_i + b_i t - \rho_i p_i), \qquad 0 \le t \le T \tag{6}$$

initial conditions taken are $I_{0bi}(0) = W_i$, $I_{0bi}(t_1) = S_{1i}$, $I_{0bi}(t_r) = W_i$, $I_{rbi}(0) = Q_i - W_i$, $I_{rbi}(t_r) = 0$, $I_{0bi}\left(\frac{T}{n_i}\right) = 0$ and $I_v(T)=0$.

These equations have solutions:

$$I_{rbi}(t) = (Q_i - W_i) - (a_i t - \rho_i p_i t + \frac{1}{2} b_i t^2) \tag{7}$$

$$I_{0bi}(t) = W_i \tag{8}$$

$$I_{0bi}(t) = S_{1i} + a_i(t_1 - t) - \rho_i p_i(t_1 - t) + \frac{1}{2} b_i\left(t_1^2 - t^2\right) \tag{9}$$

$$I_{0bi}(t) = \begin{bmatrix} a_i(t_1-t) - \rho_i p_i(t_1-t) + \frac{1}{2}a_i\theta_i\left(t_1^2-t^2\right) - \frac{1}{2}\rho_i p_i \theta_i\left(t_1^2-t^2\right) + \frac{1}{2}b_i\left(t_1^2-t^2\right) \\ + \frac{1}{3}b_i\theta_i\left(t_1^3-t^3\right) - a_i\theta_i t(t_1-t) + \rho_i p_i \theta_i t(t_1-t) - \frac{1}{2}b_i\theta_i t\left(t_1^2-t^2\right) \end{bmatrix} + S_{1i}\left[1+\theta_i(t_1-t)\right] \tag{10}$$

$$I_{0bi}(t) = \begin{bmatrix} a_i\left(\frac{T}{n_i}-t\right) - \rho_i p_i\left(\frac{T}{n_i}-t\right) + \frac{1}{2}b_i\left(\frac{T^2}{n_i^2}-t^2\right) + \frac{1}{6}a_i\theta_i\left(\frac{T^3}{n_i^3}-t^3\right) - \frac{1}{6}\rho_i p_i\theta_i\left(\frac{T^3}{n_i^3}-t^3\right) \\ + \frac{1}{8}b_i\theta_i\left(\frac{T^4}{n_i^4}-t^4\right) - \frac{1}{2}a_i\theta_i t^2\left(\frac{T}{n_i}-t\right) + \frac{1}{2}\rho_i p_i\theta_i t^2\left(\frac{T}{n_i}-t\right) - \frac{1}{4}b_i\theta_i t^2\left(\frac{T^2}{n_i^2}-t^2\right) \end{bmatrix}. \tag{11}$$

$$I_v(t) = \sum_{i=1}^{N}\left[a_i(T-t) - \rho_i p_i(T-t) + \frac{1}{2}b_i\left(T^2-t^2\right)\right]. \tag{12}$$

(by not considering higher powers of θ)

Substituting $t = t_r$, in (7) gives

$$Q_i = \left[W_i + a_i t_r - \rho_i p_i t_r + \frac{1}{2}b_i t_r^2\right]. \tag{13}$$

From equations (8) and (9), putting $t = t_r$, we have

$$I_{0bi}(t_r) = W_i \tag{14}$$

$$I_{0bi}(t_r) = S_{1i} + a_i(t_1 - t_r) - \rho_i p_i(t_1 - t_r) + \frac{1}{2}b_i\left(t_1^2 - t_r^2\right) \tag{15}$$

So from equations (14) and (15), we have

$$S_{1i} = W_i - a_i(t_1 - t_r) + \rho_i p_i(t_1 - t_r) - \frac{1}{2}b_i\left(t_1^2 - t_r^2\right) \tag{16}$$

From equations (10) and (11), putting $t = t_2$, we have

$$I_{0bi}(t_2) = \begin{bmatrix} a_i(t_1-t_2) - \rho_i p_i(t_1-t_2) + \frac{1}{2}a_i\theta_i\left(t_1^2-t_2^2\right) - \frac{1}{2}\rho_i p_i\theta_i\left(t_1^2-t_2^2\right) + \frac{1}{2}b_i\left(t_1^2-t_2^2\right) \\ + \frac{1}{3}b_i\theta_i\left(t_1^3-t_2^3\right) - a_i\theta_i t_2(t_1-t_2) + \rho_i p_i\theta_i t_2(t_1-t_2) - \frac{1}{2}b_i\theta_i t_2\left(t_1^2-t_2^2\right) \end{bmatrix} + S_{1i}\left[1+\theta_i(t_1-t_2)\right] \tag{17}$$

$$I_{0bi}(t_2) = \begin{bmatrix} a_i\left(\frac{T}{n_i}-t_2\right) - \rho_i p_i\left(\frac{T}{n_i}-t_2\right) + \frac{1}{2}b_i\left(\frac{T^2}{n_i^2}-t_2^2\right) + \frac{1}{6}a_i\theta_i\left(\frac{T^3}{n_i^3}-t_2^3\right) - \frac{1}{6}\rho_i p_i\theta_i\left(\frac{T^3}{n_i^3}-t_2^3\right) \\ + \frac{1}{8}b_i\theta_i\left(\frac{T^4}{n_i^4}-t_2^4\right) - \frac{1}{2}a_i\theta_i t_2^2\left(\frac{T}{n_i}-t_2\right) + \frac{1}{2}\rho_i p_i\theta_i t_2^2\left(\frac{T}{n_i}-t_2\right) - \frac{1}{4}b_i\theta_i t_2^2\left(\frac{T^2}{n_i^2}-t_2^2\right) \end{bmatrix}. \tag{18}$$

So from equations (17) and (18), we get

$$T = \frac{n_i}{b_i\left(\theta_i t_2^2 - 2\right)}\left(\begin{matrix} 2a_i - 2\rho_i p_i - a_i\theta_i t_2^2 + \rho_i p_i\theta_i t_2^2 \\ + \sqrt{\begin{matrix} -8b_i\rho_i p_i\theta_i t_r t_1 - 8a_i\rho_i p_i + 8a_i b_i t_r + 8a_i\rho_i p_i\theta_i t_2^2 - 4b_i\theta_i t_2^2 W_i + 8b_i\rho_i p_i\theta_i t_r t_1 \\ -4b_i\rho_i p_i\theta_i t_2^2 + 8a_i b_i\theta_i t_r t_1 + 4b_i\rho_i p_i\theta_i t_1^2 - 8a_i b_i\theta_i t_r t_2 - 4a_i b_i\theta_i t_r t_2^2 + 4a_i b_i\rho_i p_i\theta_i t_r t_2^2 \\ + 8b_i W_i + 4a_i b_i\theta_i t_2^2 - 8b_i\rho_i p_i t_r - 4a_i b_i\theta_i t_1^2 - 8b_i W_i\theta_i t_r + 8b_i W_i\theta_i t_1 \end{matrix}} \end{matrix}\right) \tag{19}$$

T is not a decision variable, since equation (19) states that W_i and t_r expresses T.

Following elements are considered for total profit:

Buyers' relevant costs:

(i) Ordering cost (OC_b) = $\sum_{i=1}^{N} n_i A_{bi}$ (20)

(ii) $$HC_b(OW) = \sum_{i=1}^{N} n_i \left[x_{bi} \left\{ \int_0^{\frac{T}{n_i}} I_{0bi}(t)dt \right\} + y_{bi} \left\{ \int_0^{\frac{T}{n_i}} tI_{0bi}(t)dt \right\} \right]$$

$$= \sum_{i=1}^{N} n_i \left[\begin{array}{l} x_{1bi} \left\{ \int_0^{t_r} I_{0bi}(t)dt + \int_{t_r}^{t_1} I_{0bi}(t)dt + \int_{t_1}^{t_2} I_{0bi}(t)dt + \int_{t_2}^{\frac{T}{n_i}} I_{0bi}(t)\,dt \right\} \\ + y_{1bi} \left\{ \int_0^{t_r} tI_{0bi}(t)dt + \int_{t_r}^{t_1} tI_{0bi}(t)dt + \int_{t_1}^{t_2} tI_{0bi}(t)dt + \int_{t_2}^{\frac{T}{n_i}} tI_{0bi}(t)dt \right\} \end{array} \right] \quad (21)$$

(iii) $$HC_b(RW) = \sum_{i=1}^{N} n_i \left[x_{2bi} \left\{ \int_0^{t_r} I_{rbi}(t)dt \right\} + y_{2bi} \left\{ \int_0^{t_r} tI_{rbi}(t)dt \right\} \right] \quad (22)$$

(iv) Deterioration Cost:

$$DC_b = \sum_{i=1}^{N} n_i c_b \theta_i \left[\left\{ \int_{t_1}^{t_2} I_{0bi}(t)dt + \int_{t_2}^{\frac{T}{n_i}} tI_{0bi}(t)dt \right\} \right] \quad (23)$$

(v) Sales Revenue:

$$SR_b = \sum_{i=1}^{N} n_i p_i \left(\int_0^{\frac{T}{n_i}} (a_i + b_i t - \rho_i p_i)dt \right) \quad (24)$$

(by not considering higher powers of θ)

(vi) Total Profit:

$$TP_b = \frac{1}{T}[SR_b - OC_b - HC_b(RW) - HC_b(OW) - DC_b] \quad (25)$$

Relevant Costs of Vendor:

(i) Cost of Ordering (OC_v)=A_v (26)

(ii) Cost of Holding:

$$HC_v = x_v \left[\int_0^T I_v(t)\,dt - \sum_{i=1}^{N} n_i \left\{ \int_0^{\frac{T}{n}} I_b(t)dt \right\} \right] + y_v \left[\int_0^T tI_v(t)dt - \sum_{i=1}^{N} n_i \left\{ \int_0^{\frac{T}{n}} tI_b(t)dt \right\} \right]$$

$$= x_v \left[\int_0^T I_v(t)dt - \sum_{i=1}^{N} n_i \left\{ \int_0^{t_r} I_{rbi}(t)dt + \int_0^{t_r} I_{0bi}(t)dt + \int_{t_r}^{t_1} I_{0bi}(t)dt + \int_{t_1}^{t_2} I_{0bi}(t)dt + \int_{t_2}^{\frac{T}{n}} I_{0bi}(t)dt \right\} \right]$$

$$+ y_v \left[\int_0^T tI_v(t)dt - \sum_{i=1}^{N} n_i \left\{ \int_0^{t_r} tI_{rbi}(t)dt + \int_0^{t_r} tI_{0bi}(t)dt + \int_{t_r}^{t_1} tI_{0bi}(t)dt + \int_{t_1}^{t_2} tI_{0bi}(t)dt + \int_{t_2}^{\frac{T}{n}} tI_{0bi}(t)dt \right\} \right] \quad (27)$$

(iii) Preservation Technology Cost (PTC_v) = m (28)

(iv) Sales Revenue:

$$SR_v = c_b\left(\sum_{i=1}^{N}\left[\int_0^T (a_i + b_i t - \rho_i p_i)dt\right]\right) \tag{29}$$

(v) Total Profit:

$$TP_v = \frac{1}{T}[SR_v - OC_v - HC_v - PTC_v] \tag{30}$$

Situation I: Buyer and vendor take independent decision:

Here the buyer and vendor make decision independently. For given value of n, TP_b can be maximized by solving

$$\frac{\partial TP_b(t_r,p_i)}{\partial t_r} = 0,\ \frac{\partial TP_b(t_r,p_i)}{\partial p_i} = 0,\ \text{where } T_b = \frac{T}{n_i}, \tag{31}$$

provided it satisfies the second order condition

$$\begin{bmatrix} \frac{\partial^2 TP_b(t_r,p_i)}{\partial t_r^2} & \frac{\partial^2 TP_b(t_r,p_i)}{\partial p_i \partial t_r} \\ \frac{\partial^2 TP_b(t_r,p)}{\partial t_r \partial p_i} & \frac{\partial^2 TP_b(t_r,p_i)}{\partial p_i^2} \end{bmatrix} > 0. \tag{32}$$

This solution (n,t_r,p_i) maximizes TP_v.

Then the total profit without collaboration is given by:

$TP = \max(TP_b + TP_v)$.

Situation-II: Joint decision of buyer and vendor:

Here joint decision is taken by buyer and vendor:

The optimum values of t_r and p_i must satisfy the following conditions which maximize total profit (TP) when buyer and vendor take joint decision.

$$\frac{\partial TP_b(t_r,p_i)}{\partial t_r} = 0,\ \frac{\partial TP_b(t_r,p_i)}{\partial p_i} = 0,\quad \text{for } T_b = \frac{T}{n_i} \text{ is a function of } t_r \tag{33}$$

provided it satisfies the second order condition

$$\begin{bmatrix} \frac{\partial^2 TP_b(t_r,p_i)}{\partial T^2} & \frac{\partial^2 TP_b(t_r,p)}{\partial p_i \partial T} \\ \frac{\partial^2 TP_b(t_r,p)}{\partial T \partial p_i} & \frac{\partial^2 TP_b(t_r,p)}{\partial p_i^2} \end{bmatrix} > 0 \tag{34}$$

where total profit (TP) with collaboration is given by:

$$TP = TP_b + TP_v \tag{35}$$

IV. Numerical Example

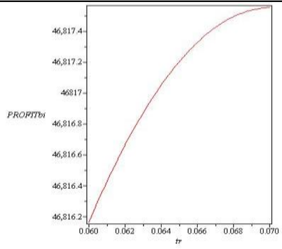
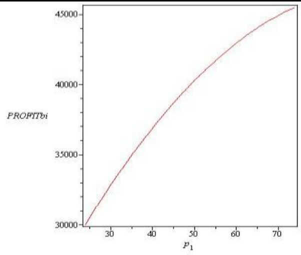
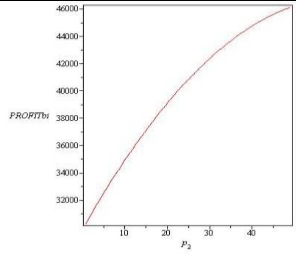
Various parameter values in appropriate units are taken for numerical illustration, A_{b1} = 85, A_{b2} = 65, W_1 = 70, W_2 = 65, a_1 = 650, a_2 = 550, b_1=0.05, b_2 =0.05, θ_1=0.06, θ_2=0.04, c_b=40, ρ_1=3.5, ρ_2=4.5, x_{1b1} =4.5 , x_{1b2} =3.5, y_{1b1}=0.04, y_{1b2}=0.04, x_{2b1} =6.5 , x_{2b2} =5.5, y_{2b1}=0.08, y_{2b2}=0.08, m = 5, A_v=2000, x_v=3, y_v=0.03, v_1=0.3, v_2=0.50, N=2 in suitable units.

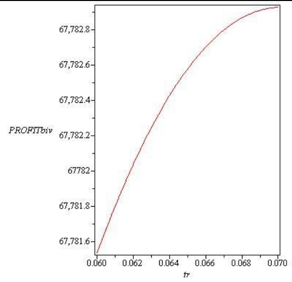
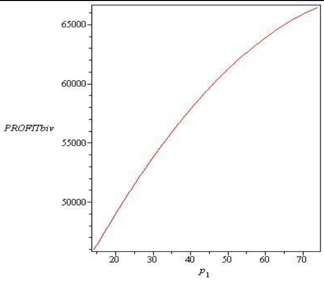
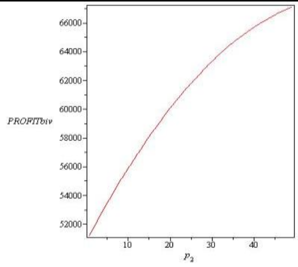
Table provides optimum independent and joint values of t_r, T, p_1, p_2 and profits for buyers and vendor. The second order conditions given in equation (32) and equation (34) are also satisfied.

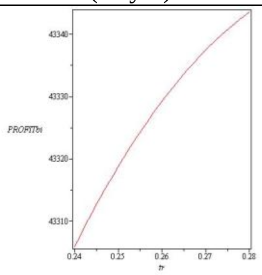
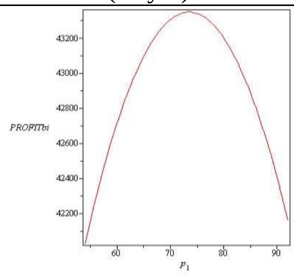
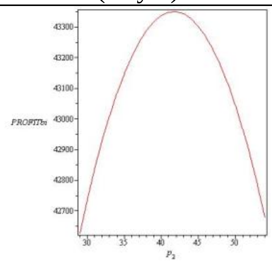
Table 1
Without collaboration and with collaboration optimum solution

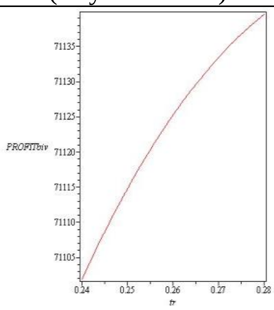
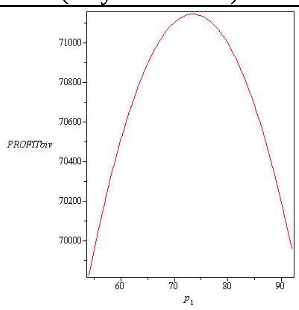
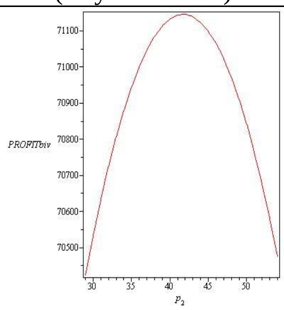
	Independent Decision	Joint Decision
n_1,n_2	n_1=5, n_2=5	n_1=3, n_2=3
t_r	0.0701	0.3082
p_1	93.2525	73.4891
p_2	61.4480	41.7307
T	2.9480	2.7926
Buyers Profit	46817.5552	43350.5664
Vendor’s Profit	20965.3715	27795.9992
Total Profit	67782.9267	71146.5656

Concavity of profit functions are shown in graphs 1 to graphs 12.

Graph Independent		
Graph of t_r and Profit (Buyer)	Graph of p_1 and Profit (Buyer)	Graph of p_2 and Profit (Buyer)
		
Graph 1	Graph 2	Graph 3

Graph Independent		
Graph of t_r and Profit (Buyer+Vendor)	Graph of p_1 and Profit (Buyer+Vendor)	Graph of p_2 and Profit (Buyer+Vendor)
		
Graph 4	Graph 5	Graph 6

Graph Jointly		
Graph of t_r and Profit (Buyer)	Graph of p_1 and Profit (Buyer)	Graph of p_2 and Profit (Buyer)
		
Graph 7	Graph 8	Graph 9

Graph Jointly		
Graph of t_r and Profit (Buyer+Vendor)	Graph of p_1 and Profit (Buyer+Vendor)	Graph of p_2 and Profit (Buyer+Vendor)
		
Graph 10	Graph 11	Graph 12

V. Sensitivity Analysis

Study of one parameter at a time, table below gives post-optimality computations.

Table 2
Sensitivity Analysis
Independent Decision

Parameter	%	n_1	n_2	Profit(b)	Profit(v)	Profit(bv)
a_1,a_2	+20%	5	5	67392.3754	25569.9743	92962.3497
	+10%	5	5	56634.6217	23266.2158	79900.8375
	-10%	5	5	37941.2264	18668.6205	56609.8469
	-20%	5	5	30034.6667	16359.7231	46394.3898
A_{b1},A_{b2}	+20%	5	5	46717.8461	21007.6241	67725.4702
	+10%	5	5	46767.1677	20987.3318	67754.4995
	-10%	5	5	46869.0825	20942.0647	67811.1472
	-20%	5	5	46921.8328	20916.8045	67838.6373
x_{2b1},x_{2b2}	+20%	5	5	46806.9786	20912.0934	67719.0720
	+10%	5	5	46811.9077	20937.0849	67748.9926
	-10%	5	5	46824.0934	20997.1248	67821.2182
	-20%	5	5	46831.7562	21034.0881	67865.8443
θ_1, θ_2	+20%	5	5	46807.0261	20956.3495	67763.3756
	+10%	5	5	46812.2823	20960.8685	67773.1508
	-10%	5	5	46822.8449	20969.8512	67792.6961
	-20%	5	5	46828.1518	20974.3287	67802.4805
ρ_1, ρ_2	+20%	5	5	38986.8898	20942.7171	59929.6069
	+10%	5	5	42546.2736	20954.2584	63500.5320
	-10%	5	5	52038.0339	20976.4638	73014.4977
	-20%	5	5	58563.6587	20988.0192	79551.6779
A_v	+20%	5	5	46817.5552	20694.9455	67512.5007
	+10%	5	5	46817.5552	20830.1585	67647.7137
	-10%	5	5	46817.5552	21100.5845	67918.1397
	-20%	5	5	46817.5552	21235.7975	68053.3527

Table 3
Sensitivity Analysis
Joint Decision

Parameter	%	n_1	n_2	Profit(b)	Profit(v)	Profit(bv)
a_1,a_2	+20%	4	4	63963.2786	32481.6338	96444.9124
	+10%	4	4	53228.6314	30131.2944	83359.9258
	-10%	4	4	38396.0554	39658.0519	60020.5245
	-20%	4	4	26675.7667	23091.1460	49766.9127
A_{b1},A_{b2}	+20%	4	4	43349.6627	27801.1779	71150.8406
	+10%	4	4	43392.5381	27791.8946	71184.4327
	-10%	4	4	43480.0237	27772.5504	71252.5741
	-20%	4	4	43524.6929	27762.4497	71287.1426
x_{2b1},x_{2b2}	+20%	4	4	43433.7262	27679.6267	71113.3529
	+10%	4	4	43434.1411	27729.4825	71163.6236
	-10%	4	4	43439.5887	27838.6466	71278.2353
	-20%	4	4	43445.8539	27898.3783	71344.2322
θ_1,θ_2	+20%	4	4	43426.7967	27769.1265	71195.9232
	+10%	4	4	43431.3279	27775.7769	71207.1048
	-10%	4	4	43440.5739	27789.0583	71229.6322
	-20%	4	4	43445.2898	27795.6901	71240.9799
ρ_1, ρ_2	+20%	4	4	34963.3730	29028.2871	63991.6601
	+10%	4	4	38843.9990	28404.8089	67248.8079
	-10%	4	4	48976.6211	27160.9607	76137.5818
	-20%	4	4	55822.0134	26540.4927	82362.5061
	+20%	4	4	43371.5617	27626.4536	70998.0153

A_v	+10%	4	4	43403.7797	27702.7538	71106.5335
	-10%	4	4	43467.6523	27866.1120	71333.7643
	-20%	4	4	43498.4257	27954.7637	71453.1894

From Tables 2 and 3 computations we observe about variations of optimal cycle time T*, t_r*, prices, p_1*, p_2* and maximum total profits for independent as well as joint decisions.

There will be increase or decrease in value of profits when parameter 'a_1, a_2' increase/ decrease independently as well as jointly, however, when A_{b1}, A_{b2},, x_{2b1}, x_{2b2}, A_v, θ_1, θ_2 and ρ_1, ρ_2 increase/decrease then total profit decrease/increase in independent and joint decision case.

VI. Conclusion

The result shows that the optimal cycle time is significantly decreased and total profit significantly increased when buyers and vendor take joint decision as compared to independent decision taken by buyers and vendor.

We can also observe that the vendor's profit is increased and number of times order placed by buyer during cycle time is decreased when buyers and vendor take joint decision.

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