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Perfect Codes In Quadratic Residue Cayley Graphs G(Zn,Q)

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ABSTRACT
In this paper the concept of perfect codes to Quadratic Residue Cayley graph modulo an odd prime p, is introduced. Let p be an odd prime & let S be the set of quadratic residue modulo p. Consider the set S^* ={ s, p-s/s ∈ S}. Then S^* is a symmetric subset of an additive abelian group of (Z_P,⊕), of integers 0,1,2,…, P-1. The cayley graph of the group (Z_P,⊕) is associated with the symmetric subset S^* of Z_P is called quadratic residue Cayley graph. Associated with odd prime P and it is denoted by G(Zp,Q), where the vertex set V=$Z_{p=}${ 0,1,2,….p-1} and edge set $E = \{(x, y)/x, y \in V$, x-y ∈ S^* or y-x ∈ S^*}. To obtain perfect codes the between the vertices of the cayley residue of the graph G(Z_p,Q) is determined, then the perfect codes are found by using quadratic residue Cayley Graph, Also we show that these perfect codes are not unique for this graph.
Keywords: - *Cayley Graph, Quadratic residue Cayley Graph, Symmetric set, Perfect code.*

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I. Introduction

Unitary Cayley graphs, generator Cayley graphs and their cycle structure were studied by Dejter [3], Berrizbeite, Giudici [1,2]. Significant contributions are made to this class of graphs in recent times. For complete graphs also this problem has been studied. In this paper we have made an attempt to study this aspect for the Cayley graphs associated with quadratic residue modulo an odd prime p. Domination theory of graphs is an important branch of Graph Theory and has many applications in Engineering, Communication Networks and many others. Allan and Laskar (1978); Allan et al. (1984); Cockayne and Hedetniemi (1977); Haynes et al. (1998) have studied various domination parameters of graphs. Graphs associated with certain arithmetical functions which are usually called arithmetical graphs have been studied extensively by many researchers. Here we consider quadratic residue Cayley graph G(Zp, Q).

Originally, perfect code was always associated with coding theory, specifically in error correcting codes [6]. This topic is then extended in other fields, including graph theory. In 1973, Biggs [7] started the idea by studying the perfect codes of distance-transitive graphs. Later in 1986, Kratochvil [8] introduced a variant of perfect codes in graphs, which is the t-perfect code. The study was done on complete bipartite graph and products of graphs, where the independent sets of the graphs are considered to determine the t-perfect codes.

II.Preliminaries

Definition 1.1

Let p be an odd prime and n a positive integer such that n ≡ 0 (modp). If the quadratic congruence,x^2 ´≡n(modp) has a solution, then n is called a quadratic residue mod p and it is written as nRp.

Let p be an odd prime and let S be the set of quadratic residues modulo p. Consider the set S^* = {s, p − s/s ∈ S}. Then S^* is a symmetric subset of the additive abelian group (Zp, ⊕) of integers 0, 1, 2, . . . , p − 1 modulo p.

Definition 1.2.

The Cayley graph of the group (Zp,(+)) associated with the symmetric subset S^* of Zp is called the quadratic residue Cayley graph associated with odd prime P and it is denoted by G ($Z_{p,}$Q). Where the vertex set V=$Z_{p=}${ 0,1,2,….p-1} and edge set E = {(x, y)/x, y ∈ V, x-y ∈ S^* or y-x ∈ S^*}.

Definition 1.3

Let p be an odd prime, S, the set of quadratic residues modulo p and let S^* = {s, p – s / s ∈S, s≠ p}. The quadratic residue Cayley graph G(Zp, Q) is defined as the graph whose vertex set is Zp = {0, 1, 2,…, (p – 1)} and the edge set E = {(x, y) / x – y or y – x is in S^*}.

Definition 1.4
Distance between the two vertices of a graph

The distance between two distinct vertices x & y of a connected simple graph is the length of the shortest path x & y.

Definition 1.5
e-error-correcting code:

If $S_e(x)$ is the set of neighborhood elements of x with distance less than or equal to e, then a code C is an e-error-correcting code if for all x and y in C, $S_e(x) \cap S_e(y) = \emptyset$. when x and y are distinct.

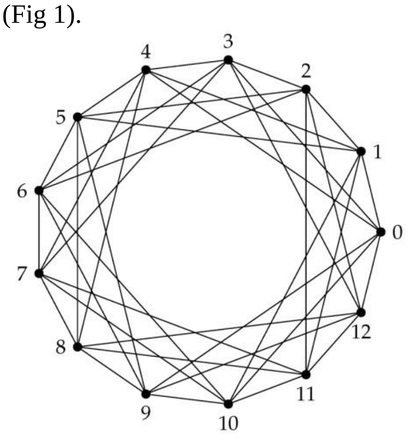
Definition 1.6
perfect code
If for an e-error-correcting code C, the union, $\bigcup_{x\in C} S_e(x) = V$, then the code is called a perfect code.

Definition 1.7
e-perfect code

If an e-error-correcting code C of a set of vertices V satisfies the union operation, i.e, $\bigcup_{x\in C} S_e(x) = V$ then C is called an e-perfect code, where e is the maximum distance between two vertices in the set V.

III.Main Results :

The graph G(Z13, Q) is given below (Fig 1).



Proposition 1.

Let **t**he graph G(Z13, Q) **be the** quadratic residue Cayley Graph. Then G(Z13, Q) has the **1-perfect code** . But it has no error correcting code.

Proof:
Suppose The graph G (Z13, Q) graph is shown figure
. Based on definition e-error-correcting codes are determined.
The set of all neighborhoods of elements of G (Z13, Q) are determined.

When e = 1, the set $S_1(x)$, which is the set of all neighborhood elements with distance less than or equal to one (namely 0 and 1) are determined for all x in G (Z13, Q) .

The sets are listed as follows.

• $S_1(0)=\{0,1,3,4,9,10,12\}$	• $S_1(7)=\{3,4,6,7,8,10,11\}$
• $S_1(1)=\{0,1,2,4,5, 10,11\}$	• $S_1(8)=\{4,5,7,8,9,11,12\}$
• $S_1(2)=\{1,2,3,5,6,11,12\}$	• $S_1(9)=\{0,5,6,8,9,10,12\}$
• $S_1(3)=\{0,2,3,4,6,7,12\}$	• $S_1(10)=\{0,1,6,7,9,10,11\}$
• $S_1(4)=\{0,1,3,4,5,7,8\}$	• $S_1(11)=\{1,2,7,8,10,11,12\}$
• $S_1(5)=\{1,2,4,5,6,8,9\}$	• $S_1(12) =\{0,2,3,8,9,11,12\}$
• $S_1(6)=\{2,3,5,6,7,9,10\}$	

Let C = {0, 1, 7} be a code. Since $S_1(0)\cap S_1(1) \neq \emptyset$, $S_1(0)\cap S_1(7) \neq \emptyset$ and $S_1(1)\cap S_1(7) \neq \emptyset$, then the code C is not 1-error-correcting code of G (Z13, Q). However, from Definition a, since $S_1(0) \cup S_1(1) \cup S_1(7) = \{0,1,2,3,4,5,6,7,8,9,10,11,12\}$ = G (Z13, Q) , then the code C = {0,1,7} is a 1-perfect code of G (Z13, Q).

Proposition 2.

Let the graph G(Z13, Q) be the commuting the quadratic residue Cayley Graph. Then G(Z13, Q) is 2-perfect code. But it has no error correcting code.

Proof:

Suppose G(Z13, Q) graph is shown figure 1. To obtain the 2-error-correcting code. The set of neighborhood of elements of the vertices **G(Z13, Q)** is firstly determined for distance less than or equal to two are listed below.

$S_2(0)=\{0,1,2,3,4,5,6,7,8,9,10,11,12\}$ =)= $S_2(1)=S_2(2)= S_2(3)= S_2(4)= S_2(5)= S_2(6)= S_2(6)= S_2(7)= S_2(8)= S_2(9)= S_2(10)= S_2(11)$)= $S_2(12)$

Let $C=\{0,1,2,3,4,5,6,7,8,9,10,11,12\}$ be a code. From the above sets $S_2(0)\cap S_2(1) \neq \emptyset$, $S_2(0)\cap S_2(2) \neq \emptyset$,… , $S_2(0)\cap S_2(12) \neq \emptyset$, $S_2(x_1)\cap S_2(x_2) \neq \emptyset$ for all x_1, x_2 in C. Therefore, C is not a 2-error-correcting code of **G(Z13, Q)**.

In addition, the union $S_2(0) U S_2(1)$ = C is also a 2-perfect code of **G(Z13, Q).**.

From Figure 1, the maximum distance of the graph is two. Therefore, when e > 2, the results of the e-perfect codes will be the same as e = 2.

II. Conclusion

The quadratic residue Cayley Graph G (Z13, Q**) constructed**, Based on the graph, the e-perfect codes are determined. It has been found that the graph G (Z13, Q) has 1-perfect code for all natural numbers i.e.n=1,2,3….. However, the graph has several 2-perfect codes for n≥ 4. Meanwhile for the graph G (Z13, Q), there is no 1-error correcting codes and 2-error correcting codes, but all singleton sets are perfect codes of the graph. From the results, it is concluded that if n is the maximum distance of a graph, then the e-perfect codes of the graph when e > n will be the same as e = n

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