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Chirped Soliton Solutions Of The Generalized Derivative Nonlinear Schrödinger Equation

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Abstract
The purpose of this paper is to search the exact chirped soliton solutions of generalized derivative nonlinear Schrödinger equation which appears in many fields of physics as various nonlinear waves, such as nonlinear Alfvén waves in space plasma, sub-picosecond or femtosecond pulses in single mode optical fibers. Firstly, we apply a special complex envelope traveling-wave method to the generalized derivative nonlinear Schrödinger equation. Complex envelope traveling-wave solution is used to reduce the governing equation to an ordinary differential equation. Secondly, we introduce a new chirping ansatz given as an expansion in powers of intensity of the light pulse and obtain both linear and nonlinear chirp contributions associated with propagating optical pulses. It is shown that the phase associated to the obtained pulses has a nontrivial form and possesses two intensity dependent chirping terms. By using this ansatz the ordinary differential equation has been reduced to a fourth-degree an elliptic differential equation. Lastly, using an auxiliary equation to construct the analytical chirped solutions of the generalized derivative nonlinear Schrödinger equation. The resulting amplitude equation is then solved to get exact analytical chirped bright solitons, dark solitons, and periodic solutions for the model. As a result, we derive families of chirped soliton solutions under certain parametric conditions.
Keywords: *The generalized derivative nonlinear Schrödinger equation, Chirped soliton, Exact solution*

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I. Introduction

The nonlinear Schrödinger equation (NLSE) is a significant model that naturally arises in fields such as nonlinear optics, plasmas, fluid dynamics, and physics such as Bose-Einstein condensation. Mainly, the NLSE describes the propagation of an optical pulse in Kerr-law or non-Kerr law media . The NLSE is the familiar nonlinear partial differential equations that defined the evolution of optical solitons in optical fiber [1-2].

Optical solitons represent a non-linear pulse or wave packet that travels without deformation over extremely large distances. Due to their remarkable stability properties, optical solitons are now at the center of an active research field of nonlinear wave propagation in optical fibers. This research field started with the result obtained by Hasegawa and Tappert [3].

One of the attractive wave phenomena in recent times is the chirped solitons. The word chirp takes its origin from the chirping sound made by birds. For example, in optical transmission systems, ultrashort pulses could exhibit chirp, which could interact with the dispersion properties of the materials in which they are propagating, to increase or decrease the total pulse dispersion. Chirp is used in spread spectrum communications and in some devices as sonar and radar [4].

This paper is concerned with the generalized derivative nonlinear Schrödinger equation:

$$iq_t + aq_{xx} + b|q|^2 q + c|q|^4 q - idq_x - ih\left(|q|^2 q\right)_x = 0, \tag{1}$$

where $q(x,t)$ is complex envolope of the physical field in the comoving frame. a, b, c, d, h are real parameters related to the group velocity dispersion, cubic nonlinearity, quintic nonlinearity, the first order derivative term and the self steepening, respectively [5].This equation was studied different authors [6-8].

II. Mathematical Analysis

It is of interest to find exact chirped soliton solutions of the generalized derivative nonlinear Schrödinger equation. In order to obtain such solutions of Eq. (1) we express the complex envolope function as [9-13]

$$q(x,t) = \rho(\xi)e^{i[\chi(\xi)-\omega t]} \quad , \tag{2}$$

where $\rho = \rho(\xi)$ is the amplitude function and $\chi = \chi(\xi)$ is the phase function of $\xi(x,t) = kx - vt$. Also, v is the wave velocity, and ω is the frequency of the wave oscillation. The associated chirp is can be written as

$$\delta\omega(x,t) = -\frac{\partial}{\partial t}[\chi(\xi) - \omega t] = -\chi'(\xi). \tag{3}$$

Substituting (2) into Eq.(1), an equation consisting of real and imaginary parts is obtained, which depends on the dependent variables ρ and χ. Real part gives

$$b\rho^3 + c\rho^5 + \rho\omega + dk\rho\chi' + v\rho\chi' + hk\rho^3\chi' - ak^2\rho(\chi')^2 + ak^2\rho'' = 0, \tag{4}$$

while imaginary part implies

$$-dk\rho' - v\rho' - 3hk\rho^2\rho' + 2ak^2\rho'\chi' + ak^2\rho\chi'' = 0, \tag{5}$$

where primes denote differentiations with respect to ξ. Multiplying both sides of (5) by ρ and integrating once time leads to

$$\chi' = \frac{dk+v}{2ak^2} + \frac{A}{ak^2\rho^2} + \frac{3h\rho^2}{4ak}, \tag{6}$$

where A is an integration constant. Therefore, the resultant chirp is obtained as

$$\delta\omega = -\left(\frac{dk+v}{2ak^2} + \frac{A}{ak^2\rho^2} + \frac{3h\rho^2}{4ak}\right), \tag{7}$$

which shows that the chirping has two intensity dependent chirping terms apart from the linear term [where $I=|q|^2=\rho^2$ being intensity]. On substituting (6) in (4) gives

$$-\frac{A^2}{ak^2\rho^3} + \frac{\left(-2Ahk + d^2k^2 + 2dkv + v^2 + 4ak^2\omega\right)}{4ak^2}\rho + \left(\frac{2abk + dhk + hv}{2ak}\right)\rho^3 + \frac{\left(16ac + 3h^2\right)}{16a}\rho^5 + ak^2\rho'' = 0. \tag{8}$$

Multiplying (8) by ρ' and integrating with respect to ξ, we get

$$(\rho')^2 = \frac{2B}{ak^2} - \frac{A^2}{a^2k^4\rho^2} + \frac{\left(2Ahk - d^2k^2 - 2dkv - v^2 - 4ak^2\omega\right)}{a^2k^4}\rho^2 - \frac{\left(2abk + h(dk+v)\right)}{a^2k^3}\rho^4 - \frac{\left(16ac + 3h^2\right)}{12a^2k^2}\rho^6. \tag{9}$$

where B is the second integration constant.
Equation (9) is a nonlinear differential equation describing the evolution of the wave amplitude in a nonlinear medium that is governed by Eq. (1).

III. Chirped soliton solutions

Next, we present various chirped soliton solutions of the model Eq.(1), for different parameter conditions. Before discussing exact solutions to Eq. (9), the change of variable for the field amplitude

$$\rho^2(\xi) = U(\xi), \tag{10}$$

transforms Eq. (9) into the following new auxiliary elliptic equation [14-18] :

$$(U')^2 = a_0 + a_1U + a_2U^2 + a_3U^3 + a_4U^4, \tag{11}$$

where

$$a_0 = -\frac{4A^2}{a^2k^4}, a_1 = \frac{8B}{ak^2}, a_2 = \frac{2Ahk - d^2k^2 - 2dkv - v^2 - 4ak^2\omega}{a^2k^4}, a_3 = -\frac{\left(2abk + h(dk+v)\right)}{a^2k^3}, a_4 = -\frac{16ac + 3h^2}{12a^2k^2}. \tag{12}$$

3.1. Bright solitons

We have found that there are three types of bright soliton solutions for Eq. (11) under the following parametric conditions:

Case-I: For $a_0 = a_1 = a_3 = 0$, $a_2 > 0$, $a_4 < 0$, $\varepsilon = \pm 1$, then Eq. (11) has a bright soliton solution:

$$U(\xi) = \varepsilon\sqrt{-\frac{a_2}{a_4}}\,\mathrm{sech}\left(\sqrt{a_2}\,\xi\right). \tag{13}$$

Based upon the above finding, we obtain a first chirped bright soliton solution for Eq. (1) of the form

$$q(x,t)=\left[\varepsilon\sqrt{-\frac{a_2}{a_4}}\operatorname{sech}\left(\sqrt{a_2}\xi\right)\right]^{\frac{1}{2}}e^{i[\chi(\xi)-\omega t]}. \tag{14}$$

The corresponding chirp is given as follows:

$$\delta\omega=-\frac{v}{2k^2}-\sqrt{-\frac{a_4}{a_2}}\frac{\varepsilon A}{k^2\operatorname{sech}\left(\sqrt{a_2}\xi\right)}+\frac{3\varepsilon\delta}{2k}\sqrt{-\frac{a_2}{a_4}}\operatorname{sech}\left(\sqrt{a_2}\xi\right). \tag{15}$$

Case-II: For $a_0=a_1=a_4=0,\ a_2>0,\ a_3<0$, then Eq. (11) has a bright soliton solution:

$$U(\xi)=-\frac{a_2}{a_3}\operatorname{sech}^2\left(\frac{\sqrt{a_2}}{2}\xi\right). \tag{16}$$

By using these findings, we can show the second family of chirped bright soliton solution of Eq. (1) as

$$q(x,t)=\left[-\frac{a_2}{a_3}\operatorname{sech}^2\left(\frac{\sqrt{a_2}}{2}\xi\right)\right]^{\frac{1}{2}}e^{i[\chi(\xi)-\omega t]}, \tag{17}$$

and its corresponding chirping is of the form

$$\delta\omega=-\frac{v}{2k^2}+\frac{a_3A}{k^2a_2\operatorname{sech}^2\left(\frac{\sqrt{a_2}}{2}\xi\right)}-\frac{3\delta a_2}{2ka_3}\operatorname{sech}^2\left(\frac{\sqrt{a_2}}{2}\xi\right). \tag{18}$$

Case-III: For $a_0=a_1=0,\ a_2>0,$ then Eq. (11) has a bright soliton solution:

$$U(\xi)=\frac{2a_2}{\varepsilon\sqrt{\Delta}\cosh\left(\sqrt{a_2}\xi\right)-a_3},\qquad \Delta=a_3^2-4a_2a_4>0,\quad \varepsilon=\pm1. \tag{19}$$

Using the results given above, we obtain the following chirped bright soliton solution for Eq. (1):

$$q(x,t)=\left[\frac{2a_2}{\varepsilon\sqrt{\Delta}\cosh\left(\sqrt{a_2}\xi\right)-a_3}\right]^{1/2}e^{i[\chi(\xi)-\omega t]}. \tag{20}$$

The corresponding chirping is obtained as

$$\delta\omega=-\frac{v}{2k^2}-\frac{A\left(\varepsilon\sqrt{\Delta}\cosh\left(\sqrt{a_2}\xi\right)-a_3\right)}{2a_2k^2}+\frac{3\delta a_2}{k\left(\varepsilon\sqrt{\Delta}\cosh\left(\sqrt{a_2}\xi\right)-a_3\right)}. \tag{21}$$

3.2. Dark solitons

We have found three types of dark soliton solutions for Eq. (11) under the following parametric conditions:

Case-I: For $a_1=a_3=0,\ a_0=\frac{a_2^2}{4a_4},\ a_2<0,\ a_4>0,\ \varepsilon=\pm1$, then Eq. (11) has a dark soliton solution:

$$U(\xi)=\varepsilon\sqrt{-\frac{a_2}{2a_4}}\tanh\left(\sqrt{-\frac{a_2}{2}}\xi\right). \tag{22}$$

Thus, the first chirped dark soliton solution to the Eq. (1) is given by

$$q(x,t)=\sqrt{\varepsilon\sqrt{-\frac{a_2}{2a_4}}\tanh\left(\sqrt{-\frac{a_2}{2}}\xi\right)}e^{i[\chi(\xi)-\omega t]}. \tag{23}$$

The corresponding chirping is given by

$$\delta\omega=-\frac{v}{2k^2}-\sqrt{-\frac{2a_4}{a_2}}\frac{A}{\varepsilon k^2\tanh\left(\sqrt{-\frac{a_2}{2}}\xi\right)}+\frac{3\varepsilon\delta}{2k}\sqrt{-\frac{a_2}{2a_4}}\tanh\left(\sqrt{-\frac{a_2}{2}}\xi\right). \tag{24}$$

Case-II: For $a_0 = a_1 = 0,\ \ a_2 > 0,\ a_4 > 0,\ \ a_3 = -2\sqrt{a_2 a_4},\ \varepsilon = \pm 1$, then Eq. (11) has a dark soliton solution:

$$U(\xi) = \frac{1}{2}\sqrt{\frac{a_2}{a_4}}\left[1 + \varepsilon \tanh\left(\frac{1}{2}\sqrt{a_2}\,\xi\right)\right]. \tag{25}$$

The second family of chirped dark soliton solution of Eq. (1) as

$$q(x,t) = \sqrt{\frac{1}{2}\sqrt{\frac{a_2}{a_4}}\left[1 + \varepsilon \tanh\left(\frac{1}{2}\sqrt{a_2}\,\xi\right)\right]}\, e^{i[\chi(\xi) - \omega t]}, \tag{26}$$

and its corresponding chirping takes the form

$$\delta\omega = -\frac{v}{2k^2} - \sqrt{\frac{a_4}{a_2}}\frac{2A}{k^2\left(1 + \varepsilon \tanh\left(\frac{1}{2}\sqrt{a_2}\,\xi\right)\right)} + \frac{3\delta}{4k}\sqrt{\frac{a_2}{a_4}}\left(1 + \varepsilon \tanh\left(\frac{1}{2}\sqrt{a_2}\,\xi\right)\right). \tag{27}$$

Case-III: For $a_0 = a_1 = 0,\ a_2 > 0$, then Eq. (11) has a dark soliton solution:

$$U(\xi) = \frac{2a_2}{\varepsilon\sqrt{\Delta}\cosh\left(\sqrt{a_2}\,\xi\right) - a_3}, \qquad \Delta = a_3^2 - 4a_2 a_4 > 0, \quad \varepsilon = \pm 1. \tag{28}$$

Hence, we obtain the following chirped dark soliton solution for Eq. (1):

$$q(x,t) = \left[\frac{2a_2}{\varepsilon\sqrt{\Delta}\cosh\left(\sqrt{a_2}\,\xi\right) - a_3}\right]^{1/2} e^{i[\chi(\xi) - \omega t]}. \tag{29}$$

For this case, corresponding chirping is given as

$$\delta\omega = -\frac{v}{2k^2} - \frac{A\left(\varepsilon\sqrt{\Delta}\cosh\left(\sqrt{a_2}\,\xi\right) - a_3\right)}{2a_2 k^2} + \frac{3\delta a_2}{2k\left(\varepsilon\sqrt{\Delta}\cosh\left(\sqrt{a_2}\,\xi\right) - a_3\right)}. \tag{30}$$

3.3. Periodic solitons

We have found that there are three types of periodic soliton solutions for Eq. (11) under the following parametric conditions:

Case-I: For $a_0 = a_1 = a_3 = 0,\ a_2 < 0,\ a_4 > 0,\ \ \varepsilon = \pm 1$, then Eq. (11) has a periodic soliton solution

$$U(\xi) = \varepsilon\sqrt{-\frac{a_2}{a_4}}\sec\left(\sqrt{-a_2}\,\xi\right). \tag{31}$$

We then obtain the following chirped periodic soliton solution for Eq. (1):

$$q(x,t) = \left[\varepsilon\sqrt{-\frac{a_2}{a_4}}\sec\left(\sqrt{-a_2}\,\xi\right)\right]^{1/2} e^{i[\chi(\xi) - \omega t]}. \tag{32}$$

The chirping is given as

$$\delta\omega = -\frac{v}{2k^2} - \sqrt{-\frac{a_4}{a_2}}\frac{\varepsilon A}{k^2\sec\left(\sqrt{-a_2}\,\xi\right)} + \frac{3\varepsilon\delta}{2k}\sqrt{-\frac{a_2}{a_4}}\sec\left(\sqrt{-a_2}\,\xi\right). \tag{33}$$

Case-II: For $a_1 = a_3 = 0,\ a_0 = \frac{a_2^2}{4a_4},\ a_2 > 0,\ a_4 > 0,\ \ \varepsilon = \pm 1$, then Eq. (11) has a periodic soliton solution:

$$U(\xi) = \varepsilon\sqrt{-\frac{a_2}{2a_4}}\tan\left(\sqrt{\frac{a_2}{2}}\,\xi\right). \tag{34}$$

We then obtain the following chirped periodic soliton solution for Eq. (1):

$$q(x,t) = \left[\varepsilon\sqrt{-\frac{a_2}{2a_4}}\tan\left(\sqrt{\frac{a_2}{2}}\,\xi\right)\right]^{1/2} e^{i[\chi(\xi) - \omega t]}. \tag{35}$$

The chirping is given as

$$\delta\omega = -\frac{v}{2k^2} - \sqrt{-\frac{2a}{a_2}}\frac{\varepsilon A}{k^2\tan\left(\sqrt{\frac{a_2}{2}}\xi\right)} + \frac{3\varepsilon\delta}{2k}\sqrt{-\frac{a_2}{2a_4}}\tan\left(\sqrt{\frac{a_2}{2}}\xi\right). \quad (36)$$

Case-III: For $a_0 = a_1 = a_4 = 0$, $a_2 < 0$, $a_4 > 0$, then Eq. (11) has a periodic soliton solution:

$$U(\xi) = -\frac{a_2}{a_3}\sec^2\left(\frac{\sqrt{-a_2}}{2}\xi\right). \quad (37)$$

Then we get the following chirped periodic soliton solution for Eq. (1):

$$q(x,t) = \left[-\frac{a_2}{a_3}\sec^2\left(\frac{\sqrt{-a_2}}{2}\xi\right)\right]^{1/2} e^{i[\chi(\xi)-\omega t]}. \quad (38)$$

And corresponding the chirping is given as

$$\delta\omega = -\frac{v}{2k^2} + \frac{a_3 A}{a_2 k^2\sec^2\left(\frac{\sqrt{-a_2}}{2}\xi\right)} - \frac{3\delta a_2}{2ka_3}\sec^2\left(\frac{\sqrt{-a_2}}{2}\xi\right). \quad (39)$$

IV. Conclusions

In this paper, we have obtained chirped bright, dark and periodic soliton solutions for the generalized derivative nonlinear Schrödinger equation. After introducing a new ansatz that includes a novel form of chirping, the solutions were investigated within the framework of a general fourth order elliptic equation involving many parameters.

The evolution of the wave amplitude is shown to satisfy a nonlinear differential equation involving two integration constants which can be readily determined with the initial parameters of the wave. We analytically solved the resulting amplitude equation and obtain results for bright, dark and periodic soliton solutions. We have further determined the nonlinear chirp associated with each of these soliton solutions.

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