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Mathematical Modeling and Spread of Disease in the Dynamics of General Prey-predator System

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Abstract: *The paper is done to study the Spread of disease in prey-predator population. For these problem a Dynamical system of differential Equations has been proposed. The Positivity, Boundedness and existence of model solutions of the Equation has been analyzed and proved. Existence of all possible Equilibrium has been checked and computed. Stability Analysis of all Equilibrium points of the model has been done. Moreover Local and global stability of disease free and endemic equilibrium points are established with concept of Jacobian matrix and Routh Hurwitz criterion respectively. Numerical simulations are presented to clarify analytical results.*

Keywords: *Spread of disease, predator-prey system, Local Stability, global stability , Simulation Study.*

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I. Introduction

There are Many prey-predator interaction of species can be observed in ecological environment throughout the world[1,2,3,6,8]. In a certain ecosystem, prey-predator species exhibit population increase and decrease in number due to predation and disease infection[1,2,3,5, 7].So Ecological prey-predator systems suffer from various infectious diseases [1,2,3,4, 7]. These diseases sometimes play a significant role in regulating size of prey-predator population [1, 7]. within a prey-predator population, it is often to see that a parasite spreads between prey to prey, prey to predator, and predator to predator and all the populations becomes disease affected. [13, 17]. The prey populations could be affected due to the presence of both parasites and predators [8].In this study, a mathematical eco-epidemiological model of prey-predator model is formulated and analyzed ,when the spread of disease from prey to predator population.

II. Model Assumptions

Suppose $x(t)$ denote the prey population, and $y(t)$ denote the predator population with total population $N(t) = x(t) + y(t)$,and Suppose also the prey grows logistically with growth rate r and carrying capacity k and only the prey population can reproduce. The predation functional response of the predator towards the prey follow holling type II functional response with predation coefficient p and half saturation constant m ,consumed prey converted into predator with efficiency q,and the predator infected at infection rate of β through contact. we have also let that death rate of the predator due to infection be denoted by μ and the prey will not die due to infection of disease but it will die only through predation. consider natural death rate of both the prey and predator neglected in the study. In this study , All parameters and Variables are assumed to be positive

Table 1 Notation and Description of model Variables

Variables	Descriptions
$\mathbf{x(t)}$	Population size of the prey
$\mathbf{y(t)}$	Population size of the predator

Table 2 Notations and Description of model parameters

Parameter	Description of parameter
$\boldsymbol{r}$	Intrinsic growth rate of the prey population
$\boldsymbol{k}$	Carrying capacity of the prey populations
$\boldsymbol{\beta}$	Infection rate of the prey , disease transmission from prey to predator
$\boldsymbol{p}$	Predation coefficient of the prey due to the predator
$\boldsymbol{q}$	Efficiency of predation
$\boldsymbol{m}$	Half-saturation constant
$\boldsymbol{\mu}$	Death rate of the predator populations

According to the above assumptions, the description of variables and parameters the present model will have the flow diagram given in Figure 1.

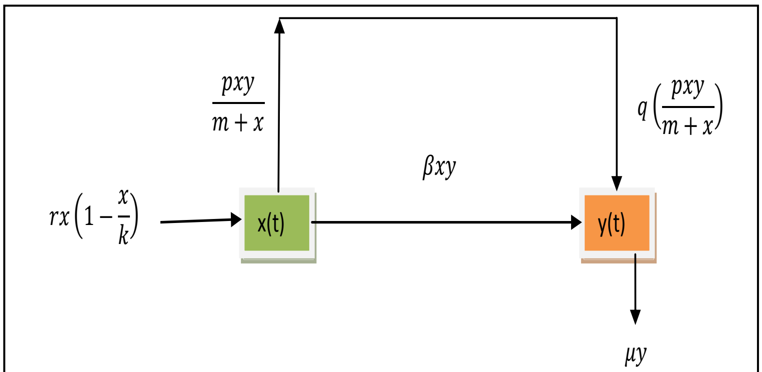

Figure 1. Model flow chart

Model Equations of the above model flow chart given as follow:

$$\frac{dx}{dt} = rx(1 - [x/k]) - [pxy/(m + x)] - \beta xy \qquad (1)$$

$$\frac{dy}{dt} = q[pxy/(m + x)] + \beta xy - \mu y \qquad (2)$$

with initial conditions here are $x(0) = x_o \geq 0$, $y(0) = y_0 \geq 0$, $p > 0$ and $0 < q \leq 1$.

III. Mathematical Analysis of the Model

Model (1) & (2) will be analyzed qualitatively . In this section, we are going to analysis the following features of the model: Positivity, Boundedness and Existence of solutions, Trivial Equilibrium point, Axial Equilibrium point, Disease free equilibrium points/boundary equilibrium points/, Endemic equilibrium points, Global stability of disease free equilibrium point and Local stability of endemic equilibrium point. All these concepts are presented and discussed in the following sub-sections.

3.1 Positivity of solutions of the Model

For model (1) & (2) to be epidemiologically meaningful and well posed, it is necessary to prove that all solutions of system with positive initial data will remain positive for all times $t > 0$. This will be established by the following theorems.

Lemma 1 (Positivity) Let $x(0) > 0$, $y(0) > 0$. Then the solutions $x(t)$, $y(t)$ of system equations (1) & (2) are positive $\forall t \geq 0$.

Proof: Positivity of the model variables is shown separately for each of the model variables $x(t)$, and $y(t)$.

Positivity of $x(t)$*:* The model equation (1) given by $dx/dt = rx\{1 - [x/k]\} - [pxy/(m + x)] - \beta xy$ can be expressed without loss of generality, after eliminating the positive terms rx which are appearing on the right hand side, as an inequality as $dX/dt \geq -\{[(rX^2)/k] + [pxy/(m + x)] + [\beta xy]\}$. This inequality can also be written $(dx/\beta xy) \geq dx/\{[(rX^2)/k] + [pxy/(m + x)] + [\beta xy]\} \geq -dt$. Then we have $dx/\beta xy \geq -dt$ using separation of variable method and on applying integration, the solution of the foregoing differentially inequality can be obtained as $x(t) \geq \exp(-\beta yt + \beta cy)$. Recall that an exponential function is always non–negative irrespective of the sign of the exponent, Hence, it can be concluded that $x(t) \geq 0$.

Positivity of $y(t)$*:* The model equation (2) given by $dy/dt = [qpxy/(m + x)] + \beta xy - \mu y$ can be expressed without loss of generality, after eliminating the positive term $[qpxy/(m + x)] + \beta xy$ which are appearing on the right hand side, as an inequality as $dy/dt \geq -\mu y$ This inequality can be written as $dy/dt \geq -(\mu)y$ hence $dy/y \geq -(\mu)dt$. Using variables separable method and on applying integration, the solution of the foregoing differentially inequality can be obtained as $y(t) \geq \exp[-\mu t + c]$. Recall that an exponential function is always non–negative irrespective of the sign of the exponent, i.e., the exponential function $\exp[-\mu t + c]$. is a non-negative quantity. Hence, it can be concluded that $y(t) \geq 0$.

3.2 Boundedness of Solution of the Model

In eco-epidemiology, the boundedness of the system implies that the system is biologically valid and well behaved. Then, we first show the biological validity of the model by providing the Boundedness of the solution of the model (1) & (2) by the following theorem

Lemma 2 (Boundedness) *All* solutions of the model (1) & (2) are uniformly bounded.

Proof: To show that each population size is bounded if and only if the total population size is bounded. Hence, it is sufficient to prove that the total population size $N = X(t) + Y(t)$ is bounded for all t. Now, summation of all the two model equations (1) & (2) $dN(t)/dt = (dX/dt) + (dY/dt)$ gives $[dN(t)/dt + \eta N(t)] \le rx + [qpxy/(m+x)] + \eta N(t) = \mu$. It can be shown that all feasible solutions are uniformly bounded in a proper subset $\Omega \in \mathbb{R}_+^2$ where the feasible region Ω is given by $\Omega = \{(x \quad y) \in \mathbb{R}+2;\ N \le \mu\eta$. Without loss of generality, after eliminating the negative terms which are appearing on the right hand side, the foregoing equation can be expressed as an inequality as $dN(t)/dt \le [\mu - \eta N(t)]$. Equivalently this inequality can be expressed as a linear ordinary differential inequality as general solution upon solving as

$0 \le N(x, \quad y) \le [\mu/\eta][1 - \exp(-\eta t)] + N(0)\exp(-\eta t)$. But, the term $N(0)$ denotes the initial values of the respective variable i.e., $N(t) = N(0)$ at $t = 0$. Thus, the particular solution can be expressed as $N(t) \le [\mu/\eta][1 - \exp(-\eta t)] + N(0)\exp(-\eta t)$. Further, it can be observed that $N(t) \to (\mu/\eta)$ as $t \to \infty$. That is, the total population size $N(t)$ takes off from the value $N(0)$ at the initial time $t = 0$ and ends up with the bounded value (μ/η) as the time t grows to infinity. Thus it can be concluded that $N(t)$ is bounded as $0 \le N(t) \le (\mu/\eta)$. Therefore, (μ/η) is an upper bound of $N(t)$. Hence, feasible solution of the system of model equations (1) & (2) remains in the positively invariant region Ω. Thus, the system is biologically meaningful and mathematically well posed in the domain Ω. Further, it is sufficient to consider the dynamics of the populations represented by the model system (1) & (2) in that domain. This proves the theorem. Therefore, it can be summarized the result of Theorem 2 as "the model variables $x(t)$, and $y(t)$ are bounded for all t.

Lemma 3 Existence Solutions of the model equations (1) & (2) together with the initial conditions $x(0) > 0$, $y(0) \ge 0$, exist in $\mathbb{R}_+^2$ i.e., the model variables $x(t)$ and $y(t)$ exist for all t and will remain in $\mathbb{R}_+^2$.

Proof: Let the system of equation (1) & (2) be as follows:

$f_1 = rx\{1 - [x/k]\} - [pxy/(m+x)] - \beta xy$

$f_2 = [qpxy/(m+x)] + \beta xy - \mu y$

According to Derrick and Groosman theorem, let Ω denote the region $\Omega = \{(x, \quad y) \in \mathbb{R}_+^2;\ N \le (\mu/\eta)\}$. Then equations (1) & (2) have a unique solution if $(\partial f_i)/(\partial x_j)$, $\forall\ i,\ j = 1, 2$ are continuous and bounded in Ω.. Here $x_1 = x$, $x_2 = y$. The continuity and the boundedness can be verified as follows:

For f_1:

$|(\partial f_1)/(\partial x)| = |r(1 - [2x]/k)| < \infty$

$|(\partial f_1)/(\partial y)| = |-(\beta + [p/(m+x)])x| < \infty$

For f_2:

$|(\partial f_2)/(\partial x)| = |(\beta + [mqp/(m+x)^2])y| < \infty$

$|(\partial f_2)/(\partial y)| = |(\beta + [qp/(m+x)])x - \mu| < \infty$

Thus, all the partial derivatives $(\partial f_i)/(\partial x_j)$, $\forall\, i,\ j = 1, 2$ exist, continuous and bounded in Ω.. Hence, by Derrick and Groosman theorem, a solution for the model (1) & (2) exists and is unique.

3.3 Equilibrium Points

Disease free equilibrium point of model (1) & (2) is obtained by solving $dx/dt = dy/dt = 0$. Model (1)& (9) possesses the following equilibrium points: (i) Trivial equilibrium point $(0, \quad 0)$, (ii) prey-free equilibrium point $(0 \quad \bar{y})$, (iii) Axial(predator-free)equilibrium point $(\bar{x} \quad 0)$, and (iv) endemic equilibrium point /positive equilibrium point/ $(\bar{x} \quad \bar{y})$ were Computed and stability analysis given as follows

IV. Stability Analysis of Equilibrium points

let us denote the model equation (1) and (2) as follows

$f(x, \quad y) = rx(1 - [x/k]) - [pxy/(m+x)] - \beta xy$

$g(x, \quad y) = q[pxy/(m+x)] + \beta xy - \mu y$

The possible critical points are ordered pair $(x, \quad y)$ such that $f(x, \quad y) = 0$ & $g(x, \quad y) = 0$. Thus are (i) Trivial equilibrium point $E_o = (0, \quad 0)$, (ii) Axial (predator-free) equilibrium point $E_1 = (k, \quad 0)$ (iii) Prey-free equilibrium point $E_2 = [0, \quad rm/(\beta m + p)]$, (iv) endemic equilibrium point /positive equilibrium point/ $E_3 = [k, \quad rm/(\beta m + p)]$, and the next generation matrix were computed $J(x, \quad y) = \begin{pmatrix} f_x & f_y \\ g_x & g_y \end{pmatrix}$

$$J(x, \quad y) = \begin{pmatrix} r - (2rx/k) & -\{\beta + [p/(m+x)]\}x \\ \{\beta + [mqp/(m+x)^2]\}y & \{\beta + [qp/(m+x)]\}x - \mu \end{pmatrix} \qquad (3)$$

Theorem 1 Trivial equilibrium point $E_o = (0, \quad 0)$ is unstable.

proof: Jacobian matrix of the model equation is given by

$$J(x, \quad y) = \begin{pmatrix} r-(2rx/k) & -\{\beta+[p/(m+x)]\}x \\ \{\beta+[mqp/(m+x)^2]\}y & \{\beta+[qp/(m+x)]\}x-\mu \end{pmatrix}$$

Evaluate the jacobian matrix at trivial equilibrium point E_0

$J(E_0) = J(0, \quad 0) = \begin{pmatrix} r & 0 \\ 0 & -\mu \end{pmatrix}$ is a diagonal matrix. so eigen values are $r, \ -\mu$, hence trivial equilibrium point is unstable.

Theorem 2 Axial(Predator -free) equilibrium point $E_1 = [k, \quad 0]$ is stable if $\mu \geq \{\beta + [qp/(m+k)]\}k$ otherwise unstable.

proof: Jacobian matrix of the model equation is given by

$$J(x, \quad y) = \begin{pmatrix} r-(2rx/k) & -\{\beta+[p/(m+x)]\}x \\ \{\beta+[mqp/(m+x)^2]\}y & \{\beta+[qp/(m+x)]\}x-\mu \end{pmatrix}$$

Evaluate the Jacobian matrix at Predator-free equilibrium point $E_2 = [k, \quad 0]$

$$J(E_2) = J(k, \quad 0) = \begin{pmatrix} -r & -\{\beta+[p/(m+k)]\}k \\ 0 & \{\beta+[qp/(m+k)]\}k-\mu \end{pmatrix}$$

which is upper triangular matrix with diagonal elements $-r, \ \{\beta+[qp/(m+k)]\}k-\mu$ and hence the predator free equilibrium point if $\{\beta+[qp/(m+k)]\}k-\mu \leq 0$ i.e $\mu \geq \{\beta+[qp/(m+k)]\}k$ otherwise unstable.

Theorem 3 Prey -free equilibrium point $E_2 = [0, \quad rm/(\beta m+p)]$ is unstable.

proof: Jacobian matrix of the model equation is given by

$$J(x, \quad y) = \begin{pmatrix} r-(2rx/k) & -\{\beta+[p/(m+x)]\}x \\ \{\beta+[mqp/(m+x)^2]\}y & \{\beta+[qp/(m+x)]\}x-\mu \end{pmatrix}$$

Evaluate the Jacobian matrix at Axial equilibrium point $E_2 = [0, \quad rm/(\beta m+p)]$

$$J(E_2) = J(0, \quad rm/(\beta m+p)) = \begin{pmatrix} r & 0 \\ [(\beta m+qp)r/(\beta m+p)] & -\mu \end{pmatrix}$$

which is lower triangular matrix with diagonal entries $r, \ -\mu$ hence axial equilibrium point or predator-free equilibrium point is again unstable.

Theorem 4 Endemic equilibrium point $[k, \quad rm/(\beta m+p)]$ is Stable if $\mu \geq [\beta k - r] + [qpk/(m+k)]$,Otherwise unstable.

proof: Jacobian matrix of the model equation is given by

$$J(x, \quad y) = \begin{pmatrix} r-(2rx/k) & -\{\beta+[p/(m+x)]\}x \\ \{\beta+[mqp/(m+x)^2]\}y & \{\beta+[qp/(m+x)]\}x-\mu \end{pmatrix}$$

Evaluate the Jacobian matrix at endemic equilibrium point $E_3 = [k, \quad rm/(\beta m+p)]$

$$J[k, \quad rm/(\beta m+p)] = \begin{bmatrix} \overbrace{-r}^{a} & \overbrace{-\{\beta+[p/(m+k)]\}k}^{b} \\ \overbrace{\{\beta+[mqp/(m+k)^2]\}k\{rm/(\beta m+p)\}}^{c} & \overbrace{\{\beta+[qp/(m+k)]\}k-\mu}^{d} \end{bmatrix}$$

$$J(E_3) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$det(J - \lambda I) = 0$$

$$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - \underbrace{(a+d)}_{T}\lambda + \underbrace{ad-bc}_{D} = 0$$

$$\lambda^2 - T\lambda + D = 0$$

$$\lambda_{1,2} = \left\{T \pm \sqrt{T^2-4D}\right\}/2$$

$$\lambda_{1,2} = \underbrace{[T/2]}_{\alpha} \pm \underbrace{\left[\left(\sqrt{T^2-4D}\right)/2\right]}_{\beta}$$

Case 1 If $T^2 - 4D < 0$, Complex eigen value $\lambda_{1,2} = \alpha \pm \beta i$ is *stable if* $\alpha < 0$, the type of equilibrium is spiral sink, unstable if $\alpha > 0$,type of equilibrium is spiral source, *stable if* $\alpha = 0$,the type of equilibrium is center i.e purely imaginary case.

$\lambda_{1,2} = \alpha = T/2 = (a+d)/2 = [(\beta k - r)/2] + [(kpq)/(2(m+k))] - [\mu/2] \leq 0$,

Therefore rearrange the terms will result $\mu \geq [\beta k - r] + [(kpq)/(m+k)]$. otherwise unstable
Case 2 If $T^2 - 4D = 0$,real and repeated eigen value $\lambda_{1,2} = \alpha$ stable if $\alpha < 0$, unstable if $\alpha > 0$.
case 3 If $T^2 - 4D > 0$,real and distinct Eigen value $\lambda_{1,2} = \alpha \pm \beta$ is stable if $\alpha + \beta < 0$ and $\alpha - \beta < 0$, Otherwise unstable. $\lambda_{1,2} = \alpha \pm \beta < 0$ which implies that $[T/2] \pm \left[\left(\sqrt{T^2 - 4D}\right)/2\right] < 0$ squaring both sides will give $[T/2]^2 < \left[\left(\sqrt{T^2 - 4D}\right)/2\right]^2$
rearranging the term , hence $D < 0$.

Theorem 5 Endemic equilibrium point $[k, \quad rm/(\beta m + p)]$ is Stable if
$\{rmk\beta^2 + (r\mu - r\beta k)(\beta m + p)\}(m+k)^3 + \{rmkp\beta - rpqk\}(m+k)^2 + r\beta kpqm^2(m+k) + rkqm^2p^2 < 0$, Otherwise unstable.

proof: Jacobian matrix at endemic equilibrium point

$$J[k, \quad rm/(\beta m + p)] = \begin{bmatrix} \overbrace{-r}^{a} & \overbrace{-\{\beta + [p/(m+k)]\}k}^{b} \\ \overbrace{\{\beta + [mqp/(m+k)^2]\}k\{rm/(\beta m + p)\}}^{c} & \overbrace{\{\beta + [qp/(m+k)]\}k - \mu}^{d} \end{bmatrix}$$

Consider $D = ad - bc < 0$ which implies that from case 3 above

$$-rk(\beta + [pq/(m+k)]) + r\mu - \{\beta k + [pk/(m+k)]\}\{rm/(\beta m + p)\}\{\beta + [mpq/(m+k)^2]\} < 0$$

$$-[r\beta k] - [rpqk/(m+k)] + r\mu + [(rmk\beta^2)/(\beta m + p)] + [(rmkp\beta)/\{(\beta m + p)(m+k)\}] + [(r\beta kpqm^2)/\{(\beta m + p)(m+k)^2\}] + [(rkqm^2p^2)/\{(\beta m + p)(m+k)^3\}] < 0$$

$$\{rmk\beta^2 + (r\mu - r\beta k)(\beta m + p)\}(m+k)^3 + \{rmkp\beta - rpqk\}(m+k)^2 + r\beta kpqm^2(m+k) + rkqm^2p^2 < 0$$

Theorem 6 (Global stability) Endemic equilibrium point E^* is globally stable.
Proof: Take an appropriate liapunove function $L(x, \quad y) = (x - x^*)^2/2 + \alpha(y - y^*)^2/2$ find the derivative of $L(x, \quad y)$ with respect of time t, $dL/dt = (x - x^*)[dx/dt] + \alpha(y - y^*)[dy/dt]$ (4)
Now substitute the model equation (1) & (2) into (4) results

$$\mathrm{dL/dt} = (x - x^*)\{\mathrm{rx}(1 - [\mathrm{x/k}]) - [\mathrm{pxy}/(\mathrm{m+x})] - \beta\mathrm{xy}\} + \alpha(y - y^*)\{[\mathrm{qpxy}/(\mathrm{m+x})] + \beta xy - \mu y\}$$

Take out x, y and put as change

$$\mathrm{dL/dt} = (x - x^*)(x - x^*)\{\mathrm{r}(1 - [\mathrm{x/k}]) - [\mathrm{py}/(\mathrm{m+x})] - \beta\mathrm{y}\} + \alpha(y - y^*)(y - y^*)\{[\mathrm{qpx}/(\mathrm{m+x})] + \beta x - \mu\}$$

By rearranging, it could be obtained

$$\mathrm{dL/dt} = -(x - x^*)^2\{-\mathrm{r}(1 - [\mathrm{x/k}]) + [\mathrm{py}/(\mathrm{m+x})] + \beta\mathrm{y}\} - \alpha(y - y^*)^2\{-[\mathrm{qpx}/(\mathrm{m+x})] - \beta x + \mu\} \leq 0$$

Thus it is possible to set α such that $\mathrm{dL/dt} \leq 0$ and endemic equilibrium point is globally stable. It is to be noted that the parameters k, m , q play a vital role in controlling the stability aspects of the system.

V. Simulation Study

In this section, Numerical simulation of model equations (1) & (2) are carried out using the software DEDiscover version: 2.6.4. Model equation and parameters were arranged for DEDiscover software in this way for simulation purpose:

```
dX/dt=r*X*(1-X/k)-beta*X*Y-p*X*Y/(m+X) // prey
dY/dt=beta*X*Y+q*p*X*Y/(m+X)-Mu*Y // predator
```

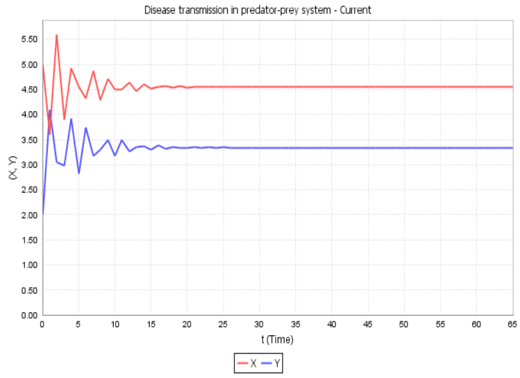

Figure 2 Time series plot

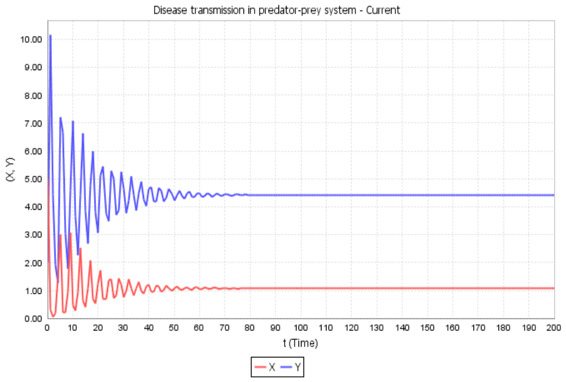

Figure 3 Time series plot

Figure 4 Time series plot with $q = \mu = 0.06$

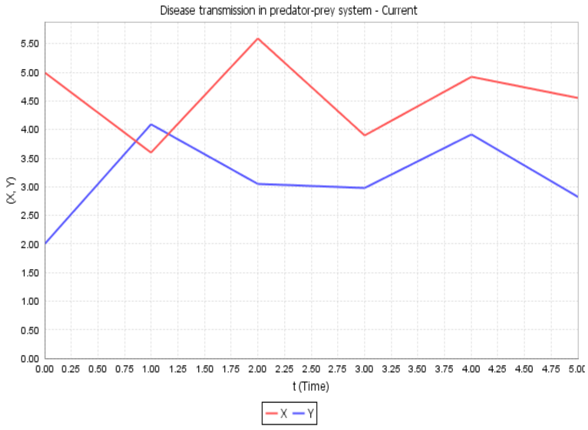

Figure 5 Time series plot with $q = 0.06, \mu = 0.07$

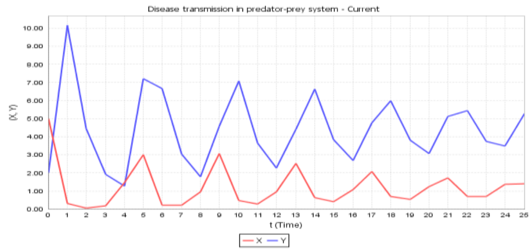

Figure 6 The system with K=25 and u=0.6

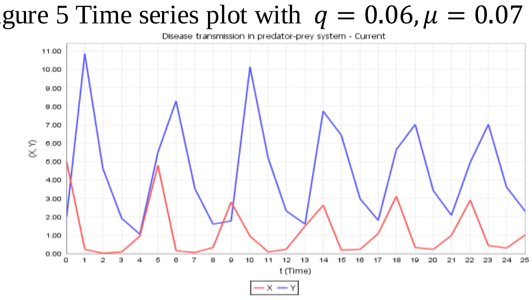

Figure 7 The system with K=50,u=0.6

VI. Conclusions

In this paper, we have formulated epidemiological model of general prey-predator system and the model is meaningful , well posed by establishing lemmas and proving that all solutions of system is exist, positive, and bounded. All possible equilibrium points are identified and From the stability analysis of Equilibrium points ,We have observed the following results: Trivial equilibrium point is always unstable, Axial(Predator -free) equilibrium point $E_1 = [k, \ 0]$ is stable if $\mu \geq \{\beta + [qp/(m+k)]\}k$ otherwise unstable, Prey -free equilibrium point $E_2 = [0, \ rm/(\beta m + p)]$ is unstable, Endemic equilibrium point $[k, \ rm/(\beta m + p)]$ is Stable if $\mu \geq [\beta k - r] + [qpk/(m+k)]$,otherwise unstable. It is proved that Endemic equilibrium point $[k, \ rm/(\beta m + p)]$ is Stable if $\{rmk\beta^2 + (r\mu - r\beta k)(\beta m + p)\}(m+k)^3 + \{rmkp\beta - rpqk\}(m+k)^2 + r\beta kpqm^2(m+k) + rkqm^2p^2 < 0$, Otherwise unstable. Global stability is stated as theorem and proved by taking appropriate liapunove function. From the simulation study in Fig 2 and Fig 3, it has been seen that prey-predator system the number initially oscillate, when the predation and disease involved in the system. Then after some time the prey-predator system gradually stable and Constant . From the fig 4-7,it can be conclude that the prey-predator system changes as the valid values of parameter varies in the system.

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