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Asymptotic synchronization of memristive high-order competitive neural networks with mixed time-varying delays

Yi Jiang
College of Science, China Three Gorges University, Yichang, Hubei, 443002, China

Abstract
The synchronization problem of memristive high-order competitive neural networks with two types of time varying delays is studied. By using Lyapunov function method and inequality method, some sufficient conditions for asymptotic synchronization of high order memory competitive neural networks are given. Finally, a numerical example is shown to verify the correctness and validity of the theoretical result.

Keywords: *memristive high-order competitive neural neural network; Lyapunov function; synchronization.*

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I. Introduction

In the past few years, neural network has been widely used in many scientific fields such as pattern recognition, combination optimization, associative memory, static image processing, signal processing and so on. So far, there are many types of neural network, such as cellular neural network [1] chaotic neural network [2] and Hopfield neural network [3]. Many scholars only consider one time scale when studying neural network. In fact, neural network has more than one time scale, for example, competitive neural network has two time scales There are two different types of state of change in competitive neural networks, one is used to describe the short-term memory of rapid neural activity, and the other is used to describe the long-term memory of unsupervised synaptic changes that are slower [4]. Since Cohen and Grossberg [5] put forward the competitive neural network model in 1983, some scholars have carried out in- depth research on this model, for example, the stability of competitive neural networks with time delay is studied [6–13], Studies on synchronization of competitive neural networks with time delays [14–20], determination of observability of competitive neural networks with different time constants [21], and multi-stability and instability of competitive neural networks with non- monotone piecewise linear activation function [22]. Before no brings forward today, usually using resistance simulated biological synapses be- tween neurons in neural network, build a network model and carries on the research as resistor is no memory function, the brain is complicated, in 1971, Chua [23] for the first time put forward today, it is considered a fourth circuit element in addition to resistance, inductance and capaci- tance.Unfortunately, it was not until 2008, when the Hewlett-Packard [24] research team realized a new prototype of the memristor, that people began to pay attention to the memristor.Because of their nanoscale, low energy consumption, and storage properties, memristors can replace re- sistors. The memristor is introduced into the neural network and becomes the memristive neural networks. Compared with the traditional neural network, the computing capacity and memory storage capacity of the memristive neural networks are improved. At present, memristive neural networks has attracted many researchers to study it [25–31]. At present, there are a lot of research achievements related to first-order synaptic connection neural network, but the traditional first-order synaptic connection neural network has some limitations in pattern recognition and optimization problems, such as limited capacity high- order terms have a strong influence on the dynamic characteristics of neural network. By adding high-order synaptic connections to the neural network, the storage capacity of neural network is significantly increased and the model is optimized. Higher order neural network has stronger approximation, faster convergence rate, larger storage capacity and higher fault tolerance than first-order neural network [32–37]. The synchronous behavior of network is a basic characteristic of dynamic network, which means that the dynamic behavior of nodes in the network is close to the same state, which can promote the network to have more efficient and coordinated operation ability [38–40], and has been widely applied in practice. At present, there are few studies on the synchronization of high- order memristive competitive neural networks, therefore, it is meaningful to study asymptotic synchronization control of high-order competitive neural networks. The main contributions of this paper are as follows:(1) in the synchronous analysis of the competitive neural network, time-varying and distributed delays are considered, as well as second-order synaptic connections;(2) Global asymptotic synchronization is achieved by adaptive control strategy. The rest of the paper is composed as follows: Section 2 provides the description of the model and some preliminaries. The main outcomes are revealed in Section 3. In Section 4,one simulation examples are given to illustrate the effectiveness of the outcomes.Ultimately,conclusions are drawn in Section 5.

II. Preliminaries

First,the following memristive high-order competitive neural networks (MHCNN) with mixd time-varying delays is considerded:

$$\begin{cases} STM:\varepsilon\dfrac{dx_i(t)}{dt}=-a_ix_i(t)+\sum\limits_{j=1}^{n}\Big[c_{ij}\big(x_i(t)\big)h_j\big(x_j(t)\big)+d_{ij}\big(x_i(t-\tau(t))\big)h_j\big(x_j(t-\tau(t))\big)\Big] \\ \qquad +\sum\limits_{j=1}^{n}\sum\limits_{k=1}^{n}W_{ijk}\big(x_i(t)\big)h_j\big(x_j(t)\big)h_k\big(x_k(t)\big) \\ \qquad +\sum\limits_{j=1}^{n}\sum\limits_{k=1}^{n}R_{ijk}\big(x_i(t-\tau(t))\big)h_j\big(x_j(t-\tau(t))\big)h_k\big(x_k(t-\tau(t))\big) \\ \qquad +b_i\sum\limits_{s=1}^{p}v_{is}(t)\varsigma_s+I_i+\sum\limits_{j=1}^{n}e_{ij}\big(x_i(t)\big)\int_{t-\tau(t)}^{t}h_j\big(x_j(\Delta)\big)d\Delta, \\ LTM:\dfrac{dv_{is}}{dt}=-c_iv_{is}(t)+\varsigma_sh_i\big(x_i(t)\big), \end{cases} \tag{1}$$

where $i=1,2,...,n,s=1,2,...,p,x_i(t)$ is the neuron state, $a_i>0$ represents the decay of the neuron, b_i is the strength of the external stimulus, $c_i>0$ denotes a disposable scaling Constant, $h_i(\cdot)$ is an activation function, υ_{is} is the synaptic efficiency, ς_s represents an external stimulus, $\tau(t)$ and $\sigma(t)$ represent the discrete time-varying delay and distributed time-varying delay respectively,and satisfy $0<\tau_0\le\tau(t)\le\tau,0<\sigma_0\le\sigma(t)\le\sigma$. I_i denotes a bias or external input, $\varepsilon>0$ represents a time scale, $c_{ij}(x_i(t))$ and $d_{ij}(x_i(t-\tau(t)))$ represent the first-order memristive connection weights and delayed connection weights,respectively, $W_{ijk}(x_i(t))$ 、 $R_{ijk}(x_i(t-\tau(t)))$ represent the second-order memristive connection weights and delayed connection weights,respectively. According to the pinched hysteretic characteristic of the memristor,the memristive connection weights can be modelled mathematically as follows:

$$c_{ij}(x_i(t))=\begin{cases}c'_{ij}, & |x_i(t)|>\chi_i,\\ c''_{ij}, & |x_i(t)|\le\chi_i,\end{cases}\quad d_{ij}(x_i(t-\tau(t)))=\begin{cases}d'_{ij}, & |x_i(t-\tau(t))|>\chi_i,\\ d''_{ij}, & |x_i(t-\tau(t))|\le\chi_i,\end{cases}$$

$$W_{ijk}(x_i(t))=\begin{cases}W'_{ij}, & |x_i(t)|>\chi_i,\\ W''_{ij}, & |x_i(t)|\le\chi_i,\end{cases}\quad R_{ijk}(x_i(t-\tau(t)))=\begin{cases}R'_{ijk}, & |x_i(t-\tau(t))|>\chi_i,\\ R''_{ijk}, & |x_i(t-\tau(t))|\le\chi_i,\end{cases}$$

and $e_{ij}(x_i(t))=\begin{cases}e'_{ij}, & |x_i(t)|>\chi_i,\\ e''_{ij}, & |x_i(t)|\le\chi_i,\end{cases}$

where $D^{-}x_i(\cdot)$,denotes the left Dini derivation of $x_i(\cdot)$.

For simplicity,we will give some notations:

$\breve{c}_{ij} = \min\{c'_{ij}, c''_{ij}\}, \widehat{c}_{ij} = \max\{c'_{ij}, c''_{ij}\}, \bar{c}_{ij} = \max\{|c'_{ij}|, |c''_{ij}|\}, \overline{C} = (\bar{c}_{ij})_{n\times n}$,

$\breve{d}_{ij} = \min\{d'_{ij}, d''_{ij}\}, \widehat{d}_{ij} = \max\{d'_{ij}, d''_{ij}\}, \bar{d}_{ij} = \max\{|d'_{ij}|, |d''|\}, \overline{D} = (\bar{d}_{ij})_{n\times n}$,

$\breve{W}_{ijk} = \min\{W'_{ijk}, W''_{ijk}\}, \widehat{W}_{ij} = \max\{W'_{ijk}, W''_{ijk_{ij}}\}, \overline{W}_{ijk} = \max\{|W'_{ijk}|, |W''_{ijk}|\}$,

$\overline{W_i} = (\overline{W}_{ijk})_{n\times n}, \varpi = \left(\overline{W_1} + \overline{W_1}^T, ..., \overline{W_n} + \overline{W_n}^T\right)^T$,

$\breve{R}_{ijk} = \min\{R'_{ijk}, R''_{ijk}\}, \widehat{R}_{ijk} = \max\{R'_{ijk}, R''_{ijk}\}, \overline{R}_{ijk} = \max\{|R'_{ijk}|, |R''_{ijk}|\}$,

$\overline{R}_i = (\overline{R}_{ijk})_{n\times n}, \Re = \left(\overline{R}_1 + \overline{R}_1^T, ..., \overline{R}_n + \overline{R}_n^T\right)^T$,

$\breve{e}_{ij} = \min\{e'_{ij}, e''_{ij}\}, \widehat{e}_{ij} = \max\{e'_{ij}, e''_{ij}\}, \bar{e}_{ij} = \max\{|e'_{ij}|, |e''|\}, \overline{E} = (\bar{e}_{ij})_{n\times n}$,

$A = diag\{a_1, a_2, ..., a_n\}$, $B = diag\{b_1, b_2, ..., b_n\}$, $C = diag\{c_1, c_2, ..., c_n\}$.

Remark 1. According to the above definition,the memristive connection weights of system (1) are binary switching. If $c'_{ij} = c''_{ij}, d'_{ij} = d''_{ij}, W'_{ijk} = W''_{ijk}, R'_{ijk} = R''_{ijk}, e'_{ij} = e''_{ij}$, $i, j, k = 1,2,...,n$,then,system (1) becomes a traditional high-order competitive neural network[].

Introducing new variables $\overline{S}_i(t) = \sum_{s=1}^{p} v_{is}\varsigma_s = \varsigma^T v_i(t)$,where $\varsigma = (\varsigma_1 \varsigma_2, ..., \varsigma_p)^T$, $v_i = (v_{i1}, v_{i2}, ..., v_{ip}(t))^T$,it is not difficult to find that $\|\varsigma\|^2 = \|\varsigma_1\|^2 + \|\varsigma_2\|^2 + ... + \|\varsigma_p\|^2$ is a constant. Without loss of generality,we assume that $\|\varsigma\|^2 = 1$,then system (1) can be simplified as:

with

$$\begin{cases} STM: \dfrac{dx_i(t)}{dt} = -\frac{a_i}{\varepsilon}x_i(t) + \frac{1}{\varepsilon}\sum_{j=1}^{n}\left[c_{ij}\left(x_i(t)\right)h_j\left(x_j(t)\right) + d_{ij}\left(x_i\left(t-\tau(t)\right)\right)h_j\left(x_j\left(t-\tau(t)\right)\right)\right] \\ \qquad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}W_{ijk}\left(x_i(t)\right)h_j\left(x_j(t)\right)h_k\left(x_k(t)\right) \\ \qquad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}R_{ijk}\left(x_i\left(t-\tau(t)\right)\right)h_j\left(x_j\left(t-\tau(t)\right)\right)h_k\left(x_k\left(t-\tau(t)\right)\right) \\ \qquad + \frac{1}{\varepsilon}b_i\bar{s}_i(t) + \frac{1}{\varepsilon}I_i + \frac{1}{\varepsilon}\sum_{j=1}^{n}e_{ij}\left(x_i(t)\right)\int_{t-\tau(t)}^{t}h_j\left(x_j(\Delta)\right)d\Delta, \\ LTM: \dfrac{d\bar{s}_i(t)}{dt} = -c_i\bar{s}_i(t) + h_i\left(x_i(t)\right), \end{cases} \tag{2}$$

the initial condition

$$\begin{cases} x_i(s) = \phi_i(s) \in C([-\tau,0],R) \\ \bar{s}_i(s) = \beta_i(s) \in C([-\tau,0],R) \end{cases}, \quad s \in [-\tau,0]. \tag{3}$$

Since the classical definition of solution is invalid due to the discontinuity of memristive connection weights,we need to introduce the following definition of solution in sense of Fillipov for system (2).

A function: $u_i(t) = (x_i(t), \bar{s}_i(t))^T : [-\tau, T) \to R^2$. $T \in (0,+\infty]$ is a solution of system (2)

with the initial value (3),if $h_i(\cdot)$ is continuous on $[-\tau,T)$ and absolutely continuous on any

closed subinterval of $[0,T)$,and for almost all $t \in [0,T)$ satisfies:

whe

$$\begin{cases} STM: \varepsilon \dfrac{dx_i(t)}{dt} \in -\frac{1}{\varepsilon} a_i x_i(t) + \frac{1}{\varepsilon} \sum_{j=1}^{n} \Big[K[c_{ij}(x_i(t))] h_j(x_j(t)) + K[d_{ij}(x_i(t-\tau(t)))] h_j(x_j(t-\tau(t))) \Big] \\ \qquad + \frac{1}{\varepsilon} \sum_{j=1}^{n} \sum_{k=1}^{n} K[W_{ijk}(x_i(t))] h_j(x_j(t)) h_k(x_k(t)) \\ \qquad + \frac{1}{\varepsilon} \sum_{j=1}^{n} \sum_{k=1}^{n} K[R_{ijk}(x_i(t-\tau(t)))] h_j(x_j(t-\tau(t))) h_k(x_k(t-\tau(t))) \\ \qquad + \frac{1}{\varepsilon} b_i \bar{s}_i(t) + \frac{1}{\varepsilon} I_i + \frac{1}{\varepsilon} \sum_{j=1}^{n} K[e_{ij}(x_i(t))] \int_{t-\tau(t)}^{t} h_j(x_j(\Delta)) d\Delta, \\ LTM: \dfrac{d\bar{s}_i(t)}{dt} \in -c_i \bar{s}_i(t) + h_i(x_i(t)), \end{cases} \tag{4}$$

re

$$K[c_{ij}(x_i(t))] = \begin{cases} c'_{ij}, & |x_i(t)| > \chi_i, \\ c''_{ij}, & |x_i(t)| \le \chi_i, \end{cases} \quad K[d_{ij}(x_i(t-\tau(t)))] = \begin{cases} d'_{ij}, & |x_i(t-\tau(t))| > \chi_i, \\ d''_{ij}, & |x_i(t-\tau(t))| \le \chi_i, \end{cases}$$

$$K[W_{ijk}(x_i(t))] = \begin{cases} W'_{ij}, & |x_i(t)| > \chi_i, \\ W''_{ij}, & |x_i(t)| \le \chi_i, \end{cases} \quad K[R_{ijk}(x_i(t-\tau(t)))] = \begin{cases} R'_{ijk}, & |x_i(t-\tau(t))| > \chi_i, \\ R''_{ijk}, & |x_i(t-\tau(t))| \le \chi_i, \end{cases}$$

and $K[e_{ij}(x_i(t))] = \begin{cases} e'_{ij}, & |x_i(t)| > \chi_i, \\ e''_{ij}, & |x_i(t)| \le \chi_i, \end{cases}$

by the measurable selection theorem of differential inclusion [],there exist measurable selection function $\mu_{ij}(t) \in K[c_{ij}(x_i(t))]$, $\upsilon_{ij}(t) \in K[d_{ij}(x_i(t-\tau(t)))]$ $\varpi_{ijk}(t) \in K[W_{ijk}(x_i(t))]$,

$\pi_{ijk}(t) \in K[R_{ijk}(x_i(t-\tau(t)))]$,and $m_{ij}(t) \in K[e_{ij}(x_i(t))]$ such that:

$$\begin{cases} STM: \dfrac{dx_i(t)}{dt} = -\frac{1}{\varepsilon}a_i x_i(t) + \frac{1}{\varepsilon}\sum_{j=1}^{n}\left[\mu_{ij}(t)h_j\left(x_j(t)\right) + \nu_{ij}(t)h_j\left(x_j\left(t-\tau(t)\right)\right)\right] \\ \qquad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}\varpi_{ijk}(t)h_j\left(x_j(t)\right)h_k\left(x_k(t)\right) \\ \qquad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}\pi_{ijk}(t)h_j\left(x_j\left(t-\tau(t)\right)\right)h_k\left(x_k\left(t-\tau(t)\right)\right) \\ \qquad + \frac{1}{\varepsilon}b_i\overline{s}_i(t) + \frac{1}{\varepsilon}I_i + \frac{1}{\varepsilon}\sum_{j=1}^{n}m_{ij}(t)\int_{t-\tau(t)}^{t}h_j\left(x_j(\Delta)\right)d\Delta, \\ LTM: \dfrac{d\overline{s}_i(t)}{dt} = -c_i\overline{s}_i(t) + h_i\left(x_i(t)\right), \end{cases} \tag{5}$$

for almost all $t \in [0,T)$ and $i = 1,2,...,n$.

If we think of system (2) as a driving system,the response system is giving by:

with

$$\begin{cases} STM: \dfrac{dy_i(t)}{dt} = -\frac{1}{\varepsilon}a_i y_i(t) + \frac{1}{\varepsilon}\sum_{j=1}^{n}\left[c_{ij}\left(y_i(t)\right)h_j\left(y_j(t)\right) + d_{ij}\left(y_i\left(t-\tau(t)\right)\right)h_j\left(y_j\left(t-\tau(t)\right)\right)\right] \\ \qquad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}W_{ijk}\left(y_i(t)\right)h_j\left(y_j(t)\right)h_k\left(y_k(t)\right) \\ \qquad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}R_{ijk}\left(y_i\left(t-\tau(t)\right)\right)h_j\left(y_j\left(t-\tau(t)\right)\right)h_k\left(y_k\left(t-\tau(t)\right)\right) \\ \qquad + \frac{1}{\varepsilon}b_i\overline{r}_i(t) + \frac{1}{\varepsilon}I_i + \frac{1}{\varepsilon}\sum_{j=1}^{n}e_{ij}\left(y_i(t)\right)\int_{t-\tau(t)}^{t}h_j\left(y_j(\Delta)\right)d\Delta + \frac{1}{\varepsilon}U_i(t), \\ LTM: \dfrac{d\overline{r}_i(t)}{dt} = -c_i\overline{r}_i(t) + h_i\left(y_i(t)\right), \end{cases} \tag{6}$$

the initial value of (6) defined as

$$y_i(t) = \gamma_i, \qquad \overline{r}_i(t) = o_i(t), \qquad -\tau \le t \le 0 \tag{7}$$

where the controller $U_i(t)$ is designed to ensure that systems (2) and (6) are synchronized, for $i = 1,2,...,n$.

According to the definition of Fillipov solution of system (6) with the initial value (7), for $t \in [-\tau, T)$, $T \in (0,+\infty]$, then it satisfifies:

By

$$\begin{cases} STM: \dfrac{dy_i(t)}{dt} \in -\frac{1}{\varepsilon}a_i y_i(t) + \frac{1}{\varepsilon}\sum_{j=1}^{n}\left[K[c_{ij}\left(y_i(t)\right)]h_j\left(y_j(t)\right) + K[d_{ij}\left(y_i\left(t-\tau(t)\right)\right)]h_j\left(y_j\left(t-\tau(t)\right)\right)\right] \\ \qquad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}K[W_{ijk}\left(y_i(t)\right)]h_j\left(y_j(t)\right)h_k\left(y_k(t)\right) + \frac{1}{\varepsilon}U_i(t) \\ \qquad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}K[R_{ijk}\left(y_i\left(t-\tau(t)\right)\right)]h_j\left(y_j\left(t-\tau(t)\right)\right)h_k\left(y_k\left(t-\tau(t)\right)\right) \\ \qquad + \frac{1}{\varepsilon}b_i\overline{r}_i(t) + \frac{1}{\varepsilon}I_i + \frac{1}{\varepsilon}\sum_{j=1}^{n}K[e_{ij}\left(y_i(t)\right)]\int_{t-\tau(t)}^{t}h_j\left(y_j(\Delta)\right)d\Delta, \\ LTM: \dfrac{d\overline{r}_i(t)}{dt} = -c_i\overline{r}_i(t) + h_i\left(y_i(t)\right), \end{cases} \tag{8}$$

the measurable selection theorem of differential inclusion [],there exist measurable selection functions $\mu_{ij}(t) \in K[c_{ij}(x_i(t))]$, $\upsilon_{ij}(t) \in K[d_{ij}(x_i(t-\tau(t)))]$,

$\overline{\omega}_{ijk}(t) \in K[W_{ijk}(x_i(t))]$, $\pi_{ijk}(t) \in K[R_{ijk}(x_i(t-\tau(t)))]$,and $m_{ij}(t) \in K[e_{ij}(x_i(t))]$,such that for almost all $t \in [0,T)$, $i = 1,2,...,n$,

$$\begin{cases} STM: \dfrac{dy_i(t)}{dt} = -\frac{1}{\varepsilon} a_i y_i(t) + \frac{1}{\varepsilon}\sum_{j=1}^{n}\left[\theta_{ij}(t)h_j\left(y_j(t)\right) + k_{ij}(t)h_j\left(y_j\left(t-\tau(t)\right)\right)\right] \\ \qquad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}\omega_{ijk}(t)h_j\left(y_j(t)\right)h_k\left(y_k(t)\right) \\ \qquad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}l_{ijk}(t)h_j\left(y_j\left(t-\tau(t)\right)\right)h_k\left(y_k\left(t-\tau(t)\right)\right) \\ \qquad + \frac{1}{\varepsilon} b_i \overline{r}_i(t) + \frac{1}{\varepsilon} I_i + \frac{1}{\varepsilon}\sum_{j=1}^{n} p_{ij}(t)\int_{t-\tau(t)}^{t} h_j\left(y_j(\Delta)\right)d\Delta, \\ LTM: \dfrac{d\overline{r}_i(t)}{dt} = -c_i\overline{r}_i(t) + h_i\left(y_i(t)\right), \end{cases} \tag{9}$$

Defifine synchronization error : $e_i(t) = (e_{1i}(t), e_{2i}(t))^T$, $e_{1i}(t) = y_i(t) - x_i(t)$,

$e_{2i}(t) = \overline{r}_i(t) - \overline{s}_i(t)$. Then,the error system can be described as the following form:

$$\begin{cases} \dfrac{de_{1i}(t)}{dt} = -\frac{1}{\varepsilon} a_i e_{1i}(t) + \frac{1}{\varepsilon}\sum_{j=1}^{n}\left[\theta_{ij}(t)h_j\left(y_j(t)\right) - \mu_{ij}(t)h_j\left(x_j(t)\right)\right] \\ \quad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\left[k_{ij}(t)h_j\left(y_j\left(t-\tau(t)\right)\right) - v_{ij}(t)h_j\left(x_j\left(t-\tau(t)\right)\right)\right] \\ \quad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}[\omega_{ijk}(t)h_j\left(y_j(t)\right)h_k\left(y_k(t)\right) - \varpi_{ijk}(t)h_j\left(x_j(t)\right)h_k\left(x_k(t)\right)] \\ \quad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}[l_{ijk}(t)h_j\left(y_j\left(t-\tau(t)\right)\right)h_k\left(y_k\left(t-\tau(t)\right)\right) - \pi_{ijk}(t)h_j\left(x_j\left(t-\tau(t)\right)\right)h_k\left(x_k\left(t-\tau(t)\right)\right)] \\ \quad + \frac{1}{\varepsilon} b_i e_{2i}(t) + \frac{1}{\varepsilon} I_i + \frac{1}{\varepsilon}\sum_{j=1}^{n}[p_{ij}(t)\int_{t-\tau(t)}^{t} h_j\left(y_j(\Delta)\right)d\Delta - m_{ij}(t)\int_{t-\tau(t)}^{t} h_j\left(x_j(\Delta)\right)d\Delta] \\ \dfrac{de_{2i}(t)}{dt} = -c_i e_{2i}(t) + h_i\left(y_i(t)\right) - h_i\left(x_i(t)\right), \end{cases} \tag{10}$$

for almost all $t \in [0,T)$, $i = 1,2,...,n$, where $g_i(e_{1i}(\cdot)) = h_i(y_i(\cdot)) - h_i(x_i(\cdot))$,

$$\xi_k = \begin{cases} \dfrac{\overline{W}_{ijk}h_k(y_k(t)) + \overline{W}_{ikj}h_k(x_k(t))}{\overline{W}_{ijk} + \overline{W}_{ikj}}, \overline{W}_{ijk} + \overline{W}_{ikj} \neq 0, \\ 0, \overline{W}_{ijk} + \overline{W}_{ikj} = 0, \end{cases}$$

$$\zeta_k = \begin{cases} \dfrac{\overline{R}_{ijk}h_k(y_k(t)) + \overline{R}_{ikj}h_k(x_k(t))}{\overline{W}_{ijk} + \overline{W}_{ikj}}, \overline{R}_{ijk} + \overline{R}_{ikj} \neq 0, \\ 0, \overline{R}_{ijk} + \overline{R}_{ikj} = 0, \end{cases}$$

Then the error system can be described as the following form :

$$\begin{cases} \frac{de_{1i}(t)}{dt} = -\frac{1}{\varepsilon}a_i e_{1i}(t) + \frac{1}{\varepsilon}b_i e_{2i}(t) + \frac{1}{\varepsilon}U_i(t) \\ \quad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\Big[(\theta_{ij} - \bar{c}_{ij})(t)h_j\big(y_j(t)\big) + (\bar{c}_{ij} - \mu_{ij}(t))h_j\big(x_j(t)\big)\Big] \\ \quad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\Big[(k_{ij}(t) - \bar{d}_{ij})h_j\big(y_j(t-\tau(t))\big) + (\bar{d}_{ij} - \nu_{ij}(t))h_j\big(x_j(t-\tau(t))\big)\Big] \\ \quad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}\Big\{[\bar{W}_{ijk} + \bar{W}_{ikj}]\xi_k g_j(e_{ij}(t)) + [\bar{R}_{ijk} + \bar{R}_{ijk}]\zeta_k g_j(e_{ij}(t-\tau(t)))\Big\} \\ \quad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}[[\omega_{ijk}(t) - \bar{W}_{ijk}]h_j\big(y_j(t)\big)h_k\big(y_k(t)\big) + [\bar{W}_{ijk} - \varpi_{ijk}(t)]h_j\big(x_j(t)\big)h_k\big(x_k(t)\big)] \\ \quad + \frac{1}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}[[l_{ijk}(t) - \bar{R}_{ijk}]h_j\big(y_j(t-\tau(t))\big)h_k\big(y_k(t-\tau(t))\big)] \\ \quad + \frac{1}{\varepsilon}\sum\sum[\bar{R}_{ijk} - \pi_{ijk}(t)]h_j\big(x_j(t-\tau(t))\big)h_k\big(x_k(t-\tau(t))\big) \\ \quad + \frac{1}{\varepsilon}\sum_{j=1}^{n}[p_{ij}(t)\int_{t-\tau(t)}^{t}h_j\big(y_j(\Delta)\big)d\Delta - m_{ij}(t)\int_{t-\tau(t)}^{t}h_j\big(x_j(\Delta)\big)d\Delta] \\ \frac{de_{2i}(t)}{dt} = -c_i e_{2i}(t) + g_i\big(e_{1i}(t)\big), \end{cases} \tag{11}$$

Defifinition 1. Systenm (6) under controller is globally asymptotic synchronized with system (2),if the error system (10),or (11) respectively satisfifies:

$$\lim_{t\to+\infty}\|e_{1i}(t)\|^2 = 0, \qquad \lim_{t\to+\infty}\|e_{2i}(t)\|^2 = 0, \qquad t \ge 0.$$

Assumption 1. For $\forall u, v \in R$, the activation function $h_i : R \to R$ satisfifies $|h_i(u)| \le m_i$, and

$|h_i(u) - h_i(v)| \le l_i|u - v|$, where $m_i > 0, l_i > 0,\ i = 1,2,...,n$.

Remark 2.Since systems (2) and (6) are state-dependent switching systems,they exhibit an unexpected parameter mismatch when difffferent initial conditions are chosen. Therefore, assumption 1 is necessary to ensure that system (6) is synchronized with system (2). This assumption is adopted to solve the synchronization problem for memristive neural network.

Remark 3. According to the Assumption 1,it is easy to see that $|\xi_k| \le M_k, |\zeta_k| \le M_k$.

Assumption 2. The discrete time-varying delay $\tau(t)$ and the distributed time-varying delay

$\sigma(t)$ respectively satisfy: $\dot{\tau}(t) \le \tau_D < 1, \qquad \dot{\sigma}(t) \le \sigma_D(t) < 1.$

Lemma 1 [41]. The inequality $2x^T y \le ax^T x + \frac{1}{a}y^T y, \forall x, y \in R^n (n \ge 1)$ holds,where $a > 0$.

Lemma 2 [40]. Suppose that Assumption 1 holds.Then system (2) with the initial condition (3) has at least one solution defifined on $(0, +\infty]$.

III. Main results

In this subsection,an adaptive state-feedback controller is designed,by using Lyapunov method,the following criteria for global asymptotic synchronization of systems (2) and (6) are presented.
The adaptive state-feedback controller is provided as follows:

$$\begin{cases} u_i(t) = -k_i(t)e_{1i}(t) - \eta_i(t)sign(e_{1i}(t)) - \varphi_i(t)x_i(t)sign(e_{1i}(t)x_i(t)), \\ \dot{k}_i(t) = p_i e_{1i}^2(t), \\ \dot{\eta}_i(t) = q_i\left|e_{1i}(t),\right| \\ \dot{\varphi}_i(t) = \psi_i\left|e_{1i}(t)x_i(t)\right|, \end{cases} \tag{12}$$

where Q_i , ω_i , n_i are all positive constants, for $i = 1,2,...,n$.

Theorem 1. Assumme that Assumption 1 and 2 hold.Then response MHCNN (6) can be globally asymptotically synchronized with drive MHCNN (2) under controller (12),if there exist positive numbers R_i , c_i , Q_i , ω_i , n_i such that

$$\lambda_{2i}c_i \geq \tfrac{1}{\varepsilon} z_{0i}^2 + z_{0i}^2, \tag{13}$$

$$\lambda_{3i}(1-\tau_D) \geq \tfrac{n}{\varepsilon} z_{0i}^2 + \tfrac{2}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}\overline{R}_{ijk}M_k l_j z_{0i}^2, \tag{14}$$

$$\begin{aligned} \omega_i \geq & \sum_{i=1}^{n}\left\{\tfrac{4}{\varepsilon}\sum_{j=1}^{n}\left|c'_{ij} - c''_{ij}\right|m_j + \tfrac{4}{\varepsilon}\sum_{j=1}^{n}\left|d'_{ij} - d''_{ij}\right|m_j\right\} \\ & + \tfrac{4}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\left\{\left|W'_{ijk} - W''_{ijk}\right|m_j m_k + \left|R'_{ijk} - R''_{ijk}\right|m_j m_k\right\} \\ & + \tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|e'_{ij} - e''_{ij}\right|\sigma m_j, \end{aligned} \tag{15}$$

$$\begin{aligned} a_i \geq & \tfrac{\varepsilon}{2}\left\{\tfrac{2}{\varepsilon}\sum_{j=1}^{n}\frac{z_{1ij}^2 + z_{0i}^2}{2} + \tfrac{2}{\varepsilon}\sum_{j=1}^{n}\frac{z_{2ij}^2}{2} + \lambda_{3i} + \sigma\tfrac{n}{\varepsilon} z_{0i}^2 + \lambda_{1i}c_i\right\} \\ & + \sum_{j=1}^{n}\sum_{k=1}^{n}\left\{\overline{W}_{ijk}M_k l_j z_{3ij}^2 + \overline{R}_{ijk}M_k l_j z_{4ij}^2\right\} + \tfrac{\varepsilon}{2} z_{6i}^2 \\ & + \sum_{j=1}^{n}\sum_{k=1}^{n}\left\{\overline{W}_{ijk}M_k l_j z_{0i}^2\right\} + \tfrac{1}{2} z_{5i}^2 + \tfrac{1}{2}\sum_{j=1}^{n}\sigma z_{7ij}^2. \end{aligned} \tag{16}$$

Proof. Consider the following Lyapunov function:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t),$$

where

$$V_1(t)=\sum_{i=1}^{n}[\lambda_{1i}e_{1i}^2(t)+\lambda_{2i}e_{2i}^2(t)],$$

$$V_2(t)=\sum_{i=1}^{n}\int_{t-\tau(t)}^{t}\lambda_{3i}e_{1i}^2(s)ds,$$

$$V_3(t)=\sum_{i=1}^{n}\int_{-\sigma(t)}^{0}\int_{t+v}^{t}\tfrac{n}{\varepsilon}z_{0i}^2e_{1i}^2(s)dsdv,$$

$$V_4(t)=\tfrac{1}{\varepsilon}\sum_{i=1}^{n}[\tfrac{1}{p_i}k_i^2(t)+\tfrac{1}{q_i}(\eta_i(t)-\omega_i)^2+\tfrac{1}{\psi_i}\varphi_i^2(t)],$$

where $\lambda_{1i}>0,\lambda_{2i}>0,\lambda_{3i}>0,n_i,\omega_i,v_i$ are not sure,for $i=1,2,...,n$.

Then,we calculate the upper right Dini derivative of V(t) along the trajectory of (11):

$$\begin{aligned}
D^{+}V_1(t)&=2\sum_{i=1}^{n}(\lambda_{1i}e_{1i}(t)\dot{e}_{1i}(t)+\lambda_{2i}e_{2i}(t)\dot{e}_{2i}(t))\\
&=\tfrac{-2}{\varepsilon}\sum_{i=1}^{n}\lambda_{1i}a_ie_{1i}^2(t)+\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_{1i}e_{1i}(t)\bar{c}_{ij}g_j(e_{1j}(t))+\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_{1i}e_{1i}(t)\bar{d}_{ij}g_j(e_{ij}(t-\tau(t)))\\
&=\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_{1i}[e_{1i}(t)(\theta_{ij}(t)-\bar{c}_{ij})h_j(y_j(t))+e_{1i}(t)(\bar{c}_{ij}-\mu_{ij}(t))h_j(x_j(t))]\\
&=\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_{1i}[e_{1i}(t)(k_{ij}(t)-\bar{d}_{ij})h_j(y_j(t-\tau(t)))+e_{1i}(t)(\bar{d}_{ij}-v_{ij}(t))h_j(x_j(t-\tau(t)))]\\
&=\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{1i}[e_{1i}(t)(\overline{W}_{ijk}+\overline{W}_{ikj})\xi_kg_j(e_{1j}(t))+e_{1i}(t)(\overline{R}_{ijk}+\overline{R}_{ikj}))\varsigma_kg_j(e_{1j}(t-\tau(t)))]\\
&=\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{1i}[e_{1i}(t)(\omega_{ijk}(t)-\overline{W}_{ikj})h_j(y_j(t))h_k(y_k(t))\\
&+e_{1i}(t)(-\overline{\omega}_{ijk}(t)+\overline{W}_{ikj})h_j(x_j(t))h_k(x_k(t))]\\
&+\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{1i}[e_{1i}(t)(l_{ijk}(t)-\overline{R}_{ijk})h_j(y_j(t-\tau(t)))h_k(y_k(t-\tau(t)))]\\
&+\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{1i}[e_{1i}(t)(-\pi_{ijk}(t)+\overline{R}_{ijk})h_j(x_j(t-\tau(t)))h_k(x_k(t-\tau(t)))]\\
&+\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\lambda_{1i}[-k_i(t)e_{1i}^2(t)-\eta_i(t)e_{1i}(t)sign(e_{1i}(t))-\varphi_i(t)e_{1i}(t)x_i(t)sign(e_{1i}(t)x_i(t))]\\
&+\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_{1i}e_{1i}(t)[\int_{t-\sigma(t)}^{t}p_{ij}(t)g_j(e_j(\Delta))d\Delta+\int_{t-\sigma(t)}^{t}(p_{ij}(t)-m_{ij}(t))h_j(x_j(\Delta))d\Delta]\\
&+\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\lambda_{1i}(b_ie_{1i}(t)e_{2i}(t))+2\sum_{i=1}^{n}\lambda_{2i}(g_i(e_{1i}(t))e_{2i}(t)-c_ie_{2i}^2(t)),
\end{aligned}\qquad(17)$$

$$D^{+}V_2(t)\le\sum_{i=1}^{n}\lambda_{3i}e_{1i}^2(t)-\sum_{i=1}^{n}(1-\tau_D)\lambda_{3i}e_{1i}^2(t-\tau(t)), \tag{18}$$

$$D^{+}V_3(t)\le\sum_{i=1}^{n}\sigma\tfrac{n}{\varepsilon}z_{0i}^2e_{1i}^2(t)-\sum_{i=1}^{n}\int_{t-\sigma(t)}^{t}\tfrac{n}{\varepsilon}z_{0i}^2e_{1i}^2(s)ds, \tag{19}$$

$$D^{+}V_4(t)\le\tfrac{2}{\varepsilon}\sum_{i=1}^{n}[k_ie_{1i}^2(t)+(\eta_i(t)-\omega_i)|e_{1i}(t)|+\varphi_i(t)|e_{1i}(t)x_i(t)|]. \tag{20}$$

According to Assumption 1 and Lemma 1,we can get:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\tfrac{2}{\varepsilon}e_{1i}(t)\bar{c}_{ij}g_j(e_{1j}(t))\le\tfrac{1}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}(z_{1ij}^2+z_{0i}^2)e_{1i}^2(t), \tag{21}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\tfrac{2}{\varepsilon}\lambda_{1i}e_{1i}(t)\bar{d}_{ij}g_j(e_{1j}(t-\tau(t)))\le\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}z_{2ij}^2e_{1i}^2(t)+\tfrac{1}{\varepsilon}\sum_{i=1}^{n}nz_{0i}^2e_{1i}^2(t-\tau(t)), \tag{22}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\tfrac{2}{\varepsilon}\lambda_{1i}e_{1i}(t)(\theta_{ij}(t)-\bar{c}_{ij})h_j(y_j(t))\le\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}(\lambda_{1i}|c'_{ij}-c''_{ij}|m_j|e_{1i}(t)|), \tag{23}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\tfrac{2}{\varepsilon}\lambda_{1i}e_{1i}(t)(\bar{c}_{ij}-\mu_{ij}(t))h_j(x_j(t))\le\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}(\lambda_{1i}|c'_{ij}-c''_{ij}|m_j|e_{1i}(t)|), \tag{24}$$

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{n}\tfrac{2}{\varepsilon}\lambda_{1i}[e_{1i}(t)(-\bar{d}_{ij}+k_{ij}(t))h_j(y_j(t-\tau(t)))+e_{1i}(t)(\bar{d}_{ij}-v_{ij}(t))h_j(x_j(t-\tau(t)))]\\&\le\tfrac{4}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}(\lambda_{1i}|d'_{ij}-d''_{ij}|m_j|e_{1i}(t)|),\end{aligned} \tag{25}$$

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\tfrac{2}{\varepsilon}\lambda_{1i}[e_{1i}(t)(\overline{W}_{ijk}+\overline{W}_{ikj})\xi_kg_j(e_{1j}(t))+e_{1i}(t)(\overline{R}_{ijk}+\overline{R}_{ikj})\zeta_kg_j(e_{1j}(t-\tau(t)))]\\&\le\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}[e_{1i}^2(t)\overline{W}_{ijk}M_kl_j(z_{3ij}^2+nz_{0i}^2)+\overline{R}_{ijk}M_kl_jz_{4ij}^2e_{1i}^2(t)+\overline{R}_{ijk}M_kl_jnz_{0i}^2e_{1i}^2(t-\tau(t))],\end{aligned} \tag{26}$$

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\tfrac{2}{\varepsilon}\lambda_{1i}[e_{1i}(t)(\omega_{ijk}(t)-\overline{W}_{ikj})h_j(y_j(t))h_k(y_k(t))+e_{1i}(t)(\overline{W}_{ikj}-\omega_{ijk}(t))h_j(x_j(t))h_k(x_k(t))]\\&\le\tfrac{4}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{1i}e_{1i}(t)|W'_{ijk}-W''_{ijk}|m_jm_k,\end{aligned} \tag{27}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\tfrac{2}{\varepsilon}\lambda_{1i}[e_{1i}(t)(l_{ijk}(t)-\overline{R}_{ikj})h_j(y_j(t-\tau(t)))h_k(y_k(t-\tau(t)))]$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\tfrac{2}{\varepsilon}\lambda_{1i}[e_{1i}(t)(-\pi_{ijk}(t)+\overline{R}_{ikj})h_j(x_j(t-\tau(t)))h_k(x_k(t-\tau(t)))]$$
$$\le\tfrac{4}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{1i}e_{1i}(t)\left|R'_{ijk}-R''_{ijk}\right|m_jm_k, \qquad (28)$$

$$\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\lambda_{1i}b_ie_{1i}(t)e_{2i}(t)\le\tfrac{1}{\varepsilon}\sum_{i=1}^{n}(z_{5i}^2e_{1i}^2(t)+z_{0i}^2e_{2i}^2(t)), \qquad (29)$$

$$\sum_{i=1}^{n}\lambda_{2i}g(e_{1i}(t))e_{2i}(t)\le\sum_{i=1}^{n}(z_{6i}^2e_{1i}^2(t)+z_{0i}^2e_{2i}^2(t)), \qquad (30)$$

$$\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_{1i}e_{1i}(t)[\int_{t-\sigma(t)}^{t}p_{ij}(t)g_j(e_j(\Delta))d\Delta+\int_{t-\sigma(t)}^{t}(p_{ij}(t)-m_{ij}(t))h_j(x_j(\Delta))d\Delta]$$
$$\le\tfrac{2}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_{1i}\left|e'_{ij}-e''_{ij}\right|\sigma m_je_{1i}(t)+\tfrac{1}{\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma z_{7ij}^2e_{1i}^2(t)+\tfrac{1}{\varepsilon}\sum_{i=1}^{n}\int_{t-\sigma(t)}^{t}nz_{0i}^2e_{1i}^2(\Delta)d\Delta, \qquad (31)$$

Then substituting (18)-(21),we can obtain

$$D^{+}V(t)\le\sum_{i=1}^{n}\{\tfrac{2}{\varepsilon}\sum_{j=1}^{n}\frac{z_{1ij}^2+z_{0i}^2}{2}+\tfrac{2}{\varepsilon}\sum_{j=1}^{n}\frac{z_{2ij}^2}{2}-\tfrac{2}{\varepsilon}a_i$$
$$+\tfrac{2}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}[\overline{W}_{ijk}M_kl_jz_{3ij}^2+\overline{R}_{ijk}M_kl_jz_{4ij}^2]+z_{6ij}^2$$
$$+\tfrac{2}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}\overline{W}_{ijk}M_kl_jz_{0i}^2+\tfrac{1}{\varepsilon}z_{5i}^2+\tfrac{1}{\varepsilon}\sum_{j=1}^{n}\sigma z_{7ij}^2+\lambda_{3i}+\sigma\tfrac{n}{\varepsilon}z_{0i}^2\}e_{1i}^2(t)$$
$$+\sum_{i=1}^{n}\{\sum_{j=1}^{n}\tfrac{4}{\varepsilon}\left|c'_{ij}-c''_{ij}\right|m_j+\sum_{j=1}^{n}\tfrac{4}{\varepsilon}\left|d'_{ij}-d''_{ij}\right|m_j$$
$$+\sum_{j=1}^{n}\sum_{k=1}^{n}\tfrac{4}{\varepsilon}\left|W'_{ijk}-W''_{ijk}\right|m_jm_k+\sum_{j=1}^{n}\sum_{k=1}^{n}\tfrac{4}{\varepsilon}\left|R'_{ijk}-R''_{ijk}\right|m_jm_k$$
$$+\tfrac{2}{\varepsilon}\sum_{j=1}^{n}\left|e'_{ij}-e''_{ij}\right|\sigma m_j-\omega_i\}\left|e_{1i}(t)\right|+\sum_{i=1}^{n}(\tfrac{1}{\varepsilon}z_{0i}^2+z_{0i}^2-2\lambda_{2i}c_i)e_{2i}^2(t)$$
$$+\sum_{i=1}^{n}(\tfrac{1}{\varepsilon}nz_{0i}^2+\tfrac{2}{\varepsilon}\sum_{j=1}^{n}\sum_{k=1}^{n}\overline{R}_{ijk}M_kl_jz_{0i}^2-\lambda_{3i}(1-\tau_D))e_{1i}^2(t-\tau(t)), \qquad (32)$$

From (13)-(16),we have

$$D^{+}V(t)\le\sum_{i=1}^{n}[-\lambda_{1i}c_ie_{1i}^2(t)-\lambda_{2i}c_ie_{2i}^2(t)]\le-\min_{1\le i\le n}c_iV_1(t), \qquad (33)$$

Integrating the (33) over interval $[t_0,t],t\ge t_0$,we can get

$$V(t)-V_0(t)\le -\min c_i\int_{t_0}^{t}V_1(s)ds,$$

$$V(t)+\min c_i\int_{t_0}^{t}V_1(s)ds\le V_0(t)<\infty,$$

from the above inequality,we can kmow

$$\lim_{t\to+\infty}\sup\int_{t_0}^{t}V_1(s)ds\le\frac{V_0(t)}{\min c_i}<\infty.$$

Therefore, $\lim_{t\to+\infty}V_1(t)=0$. From the defifinition of V1(t), obtaining that

$$\lim_{t\to+\infty}\|e_{1i}(t)\|^2=0 \text{ ,and } \lim_{t\to+\infty}\|e_{2i}(t)\|^2=0.$$

Which means the drive system (2) and the response system (6) are globally asymptotically synchronized.

IV. Numerical simulations

Example 1. Consider the delayed memrisive high-order competitive neural network with the following parameters:

$\chi_j=1$, $\varepsilon=2$, $I_1=I_2=0$, $M_1=M_2=1$, $l_1=l_2=1$, $\tau_1=\tau_2=0.1\sin(t)$, $\tau_3=\tau_4=0.2\cos(t)$,

$p_1=0.2\sin(t), p_2=0.1\sin(t), p_3=0.1t, p_4=0.5\sin^2(t)$,

$A=diag\{26,26\}, F=diag\{1.5,1.5\}, B=diag\{1,1\}$,

$$c_{11}(x_1(t))=\begin{cases}0.5, & |x_i(t)|>\chi_i,\\ 0, & |x_i(t)|\le\chi_i,\end{cases}\quad c_{12}(x_1(t))=\begin{cases}-1, & |x_i(t)|>\chi_i,\\ 0.5, & |x_i(t)|\le\chi_i,\end{cases}\quad c_{21}(x_2(t))=\begin{cases}1, & |x_i(t)|>\chi_i,\\ -1, & |x_i(t)|\le\chi_i,\end{cases}$$

$$c_{22}(x_2(t))=\begin{cases}0.8, & |x_i(t)|>\chi_i,\\ 0.2, & |x_i(t)|\le\chi_i,\end{cases}\quad d_{11}(x_1(t-\tau(t)))=\begin{cases}-1, & |x_i(t-\tau(t))|>\chi_i,\\ 0, & |x_i(t-\tau(t))|\le\chi_i,\end{cases}$$

$$d_{12}(x_1(t-\tau(t)))=\begin{cases}1, & |x_i(t-\tau(t))|>\chi_i,\\ 0.1, & |x_i(t-\tau(t))|\le\chi_i,\end{cases}$$

$$d_{21}(x_1(t-\tau(t)))=\begin{cases}-0.5, & |x_i(t-\tau(t))|>\chi_i,\\ 0.5, & |x_i(t-\tau(t))|\le\chi_i,\end{cases}\quad d_{22}(x_2(t-\tau(t)))=\begin{cases}1, & |x_i(t-\tau(t))|>\chi_i,\\ -1, & |x_i(t-\tau(t))|\le\chi_i,\end{cases}$$

$$W_{111}(x_1(t))=\begin{cases}0.1, & |x_i(t)|>\chi_i,\\ -1, & |x_i(t)|\le\chi_i,\end{cases}\quad W_{112}(x_1(t))=\begin{cases}1, & |x_i(t)|>\chi_i,\\ 0.5, & |x_i(t)|\le\chi_i,\end{cases}$$

$$W_{121}(x_1(t))=\begin{cases}0.1, & |x_i(t)|>\chi_i,\\ -0.1, & |x_i(t)|\le\chi_i,\end{cases}\quad W_{122}(x_1(t))=\begin{cases}0.1, & |x_i(t)|>\chi_i,\\ 1, & |x_i(t)|\le\chi_i,\end{cases}$$

$$W_{211}(x_2(t))=\begin{cases}-0.5, & |x_i(t)|>\chi_i,\\ 0.5, & |x_i(t)|\le\chi_i,\end{cases} \quad W_{212}(x_2(t))=\begin{cases}0.6, & |x_i(t)|>\chi_i,\\ 0.1, & |x_i(t)|\le\chi_i,\end{cases}$$

$$W_{221}(x_2(t))=\begin{cases}-0.1, & |x_i(t)|>\chi_i,\\ 0.3, & |x_i(t)|\le\chi_i,\end{cases} \quad W_{222}(x_2(t))=\begin{cases}1, & |x_i(t)|>\chi_i,\\ -1, & |x_i(t)|\le\chi_i,\end{cases}$$

$$R_{111}(x_1(t-\tau(t)))=\begin{cases}-0.2, & |x_i(t-\tau(t))|>\chi_i,\\ 0.5, & |x_i(t-\tau(t))|\le\chi_i,\end{cases} \quad R_{112}(x_1(t-\tau(t)))=\begin{cases}0.1, & |x_i(t-\tau(t))|>\chi_i,\\ 1, & |x_i(t-\tau(t))|\le\chi_i,\end{cases}$$

$$R_{121}(x_1(t-\tau(t)))=\begin{cases}0.5, & |x_i(t-\tau(t))|>\chi_i,\\ 1, & |x_i(t-\tau(t))|\le\chi_i,\end{cases} \quad R_{122}(x_1(t-\tau(t)))=\begin{cases}0, & |x_i(t-\tau(t))|>\chi_i,\\ 1, & |x_i(t-\tau(t))|\le\chi_i,\end{cases}$$

$$R_{211}(x_2(t-\tau(t)))=\begin{cases}0.1, & |x_i(t-\tau(t))|>\chi_i,\\ 1, & |x_i(t-\tau(t))|\le\chi_i,\end{cases} \quad R_{212}(x_2(t-\tau(t)))=\begin{cases}0.1, & |x_i(t-\tau(t))|>\chi_i,\\ -1, & |x_i(t-\tau(t))|\le\chi_i,\end{cases}$$

$$R_{222}(x_2(t-\tau(t)))=\begin{cases}0.1, & |x_i(t-\tau(t))|>\chi_i,\\ -0.2, & |x_i(t-\tau(t))|\le\chi_i,\end{cases}$$

$$e_{11}(x_1(t))=\begin{cases}1, & |x_1(t)|>\chi_i,\\ 0, & |x_1(t)|\le\chi_i,\end{cases} \quad e_{12}(x_1(t))=\begin{cases}0, & |x_1(t)|>\chi_i,\\ 0.5, & |x_1(t)|\le\chi_i,\end{cases}$$

$$e_{21}(x_2(t))=\begin{cases}0.5, & |x_2(t)|>\chi_i,\\ 1, & |x_2(t)|\le\chi_i,\end{cases} \quad e_{22}(x_2(t))=\begin{cases}1, & |x_2(t)|>\chi_i,\\ -1, & |x_2(t)|\le\chi_i,\end{cases}$$

According to above parameters, the Assumption 1 and Assumption 2 hold can be obtained, the initial conditions of the driving system are: $x_1=-6$, $x_2=6.5$, $\bar{s}_1=-1$, $\bar{s}_2=6$.

The initial value of the response system is: $y_1=4$, $y_2=-9$, $\bar{r}_1=-1$, $\bar{r}_2=-1$.

The activation function is: $h_j(x)=\tanh(|x|-1)$, The controller is:

$$\begin{cases}u_i(t)=-k_i(t)e_{1i}(t)-\eta_i(t)sign(e_{1i}(t))-\varphi_i(t)x_i(t)sign(e_{1i}(t)x_i(t)),\\ \dot{k}_i(t)=p_ie_{1i}^2(t),\\ \dot{\eta}_i(t)=q_i|e_{1i}(t),|\\ \dot{\varphi}_i(t)=\psi_i|e_{1i}(t)x_i(t)|,\end{cases} \tag{34}$$

Including, $p_1=1.5$, $p_2=2$, $q_1=3$, $q_2=0.5$, $\psi_1=4$, $\psi_2=2.5$. the driving system and the response system are synchronized under the controller (36) with the parameters. Fig. 3 clearly confifirms the feasibility of Theorem 1.

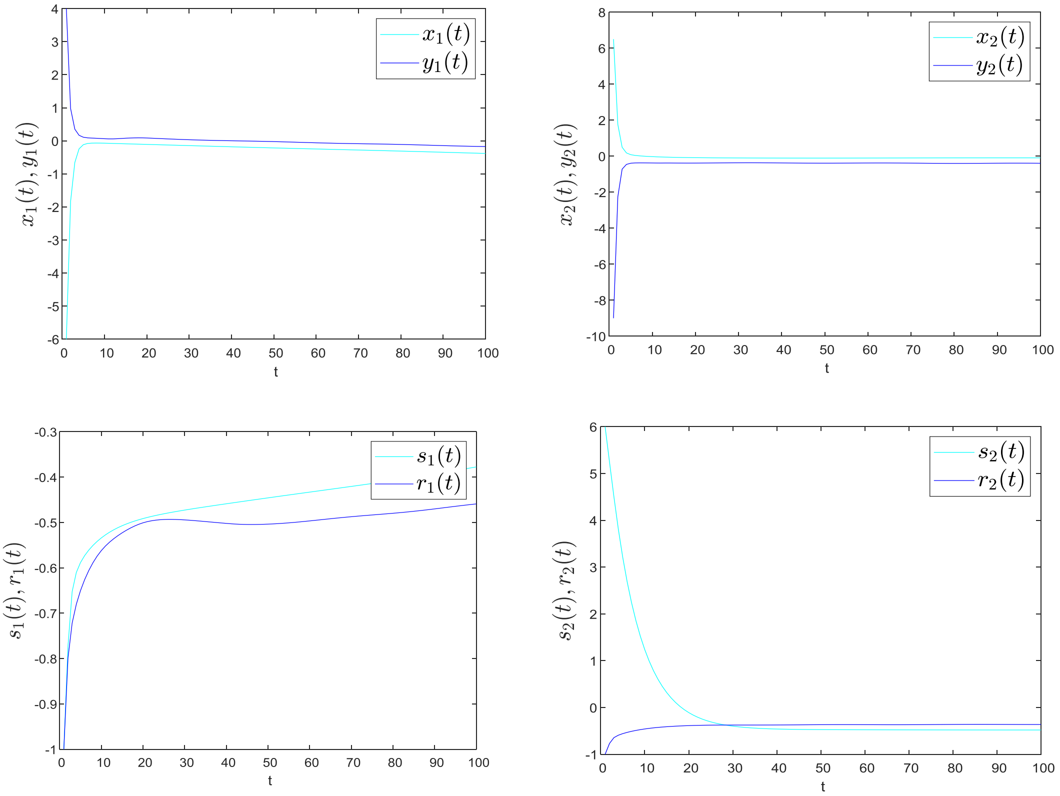

Fig.1. Time curves of states for Example without controllers.

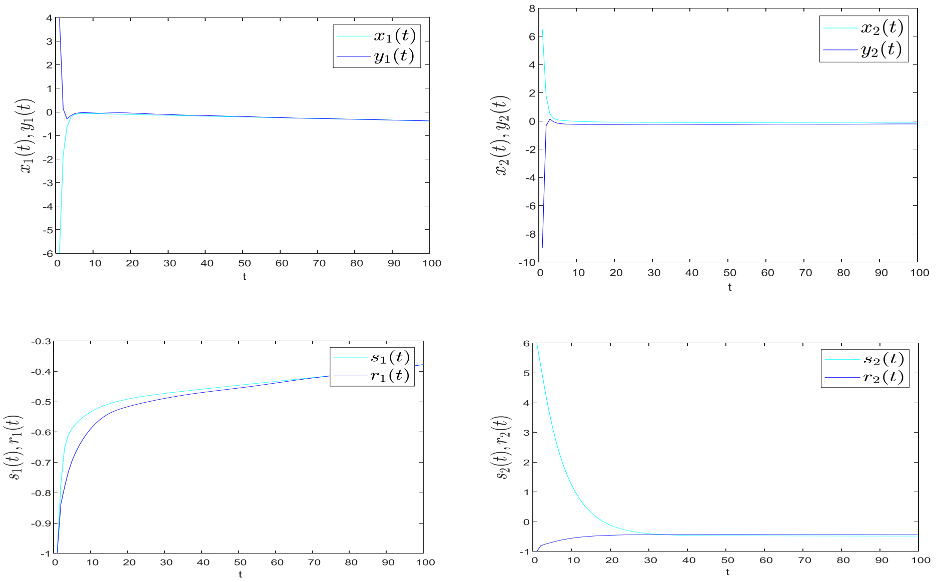

Fig.2. Time curves of states for Example with controllers.

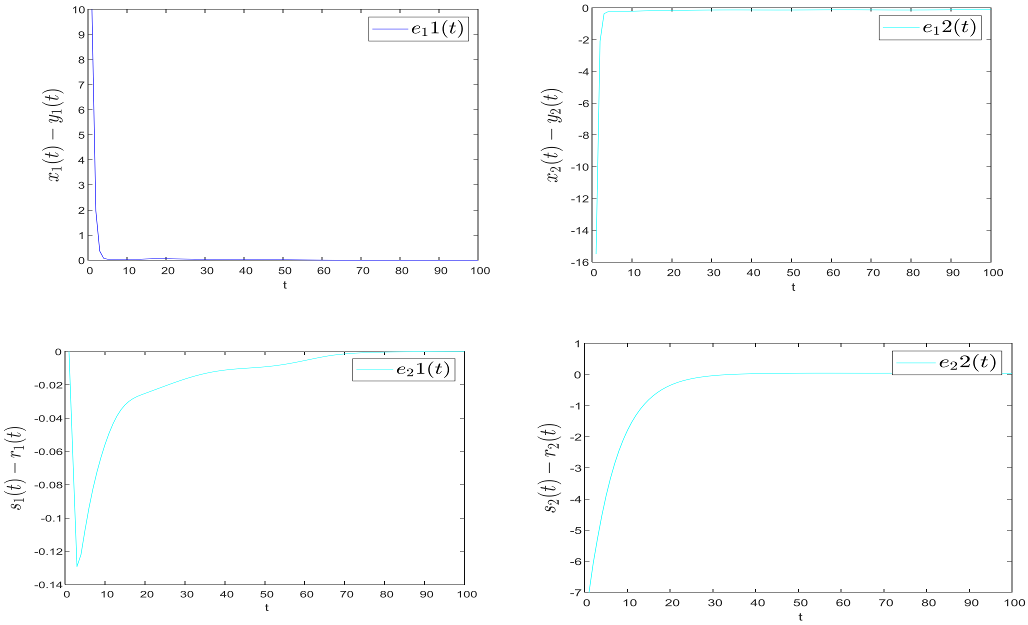

Fig.3. Time curves of states of errors for Example with controllers.

V. Conclusions

In this paper, the synchronization problem of memristive high-order competitive neural networks with mixed time-varying delays is studied. By utilizing Lyapunov method and inequality technique, some criteria for global asymptotic synchronization of the drive system and the response system are obtained.As is known to all, fifinite time synchronization and fifixed time synchronization are also hot topics in current research. Further work is to use these control strategies to solve the problems of fifinite time synchronization and fifixed time synchronization in memristor high-order competitive neural networks.

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