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Aryabhata Proven Correct

S.Haridasan

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The great Indian Astronomer and Mathematician Aryabhata was mis –interpreted for the last few centuries. Indian as well as Western scholars held a general notion that Aryabhata was a great mathematician, but some of his formulas are wrong. It was mainly due to lack of understanding of certain Sanskrit words. We are discussing two points here.

1. Aryabhata's formulas are correct.
2. There is a built in proof in his formulas

We will take up some of his formulas on Area and Volume. A book by name ARYABHATIYA, authored by great scholars V. S. Sukla and K.V.Sarma, published by Indian National Science Academy, points out some mistakes.

i). Area of rectangle.

आयामगुणं विस्तारं चतुरश्रफलशरीरम्

Ayāmaguṇam vistāram ćaturasraphalasarīram

b

l

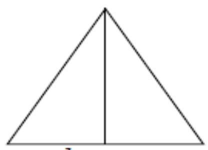
Length x Breadth = Area of rectangle.

This is a formula well in use from Vedic period. So Aryabhata skips it.

ii) Area of Triangle

त्रिभुजस्यफलशरीरं समदलकोटीभुजार्धसंवर्गः

Tribhujasyaphalaśarīram samadalakotībhujārdhasamvargah

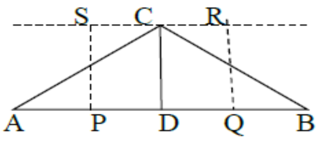


b

"The product of the perpendicular dropped from the opposite vertex to the base and half of the base is the area of triangle."
Area of triangle = ½ bh. This is how they have translated.
This is the original formula used even today. There is no mistake.
But there is something to note here. What is the meaning of the word *samadalakoṭībhujārdhasamvarga*. It could be *koṭībhujārdhasamvarga*. Then also we get the same formula.

त्रिभुजस्य फलशरीरम् = कोटीभुजार्धसंवर्गः
Tribhujasyaphalasarīram = koṭībhujārdhasamvarga
Area of triangle = ½ base x altitude
But there is a built in proof. *Bhuja* is considered as altitude and *koti* the base. That is why we have *bhujākoṭikarṇa nyāyam* (later Pythagoras Theorem).
So in a triangle when we drop a perpendicular from the opposite vertex it divides the triangle into two triangles with two *kotis*. *Koti* here used as second case dual number. कोटी (इ स्त्री द्वि द्वि) These *kotis* are divided equally by dropping perpendicular bisectors.

Thus the word *samadalakoṭī*



Samam= equal

Dalam = divided

Samadalam = equally divided (by a perpendicular bisector)

Now AB = b, CD = h, AD = x, DB = y

Area of △ ABC = Area of ▭ PQRS

= Area of ▭ PSCD + Area of ▭ DCRQ

= ½ x x h + ½ y x h

= ½ (x + y) h

= ½ b h

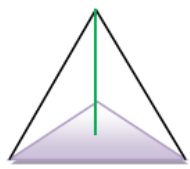
Area of triangle = equally divided base x ½ height.

Then only the translation becomes correct, hence the proof.

Actually Aryabhata gives only one formula for Area. The reason is mentioned in another sutra.

iii) Volume of Triangular pyramid

h



ऊर्ध्वभुजातत्संवर्गार्धं स घनः षडश्रिरिति ॥ ६ ॥

ūrdhvabhujātatsamvargārdham sa ghana ṣaḍasririti
"Half the product of the area of the triangular base and height is the volume of a six edged solid."
This is the meaning given by the authors. And adds, "this rule which is based on speculation on the analogy of the area of a triangle is inaccurate. The correct formula is found to occur in the *Brahmasphutasiddhanta* of Brahmagupta".
"Volume of a cone or pyramid = 1/3 base area X height."
They also observe that "Bhaskara I seems to be unaware of this formula, for he has no comments to make on the rule of Aryabhata I. Even the commentators Somesvara and Suryadeva (AD 1191) have nothing to add."
These comments clearly show that there was no mistake in Aryabhata's formula and it was profusely in use.
Now let us be familiar with the words used.
ūrdhvabhuja = height of the solid (it is not the altitude of triangle)
Tat = The area of triangle obtained from the first sutra (Area of the base of the present solid)
Samvargārdham = half the product
(Here is a problem, a typo error. *Ardhasamvarga* is used in the previous sutra. So that is carried here also. At least up to the time of Someswara the formula was correct. It could be *samvargatryamśa*, meaning one third of the product. It is no rocket science, Aryabhata could have easily tested it by filling a triangular prism by a triangular pyramid of same height.)
Sah ghana = that is the volume of
ṣaḍasri = Six edged solid (triangular pyramid)
So the actual sutra was
ūrdhvabhujātatsamvargatryamśam ṣaḍasririti
Volume of triangular pyramid = 1/3 base area X height
This is the correct formula used even today. More over Aryabhata uses the same formula in finding the volume of a sphere.
iv) Area of Circle
Aryabhata gives the formula for finding the area of a Circle:

समपरिणाहस्यार्धं विष्कम्भार्धहतमेव वृत्तफलम् ।

Samapariṇāhasyārdham viṣkambhārdhahatameva vṛttaphalam

"Half of the circumference multiplied by the semi-diameter certainly gives the area of a circle.

That is, Area of circle = ½ circumference x radius"

$$= \frac{1}{2} \times 2\pi\, r \times r$$

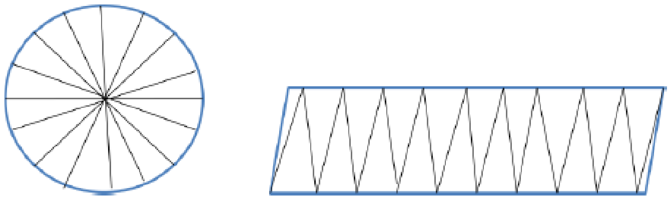
$$= \pi\, r^2$$

This is the correct formula used even today. But there is a built in proof also. First we will see the terms used:

Samapariṇāham = equaliy divided Circumference = 2π r

Ardham = half of circumference = π r

Viṣkambhasyārdham = half diameter = r



Hatam = multiplied

Vṛttaphalam = area of circle.

$$= \pi\, r \times r = \pi\, r^2$$

So, cut the circumference equally along the radii and take the two semicircles, insert together to get a parallelogram. And the area is bh = π r x r = π r^2. Hence we get the area of circle. That is the meaning of the sutra.

This is taught in the schools as such without mentioning the author's name. Next is the most wonderful and most mistaken formula.

v) Volume of a Sphere

तन्निजमूलेन हतं घनगोलफलं निरवशेषम् ॥७॥

" that area (of the diametral section) multiplied by its own square root gives the exact volume of a sphere"
This is the translation given by the authors. They continues:
" that is if r be the radius of a sphere , then according to Aryabhata I

Volume of a sphere = $\pi r^2 \times \sqrt{\pi r^2}$

Also, " Brahmagupta, who has criticized Aryabhata I even for his minutest errors, has not been able to make any improvement on Aryabhata's formula for the volume of a sphere. Still more noteworthy is the fact that mathematicians and astronomers in northern India, too, regarded Arybhata I' s formulas accurate and went on using it even in the second half of the ninth century A.D."
So they believed that Aryabhata's incorrect formula was in use for centuries. Actually we will see that Aryabhata I's correct formula is used even today without any change. Now we will go to the sutra:

तन्निजमूलेन हतं घनगोलफलं निरवशेषम् ॥७॥

तत् *tat* = that which is mentioned in the previous sutra

* **निजमूलेन** *nijamūlena* = by its own base (not *vargamulam* square root)

हतम् *hatam* = multiplied

घनगोलफलम् *ghanagolaphalam* = volume of solid sphere

निरवशेषम् *niravaśeṣam* = exactly

So what does this sutra mean? There is a sphere. It is divided into equal triangular pyramids along the radii as done in the case of area of the circle.

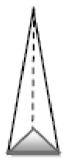
Now, volume of sphere = sum of volumes of triangular pyramids

= sum of (1/3 base area of triangular pyramids) x height

$= 1/3\ A_1 x\ h + 1/3\ A_2 \times h + 1/3\ A_3 \times h + \ldots\ldots\ldots$

$= 1/3\ (A_1 + A_2 + A_3 + \ldots\ldots\ldots\ldots) \times h$

$= 1/3\ (\text{Area of sphere}) \times h$



$= 1/3\ (4\ \pi\ r^2) \times h$

$= 4/3\ (\pi\ r^2) \times r$

$= 4/3\ \pi\ r^3$

This is exactly the correct formula used even today. Here is the formula with built in proof. Aryabhata was correct to the core thousand five hundred years ago.

Again Aryabhata says:

सर्वेषां क्षेत्राणां प्रसाध्य पार्श्वे फलं तदभ्यासः ॥९॥

"In the case of all the plane figures, one should determine the adjacent sides (of the rectangle into which that figure can be transformed) and find the area by taking their product".

It means the area of all plane figures can be found like this which I leave here as an exercise. Here is how it can be done. First we will see Aryabhata's formula for finding the sum of n terms of an Arithmetic Progression (A.P).

From sutra 16 we get the following formula.

आद्यं अन्त्यं मध्यं इष्टगुणं सर्वधनम्

ādyam antyam madhyam iṣṭaguṇam sarvadhanam

Sum =½ (First term + last term) number of terms

$S_n = ½\ (a + a_n)\ n$

This is the sum of first n terms of an A.P. This, thousand five hundred year old formula is used even today, though it is known after Carl Frederick Gauss.

Now this very formula is used in Geometry to find the area of all plane figures. We have different formula for each figure.

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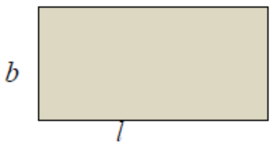
Area of Rectangle = length x breadth = lb
Area of Triangle = ½ base x altitude = ½ bh
Area of Parallelogram = base x height = bh
Area of Trapezium = ½ h(a + b)

All these can be included in one formula, the formula for finding the sum of n terms of an A.P which is = ½ n(First + Last)
Or *ādyam antyam madhyam iṣṭaguṇam sarvadhanam*
Here *ādyam* means the base or first side, *antyam* means last side and *madhyam* means half the sum and *iṣṭam* is the distance between the two opposite sides.

1. Area of rectangle = ½ ($l + l$) b

= ½ ($2l$) b

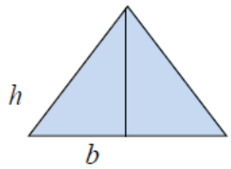
= lb



2. Area of Triangle = ½ ($b + 0$) h

= ½ b h

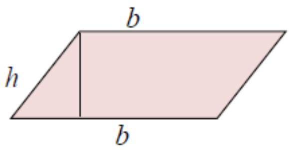
Here the second side is 0



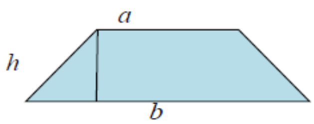
3. Area of Parallelogram = ½ ($b + b$) h

= ½ ($2b$) h

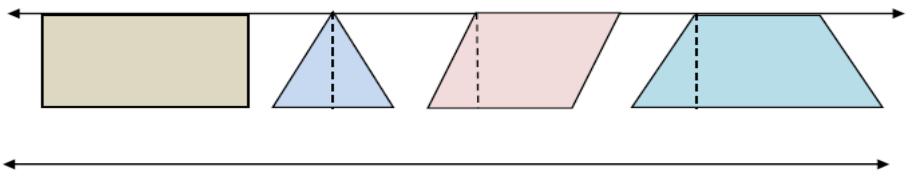
= bh



4. Area of Trapezium = ½ ($a + b$) h



So this may be the general formula Aryabhata intended. All these figures are determined by two parallel lines as follows:



Other polygons can be converted to these shapes.

Hence Aryabhata is right in all these formulas and we teach them without knowing the correct version and authenticity.

* **निजम् :- स्वके नित्ये निजं त्रिषु**

मूलम् :- मूलप्रतिष्ठायाम् (*Amarakosam*)

S.Haridasan,

Kadayil house

Paravur Road

Parippally P.O.

Kollam, 691574

Kerala, India