IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 17, Issue 5 Ser. I (Sep. – Oct. 2021), PP 57-60
www.iosrjournals.org

The Solutions to the Landau's problems in mathematics

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Abstract
The landau's problems could be addressed quickly and easily using a prime derivation formula as well as other new discoveries in mathematics which we shall know through the body of this article for the advancement of mathematics. These problems are related through these new mathematical concepts which we shall use to attack these problems.
Article subject classification: 11A41.
Keywords: *Goldbach's conjecture, Legendre's conjecture, Twin-prime conjecture, Near-square primes, Tor primes, Henry-Euchere primes.*

Date of Submission: 27-08-2021 Date of Acceptance: 11-09-2021

I. Introduction:

The landau's problems are 4 unsolved problems in mathematics concerning primes. These problems were outlined by the mathematician, Edmund landau in the year 1912 at the 5th congress of mathematicians in 1912 goes and they go as follows:

1). The problem of the Near-squre primes: Are there infinitely many primes p such that $p - 1$ is a perfect square? In other words, are there infinitely many primes of the form $n^2 + 1$?
2). The Goldbach's conjecture: Can every even integer greater than 2 be written as the sum of two primes?
3). The Legendre's conjecture: Does there always exist at least one prime between consecutive perfect squares? In other words, does there always exist at least one prime between m^2 and $(m + 1)^2$?
4).The Twin-prime conjecture: Are there infinitely many primes p such that $p + 2$ is a prime?
He presented these problems as un-attackable so now let's see how to attack them.

1. Solution 1:
1.1 The solution to the problem of the near-square primes:
To solve the 1st of these problems, I'll do so by introducing us to a theorem; the Tor prime theorem.
<u>The Tor prime theorem</u>:
The sums of 1 and the square of Toruna integers, n�, are primes which could be classified as Tor primes.
When:
Toruna Integers, n� = all even integers except those ending with 2 (excluding the integer 2 as an independent number) and/ or 8.
since there are infinite Toruna integers, infinitely many primes of the form $n^2 + 1$(tor primes) could be derived.
In this article, the examples are the proofs of the theorems they precede.
1.2. Examples:
i) when n = 10

$$n^2 + 1= p_1= 10^2 + 1= 100 + 1= 101 \tag{1}$$

ii) when n = 26

$$n^2 + 1= p_2= 26^2 + 1= 676 + 1= 677 \tag{2}$$

iii) When n= 12

$$n^2 + 1= 12^2 + 1 = 144 + 1 = 145 \tag{3}$$

iv) When n= 18

$$n^2 + 1= 18^2 + 1 = 324 + 1 = 325 \tag{4}$$

Drawing from the equations iii and iv to complete the proof of the Tor prime theorem, the products 145 and 325 were not primes because the value, n, was an even integer that ended with 2.

2. Solution 2:
2.1. The solution to the problem of the Goldbach's conjecture:
The primary odd numbers are 1, 3, 5, 7, and 9, and every prime number is a sum of an even number as ξ(including 0) and a primary odd number as χ.

The sum of any two or the same of these primary odds gives us an even number and even numbers can also be taken to be products of two or like kinds of ξs and two kinds of χs to give us two original ξs and a new ξ from the two χs which in other words, are the sums of two primes(two even numbers and two primary odd numbers to give three even numbers).
Through this, we deduce that a prime is a product of two or like kinds of ξs and three kinds of χs with two of these χs forming an ξ thus making a prime still a sum of an even number as ξ(including 0) and a primary odd number as the remaining χ.
2.2. Examples i: (p= prime)
i) When p= 17= p_1

$$17 = 10 + 7 = \xi + \chi = p_1 \tag{5}$$

ii) When p= 37= p_2

$$37 = 30 + 7 = \xi + \chi = p_2 \tag{6}$$

2.3. Examples ii:
i) When the even integer is 18:

$$18 = (0 + 5) + (10 + 3) \tag{7}$$

ii) When the even integer is 58:

$$58 = (20 + 9) + (10 + 9) \tag{8}$$

In the first of example ii, 0 and 10 are the ξs while the 3 and 5 are the χs.
From these we can conclude that all even numbers are the sums of two primes.

3. Solution 3:
3.1. The solution to the problem of the legendre's conjecture:
To solve the Legendre's conjecture problem I'll introduce us to one of new prime numbers derivation formulas by the mathematician Henry Donatus Chigozie which were derived as part or side formulas from newly discovered laws which were used to make developments to the Millennium prize problems as stated by the Clay Institute of Mathematics.
These primes are known as 'special primes' but In this article, the primes from the formula I'll introduce to us would be described as ' Henry-Euchere primes' and would be used to formulate the Euchere Theorems.
The 1st solution to the problem of the Legendre's conjecture is the Euchere Theorem 1 of the Legendre's conjecture.
<u>The Euchere Theorem 1</u>: In the formula for deriving Henry-Euchere primes i, p_{ei}

$$p_{ei} = (n(n + 2)) + (n + 1) = n^2 + 3n + 1, \tag{9}$$

When:
p_e= a prime,
n= all positive integers except, 80, 8080, and numbers that ends with 6 and/ or 1(excluding 01 as a number on it's own),

$$n + 1 = m \tag{10}$$

and,

$$n + 2 = n + 1 + 1 = m + 1, \tag{11}$$

p_e is always at in between m^2 and $(m + 1)^2$.
Although, excluding 80 and 8080, the numbers derived from the rest of the exempted numbers (numbers that ends with 6 and/ or 1(excluding 01 as a number on it's own)), when divided by 5 gives a prime number, p_x.
3.2. Examples:
Let's test this out by making n to be 3.

$$3^2 + (3 \times 3) + 1 = 19. \tag{12}$$

The prime number 19 is in between $(3 + 1)^2$ and $(3 + 2)^2$ (when 3 + 1= m and 3 + 2 = m + 1).

3.3. Solution ii of problem 3:
The solution ii of problem 3 would be given using the Euchere Theorem 2 of the Legendre's conjecture.
<u>The Euchere Theorem 2</u>: In one of the formulas used in the derivation of special primes (different from that used to formulate the Euchere Theorem 1),

$$p_{eii} = n^2 + n + 1 \tag{13}$$

when:
n= all positive integers except 7, and,
p_{eii}= prime,
p_{eii} is always in between n^2 and $(n + 1)^2$.

3.4. Examples:
i) When n= 5

$$5^2 + 5 + 1 = 31 \tag{14}$$

In the equation 14, the 31 is in between 5^2 and $(5 + 1)^2$
ii) When n= 6

$$6^2 + 6 + 1 = 43 \tag{15}$$

In this fifteenth equation(eq. 15), the 43 is in between 6^2 and $(6+1)^2$ which justifies the Legendre's conjecture together with eq. 14(equation 14) and the Euchere Theorem 2.

4. Solution 4:
4.1. The solution to the problem of the Twin-prime conjecture:
This solution would be given through the Euchere Theorem 3; the Euchere Theorem of the Twin-prime conjecture.
<u>The Euchere Theorem 3</u>: Naturally, the sums of all $p_{ei} + 2$ is a Henry-Euchere prime iii, p_{eiii}, a twin-prime, $p_{\square}$.

$$p_{ei} + 2 = p_{eiii} = p_{\square}$$

Since there are infinite p_{ei}, then there should be infinitely many primes p such that p + 2 is twin prime, $p_{\square}$.
This doesn't work out when the function f(n)= 3 in the equation

$$p_{ei} = (n(n + 2)) + (n + 1) = n^2 + 3n + 1,$$

because the p_{ei} derived won't give us a twin prime, p_{eiii}, when summed with 2.
Note: The prime numbers p_x when added to 2 also gives us a twin-prime, $p_{\square x}$.
4.2. Examples:
As proofs of this theorem,
When f(n)=5,

$$n^2 + (3n) + 1 = 5^2 + (3 \times 5) + 1 = 41 = p_1 \tag{16}$$

$$41 + 2 = p_1 + 2 = 43 = p_2 \tag{17}$$

When f(n)= 6,

$$(6^2 + (3 \times 6) + 1) \div 5 = 11 = p_x \tag{18}$$

$$p_x + 2 = 13 = p_{\square x} \tag{19}$$

4.3. Solution ii of problem 4
This would also be given using a theorem.
<u>The Euchere Theorem 4</u>:
In the formula

$$p_{ei} = n^2 + n + 1 \tag{13}$$

when n= all positive integers except 7 and those ending with 2 except 02 as an independent number, the primes derived minus 2 is a twin prime, $p_{\square}$.
Since there are infinite positive integers, then there are also infinite twin-primes.
4.4. Examples:
i) when f(n)= 5

$$n^2 + n + 1 = 5^2 + 5 + 1 = 31 = p_{ei} \tag{20}$$

By equating the equation 20 into the equation

$$P_{ei} - 2 = p_{\square} \tag{21}$$

when $p_{\square}$= twin-prime, then

$$p_{\square} = n^2 + n - 1 = 5^2 + 5 - 1 = 31 - 2 = 29 \tag{22}$$

5. Conclusions:
5.1. There are infinitely many primes p such that p − 1 is a perfect square. In other words, there are infinitely many primes of the form $n^2 + 1$.
5.2. Every even integer greater than 2 be written as the sum of two primes.
5.3. There always exist at least one prime between consecutive perfect squares. In other words, there always exist at least one prime between m^2 and $(m + 1)^2$.
5.4. There are infinitely many primes p such that p + 2 is a prime.
6. Conflicts of interests: The author declares that there are no conflicts of interests regarding the publication of this work.

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Heto Toruna Euchere Elba. "The Solutions to the Landau's problems in mathematics." *IOSR Journal of Mathematics (IOSR-JM)*, 17(5), (2021): pp. 57-60.