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The African Roots and Contemporary Study of Complex Geometry

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Abstract. *Mainstream (Eurocentric) texts on Complex Analysis, including one of the most popular texts used in undergraduate courses on the subject (James Ward Brown and Ruel V. Churchill's Complex Variables and Applications) and one of the most popular texts used in graduate courses on the subject (Dennis Zill and Patrick Shanahan's Complex Analysis: A First Course with Applications) say nothing about the African origins and contemporary study of the subject. This paper seeks to fill a part of this gap by providing evidence on Complex Geometry gleaned from non-mainstream mathematical works. The thesis that underlies this paper is therefore as follows: Any study of Complex Analysis which ignores its African origins and the continuation of its study to this day will miss a very significant aspect of the subject. Thus, the essence of this paper hinges upon how autonomous evidences from different sources are marshalled into a cohesive whole.*

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I. Introduction

Mainstream (Eurocentric) texts on *Complex Analysis* (defined in the next paragraph) begin by tracing the origins of the subject to the 16th Century AD when European mathematicians were forced to admit that other kinds of numbers existed that were not positive integers. These books then add that no one person "invented" *Complex Numbers* (also defined in the ensuing paragraph), which led to controversies surrounding the use of these numbers [**1**, **6**]. Consequently, these texts miss including the origins of the subject in Ancient Kemet/Egypt that dates back to 17000 BCE and its study to the present day. Thus, the thesis of this paper is that the study of *Complex Analysis* which ignores its African origins and the continuation of its study to this day will miss a very significant aspect of the subject. In this paper, my aim is to fill a part of this gap by providing evidence on *Complex Geometry* culled from non-mainstream mathematical works. The significance of this paper therefore lies upon how attestations from independent sources are brought together to provide an interconnected portrait of the African roots and contemporary study of *Complex Geometry*. Before doing all this, however, I will first end this section with a definition of *Complex Analysis* and what it covers for those readers who may not be familiar with the subject.

Complex Analysis is generally defined as the study of *Complex Numbers*, which are numeral values in the following form: "$z = a = ib$ where a and b are real numbers and i is the imaginary unit" [**6**]. Thus, the notations $a + ib$ and $a = bi$ are utilized interchangeably. The real number a in $z = a + ib$ is referred to as the *real part* of z, while the real number b is referred to as the *imaginary part* of z. The real part of the *Complex Number* is abbreviated Re(z) and the imaginary part is abbreviated lm(z) [**1**, **6**].

The major areas of focus of *Complex Analysis* are the following: (a) *Complex Arithmetic* which involves addition and multiplication operations that are defined on complex numbers; thus, subtraction and division must, in some way, be defined in terms of these two operations; (b) *Complex Algebra* which computes a set of complex numbers that can be defined as the set of two-dimensional real vectors (x, y) with one extra operation—complex multiplication; (c) *Complex Geometry* which concerns the properties and relations of points, lines, surfaces, solids, and higher dimensional analogs complex manifolds, complex algebraic varieties, and functions of several complex variables; included in *Complex Geometry* are the applications of transcendental methods to algebraic geometry, as well as the more geometric aspects of *Complex Analysis*; (d) *Elementary Functions* which engage functions of single variables (either real or complex) that are denoted as taking sums, products, and compositions of finitely many polynomial, rational, trigonometric, hyperbolic, and exponential functions, including possibly their inverse functions; and (e) *Integrals* which deal with functions of which given functions are the derivatives, i.e. which yield those functions when differentiated, and which may express the areas under the curves of the graphs of the functions [see **1** & **6**].

II. EVIDENCE FROM CHEIKH ANTA DIOP.

In his book titled *Civilization or Barbarism* (1981), Cheikh Anta Diop addresses several issues. The first issue is that Egypt was not a white civilization which was made up by Europeans in the 19th Century to reinforce racism and imperialism. Another issue is that Egypt was a black society and Blacks should be heirs to Egypt's legacy. The other issue is that Greek civilization owes its success and knowledge to Ancient Egyptian ideas and accomplishments. In Chapter 16 titled "Africa's Contribution: Sciences," Diop devotes three large sections to (1) Egyptian Mathematics: Geometry, (2) Algebra, and (3) Arithmetic, respectively, with the other sections dealing with Astronomy, Medicine, Chemistry, and Architecture, all of which are undergirded with their mathematics [**2**]. The following subsections entail evidence pertaining to *Complex Geometry* in the book.

Problem 10 of the *Papyrus of Moscow*. This is the first text in the Papyrus of Moscow and the first six lines are transcribed from Ancient Egyptian hieroglyphics. The very last line which is line six is controversial because it says *ges pw n inr*, which translates into English as "half of an egg." The sequence of equations is as follows [**2**]:

$$9 - 1 = 8$$
$$\frac{8}{9} = \frac{2}{3} + \frac{1}{6} + \frac{1}{18}$$
$$8 - \frac{8}{9} = 8 - \left(\frac{2}{3} + \frac{1}{6} + \frac{1}{18}\right) = 7 + \frac{1}{9}$$
$$\left(7 + \frac{1}{9}\right) \text{ x } \left(4 + \frac{1}{2}\right) = 32 = \text{surface}$$

Problem 14 of the *Papyrus of Moscow*. This exercise deals with the volume of a truncated pyramid which shows us that the Ancient Egyptians knew the exact formula. If not for this exercise, there would be a debate to this day about whether the Ancient Egyptians knew the volume of a pyramid, despite the materiality of Egypt's pyramid. This formula is for one of the complex expressions that were vandalized [**2]**:

$$V = \frac{h}{3}(a^2 + ab + b^2)$$

The Ancient Egyptians knew all the more that

$$V = \frac{h}{3}a^2$$

Problems 56 to 60 of the *Rhind Papyrus*. These are the most complex Ancient Egyptian mathematical expression that survived the vandalism of the conquerors. They show us that 2,000 years before the Greeks, the Ancient Egyptians studied the mathematics of the cone and used different trigonometric lines. The tangent, sine, cosine, and cotangent were used to calculate slopes. The Ancient Egyptian mathematical equation for the *pi* or π, which is the ratio of the circumference of any circle to the diameter of that circle, is as follows [**2**]:

$$\pi = 3.16$$

Archimedes represented this as π = 3.14. He did not explicitly calculate the value 3.1416; instead, he showed that the ratio of the circumference is situated between 3.1/7 and 3.10/71. He also did not credit the Ancient Egyptians, unbeknown to him that the ancient papyrus would one day reveal the truth to posterity [**2**].

Cylinder tangential to a sphere. This is the only case where there is equality between the height of a cylinder and the diameter of the circle at the base. This happens to be that of the inscribed sphere calculated by an Ancient Egyptian scribe. This is also the same figure that Archimedes used as an epitaph and as his best discovery. The equation derived after the literal notation is as follows [2]:

$$S\frac{1}{s} \text{ sphere} = 2 \text{ x } \frac{256}{81} \text{ x } r^2 = \underline{2\pi r^2}$$

Egyptian balance with cursors. The Ancient Egyptians are the inventors of the scale where the manipulator acts on slides in an initial position symmetrical to the central support. They used a lever with two unequal arms and a counterweight to easily draw water. The "part of support" from Archimedes was already there 2,000 years before he was born [**2**]. The equation to the English literature the phrase "Egyptian balance with cursors" is found is the following [**5**]:

$$r = a(\text{i} + \cos^6)$$

The watering of a garden using the *shadoof*. This was already in Ancient Egypt a mechanical application of the lever with unequal arms. It was the instrument that would allow Archimedes to "lift up the Earth, if he had a point of support." It was invented by the Ancient Egyptians 1,000 years before he was born. The Ancient Egyptians, however, used it to water their gardens [**2**].
The *shadoof* has been explained in terms of the dotted figures that embody a number of important notions. To begin with, the Ancient Egyptian symbol *shen* means eternity, and it also well known as the Ancient Egyptian cartouche. Next, there are two signs of equality: one of these signs is directed from ring of Y Holes to South Barrow; the other sign is directed tangentially to a ring of Z Holes. This symbolism is intended to describe the same mathematical expression with numerator and denominator, which denotes the name *Atum*. All this has been explained as follows:

The ring of X Holes symbolizes "big five" number and "big ring":

$$\pi.\Phi = 5.08$$

The ring of Z Holes symbolizes "small five" number and "small ring":

$$e^{\Phi} = 5.04$$

The ring of Y Holes symbolizes result of division ("big five" over "small five")"

$$\frac{\pi.\Phi}{e\Phi} = j$$

where the value $j = 1.0079\ldots$ [**7**].

Problem 53 of the *Rhind Papyrus*. This problem shows a figure derived from the "Theorem of Thales," albeit 700 years before Thales. Thales had to have been a pupil of Ancient Egyptian priests. This is because the shadow cast by a stick meets at the end of the shadow cast by the Great Pyramid and, therefore, this formula [**2**]:

$$0 = (A - B) \text{ x } (B - C) = (A - B) \text{ x } (B + A) = |A|2 - |B|2; \text{ therefore, } |A| = |B|$$

meaning that A and B are equidistant from the origin: i.e. from the center of M.

Ellipse drawn on a wall of the Temple of Luxor. On the wall of the Temple of Luxor is an ellipse drawn which was built under Ramses III, around 1200 BC. While doubt remains as to the problem that the Ancient Egyptian technician wanted to solve, it definitely relates to the property of a ellipse. The figure presents itself as looking for the surface (S) of an ellipse; thus, the following equation [**2**]:

$$S = \pi ab = 1 \text{ x } 1\tfrac{1}{2} \text{ x } \pi = 4.71 \text{ (exact value)}$$

In taking the diameters, which are 2 and 3, instead of the demi-axes, the formula yields the following [**2**]:

$$S = \left(2 - \frac{2}{7}\right) \text{ x } \left(3 - \frac{2}{7}\right) = 4.65$$

and the error of the Ancient Egyptian architecture would be $\frac{6}{471}$ or $\frac{1}{8}$

Surface of the sphere. Problem 10 of the Papyrus of Moscow deals with the curved surface of a half-sphere. T. Eric Peet did not hesitate with dishonesty to propose irrational changes of the text of the problem by confusing a half-sphere with a half-cylinder. Richard J. Gillings did not doubt that Problem 10 dealt with the surface of a half-sphere and that Ancient Egyptians established the equation by considering that the quantity of bark used to make a basket is double that to make a lid, which is a circle whose surface they knew how to calculate using the following equation [**2**]:

$$S = 4\pi R^2$$

Square root, so-called Pythagoreum Theorem and irrational numbers. The Ancient Egyptians knew how to extract the square root. They even knew how to do it for complicated whole or fractional numbers. The term that designates the square root also means the right angle of a square, *knbt*, to extract the square root. The Ancient

Egyptians defined a basic unit of length referred to as "double remen, which is equal to the diagonal of a square of little side a = one cubit (royal). Stated differently, if *d* is the diagonal, then the length itself is the "double remen." The Ancient Egyptians calculated the "remen" as follows and the "double remen" as shown in the subsequent subsection [**2**]:

$$d = a\sqrt{2} = (\sqrt{2} \text{ x } 20.6) = 29.1325 \text{ inches}$$

The royal cubit = 20.6 inches

The remen = $\frac{d}{2} = \frac{\sqrt{2}}{2}a$ = 14.6 inches

Royal cubit of May. The Ancient Egyptians defined a fundamental unit of length called "double remen," which is equal to the diagonal of a square of little side a = one cubit (royal). If *d* is that diagonal, then one has "double remen." The definition of the "double remen" shows that Pythagoras was nether the inventor of irrational numbers nor of the theorem with his name. The calculation of the "double remen" is as follows [**2**]:

$$S = \frac{1}{2} a^2 = \frac{1}{4} d^2 \rightarrow a^2 = \frac{1}{2} d^2 \rightarrow 2a^2 = d^2$$

where $a\sqrt{2} = d = a$ double remen.

Quadrature of the circle. The Ancient Egyptians knew the problem of the circle's quadrature. They also were the first to pose it in the history of mathematics. In Problem 48 of the Rhind Papyrus, it is a matter of comparing the surface of a square of a side of nine units to that of an inscribed circle with a diameter of nine units, represented by the following formula [**2**]:

$$A = \pi r2$$

where A is the area of a circle and *r* is the radius of the circle.

Problem 48 of the *Rhind Papyrus*. This problem deals with the squaring of the circle. It compares the area of a circle with a diameter of nine to a square with a side of nine. It is somewhat exceptional within the papyri because it does not contain written instructions that tell the scribe what to do. The formula is presented in the explication of the quadrature of the circle [**2**].

Problem 50 of the *Rhind Papyrus*. The problem involves with the area of a circle with a dimeter of nine by squaring the circle. The problem asks: "A circular field has a diameter nine *khe*t. What is the area?" A *khet* is a length of measurement of about 50 meters [**2**]. The following is the procedure for calculating the area of a circle with a diameter of nine by squaring the circle:

$$A = \pi r^2$$
$$d = 2r$$

Solving for A

$$A = \frac{1}{4}\pi d^2 = \frac{1}{4} \text{ x } \pi \text{ x } 9^2 \approx 63.61725$$

Problem 51 of the *Rhind Papyrus*. This problem concerns the area of a triangle with a height of 13 and a base of five. The Ancient Egyptians used this to find the area or a triangular piece of land. In Problem 52, discussed next, a scribe checks to see if the numbers are correct that seem to be absent. The median *AB* for the problem is derived with the following equation [**2**]:

The angle $a = ABC$ (with apex B)

Problem 52 of the *Rhind Papyrus* The problem is about the area of a trapezoid with a large base of six, small base of four, and a height of 20. The Ancient Egyptians knew how to find the area of a trapezium by taking ½ and multiplying it by the sum of the bases, by the length of a line (*meret*). The length of a line could be the side or a line that represents the altitude. Thus, the calculation for the trapezoid area is as follows [**2**]:

Trapezoid area = ((sum of the bases) ÷ height

Surface of the Circle: $S = \pi R^2$. The formula for the surface of a circle which was already known by the Ancient Egyptians is as follows [**2]**:

$$S = \left(\frac{8}{9}d\right)^2$$

which is equivalent to the following formula:

$$S = \pi\frac{d^2}{4} \text{ or } \pi R^2$$

Extracted from the Egyptian formula, π (pi) happens to be 3.1605, unlike in Babylonian formulae when π was 3. This is because irrational numbers were not perceived or thought of yet in Babylon.

Surface of the rectangle: S = L X *l*. Two problems that deal with the surface of a rectangle from two different viewpoints are Problem 49 of the Rhind Papyrus and Problem 6 of the Papyrus of Moscow. Problem 49 gives two different dimensions and asks for the surface. Problem 6 gives the surface, but it also gives the width as a fraction of the length. This is expressed as follows [**2**]:

$$S = 12,\ l = \frac{1}{2} + \frac{1}{4} \text{ of } L$$

When L and *l* are calculated, one gets L = 4; *l* = 3.

Surface of the triangle. The Ancient Egyptians knew about the surface of the triangle well before other civilizations and that the approximate formula is as follows [**2**]:

$$S = \left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right)$$

The Ancient Egyptians used *k3w* for heights of three-dimensional objects and *meryt* for heights of plane figures. The inscription in the Temple of Edfu, which was constructed by Ptolemy XI, is often used to discredit the Egyptians by saying that they did not know the formula for the triangle's surface [**2**].

Surface of the trapezium. The formula for the surface of the trapezium is the following [**2**]:

$$S = \frac{A+B}{2} \text{ x } h \text{ (or the half-sum of the bases times the height)}$$

This formula is used in Problem 52 of the Rhind Papyrus and dimensions given in terms of *khets* to find the surface of the trapezium. The problem gives the length of the parallel sides and the distances or bases between them [**2**].

Volumes of the cylinder, of the parallelepiped, and of the sphere. Three problems (41, 42 & 43) in the Rhind Papyrus deal with the volume of a cylinder represented as *š,*□, not *ipt*. Some Western mathematicians deliberately confused *dbm*, meaning cylinder, and *ifd*, meaning cube or parallelpiped, depending on the case, to discredit Ancient Egyptian Mathematics. Problem 41 shows the volume of a cylinder of a diameter 9 and a height of 10 units.

Problem 44 of the Rhind Papyrus calculates a cube—i.e. square base, *ifd*; three equal sides as follows [**2**]:

š*3*□ *dbn* = cylinder
š*3*□ *ifd* = cube or parallelopiped, depending on the case

The following is the formula for the volume of the cylinder:

$$V = \pi R^2 h = \left(\frac{16}{9}R\right)^2 \text{ x } h = \left(\frac{16}{9}\right)^2 \text{ x } R^2 \text{ x } h = S \text{ x } h$$

Volume of the sphere? The problem of Plate VIII of the Kahun Papyrus comes into play when figuring out what the problem is the scribe was asked to do. It either asked for the volume of a sphere or the volume of a cylindrical silo [**2**]. The formula for the volume of a sphere is as follows:

$$V = \frac{4}{3}\pi r^3$$

III. Evidence From Ron EGLASH.

The book titled *African Fractals: Modern Computing and Indigenous Design* (1999) by Ron Eglash is about fractals and how they can be seen in every part of African culture. This includes the layout of settlements, painting, hairstyles, fractal geometry, and many other aspects. It also examines the political and social applications of fractals since the existence of Africans. Part II of the book is on "African fractal mathematics," which explores geometric algorithms, scaling, numeric systems, recursion, infinity, and complexity [**3**]. In the subsections that ensue, evidence about *Complex Geometry* in the book is presented.

The Jola settlement of Mlomp, Senegal. By using a combination of imaginary and repeating processes, you get the Mlomp model which shows the layout of the Jola settlement in Senegal from an aerial view. The building complexes were not preplanned, but nobody knows the sequence of construction because there are not any oral historians. The spiral structures are symbolic of the chief's drinking vessel and it was based on the maintenance of the sacred forests around each of the neighborhoods [**3**]. The equation for a spiral is as follows:

$$r = ae^{\Theta \cot b}$$

where r = the radius of each turn of the spiral; a and b = constants that depend on the specific spiral; Θ = the angle of rotation as the curve spirals; and e = the base of the natural algorithm.

Neural network model for the Jola funeral ritual. The Jola (in Senegal) funeral starts with a deceased body placed on a platform and a post on each corner held by pallbearers. A priest then asks questions based on critical knowledge, perhaps murder, and the pallbearers move the platform right for yes, left for no, and forward for unknown based on the force of the deceased person. The pallbearers exert force, whether intentionally or unintentionally, which is considered to be more effective than a vote. By modeling each force as direction and magnitude, the neural network is shown where all four pallbearers and inputs are all connected, which means that each of them would have to exert force and sense each other's force [**3**]. The following is the simple formula for a neural network model:

$$y = b + \Theta + x$$

where y = the output layer or vector, b = units in the layer, Θ = weight, and x = the input layer or vector.

Lusaka. The lusaka was developed by the Baluba of the Congo to be used as a visual memory board, but it shows fractal scaling. It is mostly based on physical coding like color, but it is also mostly a "geometry of ideas," whereby each structure corresponds to a historical event. In addition, there is coding variation whereby single beads are used for individuals, groups of beads for royal courts, and larger bead arrangements for sacred forests that have been growing for years [**3**]. The fractal dimension (D) of geometric objects is calculated as follows:

$$D = \log(N) \div \log(r)$$

where $\log(N)$ = the logarithm of the number of units; $\log(r)$ = logarithm of the magnification factor

From order to disorder in a Bakuba cloth. The Bakuba of the Congo make cloth designs that are somewhat similar up and down, but constantly change from right to left which happens to suggest order-disorder. A computer scientist by the name of Clifford Pickover developed a similar pattern to show how order to disorder can be seen by taking a variable chosen at random and giving it influence over the equation of the graph. By doing this, he noticed that the design typically moved between 1 and 1.5, from periodic to fractal, as opposed to going all the way to disorder [**3**]. A simple equation for fractal order-disorder is the following:

$$Z_{n+1} = Z_n^{\ 2} + C$$

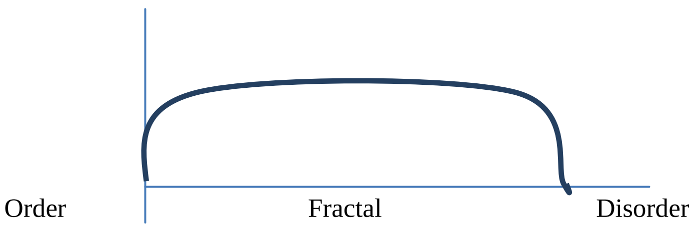
where C and Z = complex numbers; n = zero or positive integer (natural number).

Block print textile. An African textile that goes from order to full disorder is the block print. Chinua Achebe's famous novel, *Things Fall Apart*, is said to be similar to the block pattern [**3**]. For instance, while examining periodicity and fundamental blocks in African print textiles, Paulus Gerdes discovers the existence of Latin squares, magic squares, and Arithmetic modulo n. He calculates the Arithmetic modulo n as follows [**4**]:

$(1 + 2 \text{ x } 3) + (3 + 2 \text{ x } |-1|) = 3$ (modulo 5)
or

$$7 + 1 = 3 \text{ (modulo 5)}$$

African complexity concepts in religion. *Complex Geometry* has been shown to exist in African concepts of religion. One of these is Nyame's power of life turbulent waters of Tanu. The other is Mawu (acts through lower gods). Another is Amma (an expanding spiral like a whirlwind). By looking at the central peak of spiritual power and computational power in the Crutchfield-Smale complexity measure, the concepts have been determined to be equivalent represented as follows [**3**]:



Source: Self-generated by this author

Geometric design in Mangbetu personal adornment. The decorative end of a Mangbetu ivory hairpin is a scaling design (of the Congo), but taking a closer look reveals a geometric process. The Mangbetu hairstyle features a disk vertically angled at 45 degrees. Just as the heads on the ivory hairpin are angled at 45 degrees, the Mangbetu hair dresses are also at 45 degrees. By taking a look at the geometric relations of the hairpin, one can sees the alternate interior angles of a transversal that intersects two parallel lines. The geometric construction algorithm of the Mengbetu sculpture is as follows [**3**]:

$$\tan \Theta_1 = \frac{\frac{\sqrt{2}}{2}}{\frac{3}{\sqrt{2}}}$$

$$\Theta_1 = \arctan \frac{1}{3} \approx \ 18^0$$

IV. EVIDENCE FROM PAULUS GERDES.

The mathematician Paulus Gerdes in his book, *Geometry from Africa: Mathematical and Educational Explorations* (1999), examines African geometry and patterns and how they can be used to teach mathematics. He shows geometry in carving, weaving, and various aspects of art and how they can be used educationally. He also shows that one can detect the Pythagorean Theorem and how to find proofs of it by using African ornaments and artifacts. The book is cleaved into four major sections that examine the following topics: (1) "On geometrical ideas in Africa south of the Sahara"; (2) "from African designs to discovering the Pythagorean Theorem"; (3) Geometrical ideas in crafts and possibilities for their educational exploration"; and (4) "The 'sona' sand drawing tradition and possibilities for its educational use" [**4**]. Presented in the subsections that follow is evidence relating to *Complex Geometry* in the book.

From woven buttons to the Theorem of Pythagoras. In the south of Mozambique in order to attach a cover to a basket, two strips that are plaited with loops at the end are attached to the cover, and two cube shaped buttons are fastened to opposite sides. To close the basket, each loop is pulled around each button. By looking at the button design, the conclusion reached is the following [**4**]:

$$C = A + B$$

which is the Pythagorean Theorem. By joining the square designs to form a pattern then comparing the areas, one may find:

$$S + T = U$$

From decorative designs with fourfold symmetry to Pythagoras. Rotational symmetry of 90 degrees, which is called a fourfold symmetry, occurs frequently in African decorations. Under a fourfold symmetry, four points that correspond to one another always create the vertices of a square. Therefore [**4**]:

$$(a + b)^2 = 2ab + c^2$$

Taking into account the equality

$$(a + b)^2 = 2ab + a^2 + b^2$$

one may find

$$a^2 + b^2 = c^2$$

which happens to be the Pythagorean proposition.

From a widespread decorative motif to the discovery of Pythagoras' and Pappus' theorems. A decorative motif that was well known in ancient Egypt and was on a painted basket in the tomb of Rekhmire has a long tradition all over Africa. For example, in such a motif, it is possible to transform a toothed square into a real square with the same area. The area of a toothed square (T) is equal to the sum of the area of two smaller squares (A and B) and, thus, the following equation [**4**]:

$$T = A + B$$

This leads to the conclusion that the toothed square area (T) is equal to the "real" square area (C). Since $C = T$, the conclusion that is reached is the following:

$$A + B = C$$

From mat weaving patterns to Pythagoras, and Latin and magic squares. Weaving patterns are usually represented on squared paper by brown and white diagrams. In Chokwe fabric, a white square correlates to where the white (horizontal) strand passes over the brown (vertical) strand. Therefore, a brown square correlates to where the brown (vertical) strand passes over a white (horizontal) strand. If an oblique grid is composed with a brown square at the center, then it is surrounded by four congruent quadrilaterals. Consequently, the area of an oblique square (C) is equal to the sum of the areas of a brown square (A) and a white square (B). So, again, $A + B = C$ [**4**].

Rolling up mats. In Mozambique and other parts of Africa, woven mats for sitting or sleeping in the shape of a rectangle are rolled up when not in use. Rolling up one of those mats in the shape of a cylinder does not change the weight or volume of the map, allowing for the determination of the area of a circle and the volume of a cylinder as shown in the following two subsections [**4**].

Area of a circle. In the north of Mozambique, Makhuma artisans also produce circular mats in addition to rectangular mats. A circular mat is made by braiding a strand of sisal (a type of plant with stiff fibers) which is then rolled into a coil. Each circuit of coil is then sewn together and the end is then cut off a little to look like a perfect circle. This means that [**4**]:

$$A \approx \left(\frac{8d}{9}\right)^2$$

which is similar to the area of a circle as calculated in Ancient Egypt.

Volume of a cylinder. The volume and area of a cylinder may be decided similarly. By taking the thickness of the mat as a linear measurement, one finds that the volume is equal to length (L) times width (W). After the mat is rolled up, the top area (A) is equal to L. The volume of a cylinder is derived by the following equation [**4**]:

$$V \approx L \text{ x } W \approx A \text{ x } W \approx A \text{ x } h$$

where h is the height of the cylinder (which is the width of the mat). In essence, the volume of a cylinder by advancing with the limit is as follows: $\pi 2^2 h$.

Exploring rectangle constructions used in traditional house building. Most Africans south of the Sahara build houses with a circular or rectangular base. In Mozambique, two methods are used for rectangular bases. One from a different cultural environment would be surprised to see that rectangles can be formed without constructing right angles one by one. Rectangle constructions are equivalent to the following two Euclidean geometry theorems [**4**]:

Theorem 1: A parallelogram with congruent diagonals is a rectangle.

Theorem 2: A quadrilateral with congruent diagonals that intersect at their midpoints is a rectangle.

The preceding theorems can be linked to rectangle axioms and vector geometry as shown in the following two subsections.

Rectangle axioms. When looking for possible alternatives of axiomatic constructions for Euclidean geometry, the "rectangle axiom" by Alexandrov of the Soviet Union is a substitute for Euclid's fifth postulate. Alexandrov's "rectangle axiom" says the following [**4**]:

Rectangle Axiom: If in a quadrilateral $ABCD$, $AD = BC$ and $\angle A$ and $\angle B$ are right angles, then $AB = DC$ and $\angle C$ and $\angle D$ are also right angles.

Therefore, other alternative "rectangle axioms" can be formulated by using the knowledge underlying the traditional Mozambican house building techniques.

Link with vector geometry. The knowledge involved in the construction methods of the rectangular bases of traditional houses can also be translated into the language of vector geometry and linear algebra, which gives a sufficient and necessary condition for the perpendicularity of two vectors. For example, in a construction, a projective space (P) can be the following [**4**]:

$$\text{P: } |p + q| = |p - \text{q}| \Leftrightarrow p \perp q$$

where p and q represent vectors.

Exploring a woven pyramid. Pyramidal baskets are woven in the north of Mozambique, in the south of Tanzania, in the Congo region, and in Senegal. The base of the basket is an equilateral triangle and the other four sides are isosceles right triangles, which form a triangular pyramid. The form and production process have been explored in the context volumes of pyramids and regular polyhedra as shown in the subsection that follows [**4**].

Volumes of pyramids and regular polyhedra. By showing an *eheleo*-pyramid (woven basket) as part of a cube and letting s be the side length of the cube, it is not necessary to know the formula for the volume of a pyramid to determine the volume of an *eheleo*-pyramid. The volume is equal to one half $ABCDH$ with a similar height (DH) and the cube's base as base, by comparing the corresponding horizontal layers of both pyramids. Since the last pyramid fits into the cube three times, the volume of the *eheleo*-pyramid (V_E) is one sixth the volume of the cube and, thus, the following equation [**4**]:

$$V_E = \frac{s^3}{6}$$

Four *eheleo*-pyramids—*ADCH*, *ABCF*, *CGHF*, and *HEAF*—surround the regular tetrahedron and together fill the cube. The volume (*V*) of the regular tetrahedron is two times the volume of the *eheleo*-pyramid or one third of the cube's volume. Therefore, the following equation holds [**4**]:

$$V_0 = \frac{8s^3}{6} = \frac{t^3\sqrt{2}}{3}$$

Exploring square mats and circular basket bowls. A type of basket bowl that is used as a sieve or as a dish for food is common in Eastern and Southern Africa. In the north of Mozambique, the Makonde call it *chelo*. A weaver makes the bottom by plaiting a square mat. To create the border a wooden board is blended and the two sides are fastened together. Then the sides of the mat are fastened to the respective midpoints of the border. After that, the basket is wet to make it more flexible and stepped on in the center by the weaver to create a basket [**4**]. The *Complex Geometry* for this procedure is explained in the subsection that ensues.

Discovering an approximation of the area of a circle. The designs of basket bowls can be used to discover an interesting approximation formula for the area of a circle. When the circle radius (r) measures four times the half diagonal of the first square, the area of the circle (A_c) will appear almost equal to the area of the square (A_s) of which the half diagonal is five times the half diagonal of the smallest square. This is stated formulaically as follows [**4**]:

$$A_c \approx A_s$$

Since A_s is twice the square of the semi diagonal and the semi diagonal is five fourths of the radius of the circle, it follows that [**4**]:

$$A_c \approx A_s = 2\left(\frac{5}{4}r\right)^2 = \frac{25}{8}r^2$$

which is equivalent to the ancient Babylonian formula for the precise determination for the area of a circle: i.e. $\pi \approx 25/8 = 3\ 1/8 = 3{,}125$ (relative error is approximately half a percent).

Exploring hexagonal weaving. Artisans make hats in Mozambique by knitting several circuits of a spiral together. The spiral consists of a woven zigzag band which is made out of two long strips of palm with the same width. The same technique is used to make handbags. In Nigeria, the zigzag band is used for making hats and in Kenya it is used as a decoration for circular mats. A geometric example of this procedure can be seen in the tiling discussed in the subsection that follows [**4**]. A trigonometric function and a Hexastrip weavable polyhedra and Euler's theorem are presented thereafter.

Tiling. Once one finds the regular hexagonal tiling, other tilings can be discovered by subdivision. By finding a tiling with equilateral triangles, one can construct other polygons. One can therefore formulate the following three conjectures [**4**]:

(1) The sum of the measures of the internal angles of an n-gon is equal to $3(n - 2) \times 60$.

(2) Areas of similar figures are proportional to the squares of their sides, and the sum of the first n odd numbers is n^2.

(3) The sum of the first n odd numbers is n^2.

A trigonometric function. In African weaving, the angle of 60^0 (with its supplementary, which is 120^0) plays an essential role. This angle is materially enforced when using two strips of the same width. If one takes the width of the strip to be wrapped around the "horizontal" strip as a unit of measurement, and let the "horizontal" strip have a width a, the wrapping angle α will depend on a as follows [**4**]:

$$\alpha = \text{hex}(a)$$

Hexastrip weavable polyhedra and Euler's theorem. In the case of isomers of fullerenes C_n for which a hexastrip model can be constructed, it is easy to establish that there are always 12 pentagonal rings, which are independent of the number of hexagonal rings. By letting N_5 be the number of pentagonal holes in the hexastrip model, and letting N_6 be the number of hexagonal holes, the number of vertices (V), faces (F), and edges (E), one gets the following formula [**4**]:

$$V = 5N_5 + 6N_6$$
$$F = N_5 + N_6 + \frac{V}{3}$$
$$E = \frac{3V}{2}$$

The implication here is that $V + F - E = N\sqrt{6}$. Alternatively, $V + F - E = 2$, which is in agreement with Euler's theorem. This means that, just like the pentagonal rings, the number of hexagonal holes is equal to 12 [**4**].

From diagonally woven baskets and bells to a twisted decahedron. A geometrically interesting way of weaving baskets comes from the Makonde artisans who weave baskets with standardized sizes used to measure the production of maize, sorghum, and beans before taking the harvest to the storage houses. To make the *likalala* basket, an artisan weaves a square mat with visible mid lines. With two sets of six strands, the artisan weaves the center of the square called *yuyumunu*, meaning the mother who bears the whole square mat. A geometric example is shown in the ensuing subsection.

Volume of decahedral mini-bamboyo bell. A decahedron may be considered as a truncated pyramid of which four congruent wedges have been cut off. Since the bottom square has the same length as the squares of the decahedron, the volume is given by the following equation if integral calculus is used. By letting $A(y)$ be the area of a semi-rectangular horizontal section, where y is the distance between the section and base of the decahedron (d), the volume of the decahedron would be given by [**4**]:

$$V = \int_0^2 A(y)dy$$

V. SUMMARY AND FINAL REMARKS

The preceding sections clearly demonstrate that the exploration of *Complex Geometry* has its genesis in Ancient Kemet/Egypt. They also show that the exploration continues in Africa to this day. Thus, the thesis that underlies this paper, i.e. any study of *Complex Analysis* which ignores its African origins and the continuation of its study to this day will miss a very significant aspect of the subject, is valid.

In terms of my final remarks, there should be more studies to show how *Complex Analysis* in general is used as societies emerged and continue to evolve. This would be important because it would reveal the continuation or disruption of knowledge and culture. *Complex Analysis* should be taught with real-world examples. This is because not only would students learn the subject matter, they would also learn about its roots. With these examples, interest in mathematics would surge and students would know that most of the Greek mathematicians they heard about appropriated knowledge from other civilizations and capitalized on it to make themselves famous with schools named after them.

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