*IOSR Journal of Mathematics (IOSR-JM)*
*e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 18, Issue 3 Ser. I (May. – June. 2022), PP 51-55*
*www.iosrjournals.org*

# Versatile Geometric Distribution: A Potential Platform To Cover Frequency Aspect Of Non-Life Insurance Portfolio

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***Abstract***
*The well-known single parameter Geometric Distribution arising in the context of Bernoulli Trials has the negative exponential pattern for the underlying probability distribution and is generally considered suitable for individual non-life insurance policies. Portfolios covering a large number of policies pertaining to individual portfolios are likely to require different patterns for the number of claims. In the following Sections, we suggest a Multi-parametric Discrete Probability Distribution holding potential for Exponential, Negative Exponential, Double Exponential, Uniform, Peaked, and U-shaped patterns. While Modular Expressions are suggested for the end-user for the VGD, Explicit Expressions are obtained for the Characterizers' - Mean, Variance, and Median of the Simple Variants.*
***Key Words***
*Non-Life Insurance Portfolio. Probability Distribution of the Number of Claims. VGD- Versatile Geometric Distribution. Simple Variants of VGD. Contextual Characteristics of VGD and its Variants.*

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Date of Submission: 01-06-2022 Date of Acceptance: 13-06-2022

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## I. Introduction

Frequency-Number of Claims, Severity- Amount relating to the Compensation for the Incurred Loss, Management Expense and Safety / Profit Margin are the fundamental inputs in underwriting / pricing an insurance contract. In general, a non-life insurance company operates a portfolio with sections that involve inherently different characteristics. In the following sections we suggest a unified (parametric) approach to cover the frequency aspect of the expected claim experience of a non-life insurance portfolio.

## II. Versatile Geometric Distribution (Covering Different Claim Patterns)

Limitation on the Progression of Probability function considered in Section I can be taken care by involving, appropriately chosen, more parameters. A plausible choice for the number of claims in a portfolio of a non-life insurance company will be:

$$f(x) = PQ^{|x-m|},\ 0 \le x \le n, \tag{1}$$

with the provisions $0 \le f(x) \le 1$, $\sum_{x=0}^{n} f(x) = 1$ and real $m > 0$, $0 < P < 1$ and $Q > 0$.

*Figure 1* demonstrate Versatility of the proposed Probability Function $f(x)$.

It is well recognized that the probability function suggesting one or more of the patterns may not be suitable for any of the commonly run portfolios, though these may be acceptable in any other context.

With $M = [m]$, $f(x)$ can be expressed as under:

$$f(x) = \begin{cases} PQ^{m-x} & 0 \le x \le M \\ PQ^{x-m} & M+1 \le x \le n \end{cases} \tag{2}$$

Corresponding split approach can be adopted to obtain explicit expression for the MGF:

$$M_X(t) = \sum_{x=0}^{M} PQ^{m-x}e^{tx} + \sum_{x=M+1}^{n} PQ^{x-m}e^{tx} \quad (3)$$
$$= M(1,t) + M(2,t)$$

with

$$M(i,t) = \frac{A(i) - B(i)e^{C(i)t}}{1 - D(i)e^t}$$

where

$$A(i) = \begin{cases} PQ^m e^{-t} & i = 1 \\ -PQ^{1-m+n}e^{-n\,t} & i = 2 \end{cases} \qquad B(i) = \begin{cases} PQ^{-1+m-M} & i = 1 \\ -PQ^{1-m+M} & i = 2 \end{cases}$$

$$C(i) = \{1 + M \quad i = 1,2 \qquad \text{and} \qquad D(i) = \begin{cases} 1/Q & i = 1 \\ Q & i = 2 \end{cases}$$

Application of the model entails obtaining values of the parameters. To this end, expressions for Mean, Variance, Median, besides normalization equation may be useful. Modular expressions for Mean and Variance can be obtained in terms of $A(i)$, $B(i)$, $C(i)$, $D(i)$ and the end user can obtain numerical values for Mean and Variance by inputting the requisite values in the following expressions:

Mean: $\mu_X = u(1,1) + u(1,2)$ and
Variance: $\sigma_X^2 = \{u(2,1) + u(2,2)\} - \{u(1,1) + u(1,2)\}^2$

where $u(1,i) =$ coefficient of $t$ in $M(i,t)$ and $u(2,i) =$ coefficient of $t^2/2!$ in $M(i,t)$.

## III. Versatile 2 Parameter, Finite Range Geometric Distribution

In the present case progression of the probability ordinates remains akin to negative exponential. The two-way exponential versatility can be retained by the following 2-parameter, finite range probability function:

$$f(x) = PQ^x,\ x = 0(1)n \quad (4)$$

Versatility of the distribution in respect of progression of ordinates lies in the choice of $Q$ and $m$ ( *Table 2* and *Figure 1*).

- $Q = 1$ will render the progression of probability ordinates to be *uniform* [*Figure 1 (9)*]
- $Q < 1$ & $m = 0$ will render the progression of probability ordinates to be *negative exponential* [*Figure 1(1)*]
- $Q < 1$ & $m > 1$ yields shapes ranging from skewed to symmetrical for increase in value of $m$ and given $n$ [*Figure 1 (3), (5) and (7)*]
- $Q > 1$, on the other hand, will ensure the progression of probability ordinates to be *exponential* to symmetric bimodal for increase in value of $m$ [*Figure 1 (2), (4), (6) and (8)*].

Further, $\sum_{x=0}^{n} f(x) = 1$ will require

$$PQ^{n+1} - (P + Q) + 1 = 0 \quad (5)$$

$P$ being a probability ordinate, $P < 1$ for $n > 0$ and $P = 1$ when $n = 0$.

Characterizing expressions below, for the VGD, are obtained on replacing $Q^{n+1}$ from the equation (5).

Mean: $\mu_X = \frac{1-P}{1-Q} + \frac{1-P-Q}{1-Q}n$

Variance: $\sigma_X^2 = \frac{Q+(n+1)^2P\,(1-P-Q)}{(1-Q)^2}$ (6)

Median: $Md_X = \left[\log\left\{\frac{2P+Q-1}{2PQ}\right\}/\log\,(Q)\right]$

In particular, for the case $Q = 1$, with $P = \frac{1}{n+1}$, [*Figure 1 (9)*] the mean and variance yield to $\frac{n}{2}$ and $\frac{n(n+2)}{12}$ respectively.

## IV. Versatile Single Parameter, Finite Range Geometric Distribution

Following expressions for two Single Parameter Finite Range Distributions of interest are:

**Case:** $Q = P$ and $n-1$ for $n$:

Mean: $\mu_X = \dfrac{n + P - 2\,n\,P}{1-P}$

Variance: $\sigma_X^2 = \dfrac{P + n^2\,(1-2P)P}{(1-P)^2}$ (7)

Median: $Md_X = \log\left\{\dfrac{3P-1}{2P^2}\right\}/\log\,(P)$

**Case:** $Q = 1-P$, for $n = \infty$: this case reverts to the well-known geometric distribution referred to in Section V

Mean: $\mu_X = \dfrac{1-P}{P}$

Variance: $\sigma_X^2 = \dfrac{1-P}{P^2}$ (8)

Median: $Md_X = \log\left\{\dfrac{1}{2(1-P)}\right\}/\log\,(1-P)$

Examination of the requisite equation $P^{n+1} - 2P + 1 = 0$ in this case leads to the following interesting observations [1]:

- The following graph (*Figure 2*) leads to the conclusion that for a given $n$ and $P_0 = \left(\frac{2}{n+1}\right)^{1/n}$, the only stationary point over $(0,1)$ is $(P_0, f(P_0))$.
- $n$ and $P$ observe an interesting inequality [1]:

$$\left(\frac{n}{2}\right)^{n/(n-1)} < (n-1)$$

- *Table 1* for various $n$ suggests the interesting converging pattern of $P$.

## V. The Well-Known Geometric Distribution (GD)

The single parameter Probability Distribution $f(x) = PQ^x$, $x = 0(1)\infty$, $P, Q > 0$, $P + Q = 1$, usually studied in the context of Bernoulli trials [2] can be used to express, in the present context, the probability of $X$ claims arising over the cover period of the insurance contract under examination (in the context of non-life insurance business, it will be reasonable to assume a countably large value for the end value, $n$; though, rarely realized in the case of disasters – natural, e.g. floods / cyclones / earth quakes / epidemics or man-made, e.g. gang wars / riots / civil upsurges / world wars, large group insurance policies, e.g., Bancassurance and a common feature in the case of a Re-Insurer.

Contextual Characteristics of the distribution can be enunciated as under:

Progression of the Probability function is negative exponential and

Probability of no claim $= P$
Mean number of claims $= Q/P$
Variance $= Q/P^2$ (9)
Median $= \left[-\dfrac{log(2)}{log(1-P)} - 1\right]$

## References


[1]. Y. P. Sabharwal and S. Pal, "Equispaced Discrete Finite Range Geometric Distribution," The Actuary, 2000.

[2]. W. Feller, Introduction to the Theory of Probability and its Applications, Volume 1, 3 ed., New York: Wiley Series in Probability and Statistics, 1968.

**Table 1:** Convergence of $P$ for $n$

| n | 2 | 5 | 8 | 11 | 14 | 17 | 20 |
|---|---|---|---|---|---|---|---|
| P | 0.618034 | 0.508660 | 0.500994 | 0.500122 | 0.500015 | 0.500002 | 0.500000 |

**Table 2**: Choice of parameters P, Q, m

| Sl.No. | P | Q | m |
|---|---|---|---|
| 1 | 0.5 | 0.5 | 0 |
| 2 | 0.0000183158 | 1.2 | 0 |
| 3 | 0.221076 | 0.7 | 2 |
| 4 | 0.000943842 | 1.1 | 2 |
| 5 | 0.334204 | 0.5 | 7 |
| 6 | 0.00150812 | 1.1 | 7 |
| 7 | 0.111485 | 0.8 | 25 |
| 8 | 0.000882025 | 1.2 | 25 |
| 9 | $\frac{1}{51}$ | 1 | 0 |

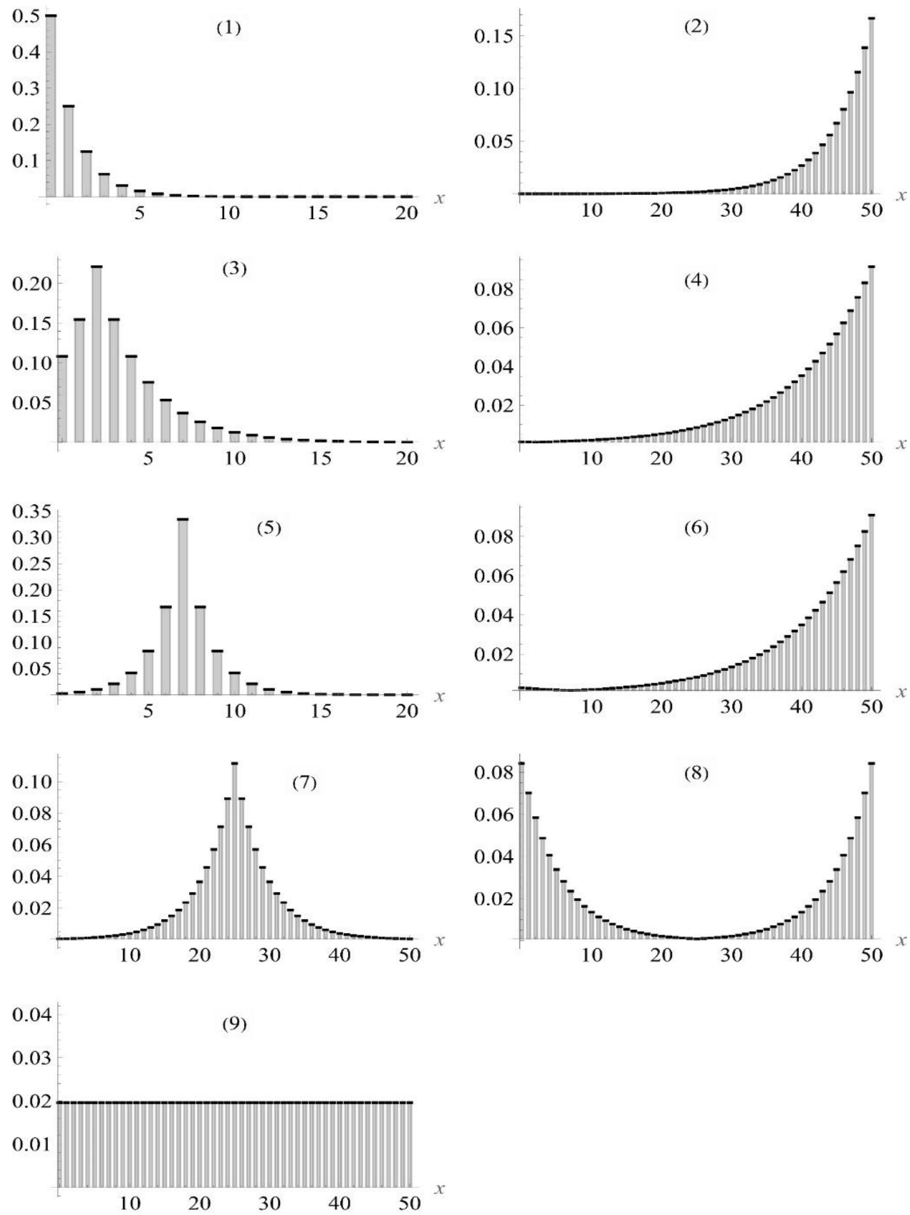


**Figure 1:** Probability function plot for choices of parameters in *Table 2*

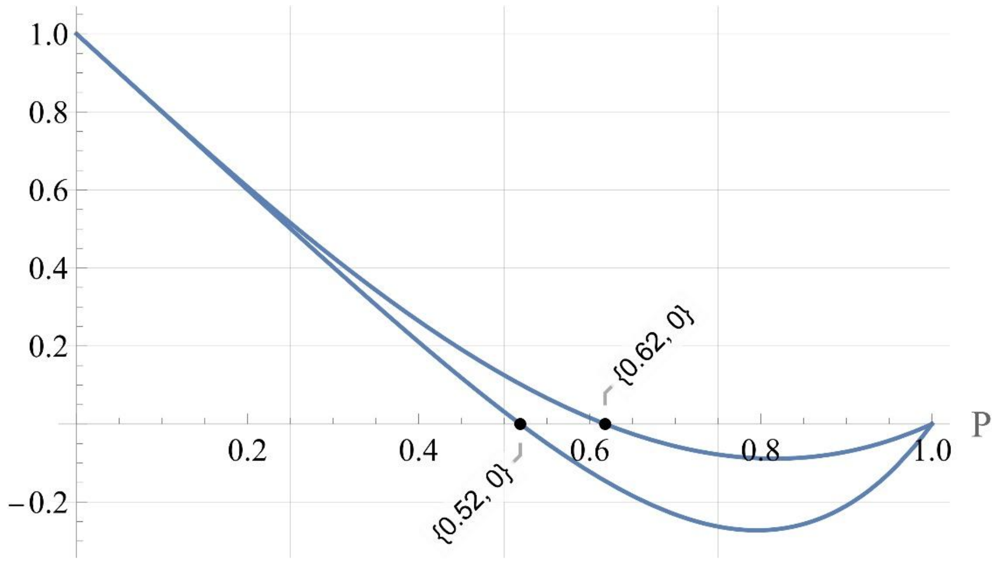


**Figure 2:** Stationary point over $(0,1)$ for values of $n = 2,4$

Dr. Y. P. Sabharwal, *et. al*. "Versatile Geometric Distribution: A Potential Platform To Cover Frequency Aspect Of Non-Life Insurance Portfolio." *IOSR Journal of Mathematics (IOSR-JM)*, 18(3), (2022): pp. 51-55.