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On Supra Generalized Pre-Regular Closed Graphs

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Abstract:
In this paper, the author introduces supra gpr-closed graphs and strongly supra gpr-closed graphs. The basic properties of these graphs by utilizing gpr^{μ}-open sets, gpr^{μ^*}- continuous functions, supra gpr-irresolute functions and weakly supra gpr irresolute functions have been discussed.
Keywords And Phrases: *gpr^{μ}-closed sets, gpr^{μ}-open sets, gpr^{μ^*}-continuous functions, supra gpr- irresolute functions, supra gpr- closed graphs, strongly supra gpr- closed graphs.*

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1. Introduction

In 1969, Long [6] introduced the notion of closed graphs in topological spaces. The same author along with Herrington [7] introduced strongly closed graphs in 1975. This invention made a large number of topologists to initiate different types of closed graphs and characterize its properties. In 1979, Kasahara [4] introduced α -closed graphs, which generalizes the concepts of closed, strongly-closed and almost-strongly-closed graph of a function. In 1983, Dube et.al [1] introduced the notion of semi closed graphs. In 2005, Nandhini Bandyopadhya and Bhattacharyya [10] investigated pre-closed graphs. They [11] further extended the work by introducing strongly pre-closed graphs.

In 1983, Mashour et.al. [8] introduced supra topological spaces where the study of *S*-continuous, *S**- continuous maps , *S*-closed graphs and Strongly *S*-closed graphs were made. Since the advent of these spaces, several research papers with interesting results in different respects came to existence. Vidhya Menon [14] introduced the notion of gpr^{μ}-closed sets and gpr^{μ}- continuity in supra topological spaces. The research then progressed and in 2016 , Vidhya Menon et.al. [13] made an extensive study of gpr^{μ}-closed sets. This paper, initiates the notion of supra generalized pre-regular closed graphs and strongly supra generalized pre-regular closed graphs . We also study some of the properties of supra generalized pre-regular closed graphs, strongly supra generalized pre-regular closed graphs with the help of gpr^{μ}-open sets, gpr^{μ^*}-continuous functions, supra *gpr*- irresolute functions and weakly supra *gpr* irresolute functions. Throughout this paper (X, τ) and (Y, σ) represents the non empty topological spaces on which no separation axioms are assumed unless explicitly stated.

2. Preliminaries

Definition 2.1. Let (X, τ) be a topological space . A sub collection $\mu \subset P(X)$ is called a supra topology on X if $X \in \mu$ and μ is closed under arbitrary union. (X,μ) is called a supra topological space. The elements of μ are said to be supra open in (X, μ) and the complement of a supra open set is called supra closed set. We call μ a supra topology associated with τ if $\tau \subset \mu$.
Let (X, τ) be a topological space with supra topology μ associated with τ for the following definitions.

Definition 2.2. [14] A subset A of (X, μ) is called supra pre-closed if $cl^{\mu}(int^{\mu}(A)) \subseteq A$ .The complement of a supra pre-closed set is called supra pre-open set .

Definition 2.3. [14] Let A be a subset of a supra topological space (X , μ). Then
i) supra closure of a set A is defined as $cl^{\mu}(A) = \cap (B : B$ is a supra closed set and $A \subseteq B)$
ii) supra interior of a set A is defined as $int^{\mu}(A) = \cup (B : B$ is a supra open set and $B \subseteq A)$
iii) supra pre-closure of a set A is defined as $pcl^{\mu}(A) = \cap (B : B$ is a supra pre-closed set and $A \subseteq B)$
iv) supra pre-interior of a set A is defined as $pint^{\mu}(A) = \cup (B : B$ is a supra pre-open set and $B \subseteq A)$

Definition 2.4 . [14] A subset A of a supra topological space (X, μ) is called
(i) supra generalized closed (briefly g^{μ} -closed) if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X,μ).

(ii) supra generalized pre-closed (briefly gp^{μ} -closed) if $pcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X,μ).
(iii) supra generalized pre-regular closed (briefly gpr^{μ} -closed) if $pcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra regular open in (X,μ).

The complement of gpr^{μ} -closed (resp. g^{μ}-closed, gp^{μ} -closed) is said to be gpr^{μ} - open (resp. g^{μ}-open, gp^{μ}-open). The collection of all supra generalized pre-regular closed and supra generalized pre- regular open subsets of X will be denoted by $GPRC^{\mu}(X)$ and $GPRO^{\mu}(X)$ respectively.

Definition 2.5 . [13 **]** A subset A of a supra topological space (X,μ) is called
$gpr^{\mu}\text{-}cl(A) = \cap\ [F: A \subset F, F$ is gpr^{μ}-closed set in $(X,\mu)]$.
$gpr^{\mu}\text{-} int(A) = \cup\ [M: M \subset A , M$ is gpr^{μ}-open set in $(X,\mu)]$.

Definition 2.6. [2 **]** Let (X, μ) be a supra topological space . Then X is
(i) $S\text{-}T_0$ if for every two distinct points x and y in X, there exists a supra open set U containing one of them but not the other.
(ii) $S\text{-}T_1$ if for every two distinct points x and y in X , there exists a pair of supra open sets U and V such that $x \in U, y \notin U$ and $y \in V , x \notin V$.
(iii) $S\text{-}T_2$ if for every two distinct points x and y in X , there exists a pair of disjoint supra - open sets U and V such that $x \in U$ and $y \in V$.

Let (X, τ) and (Y, σ) be two topological spaces with supra topologies μ and λ associated with τ and σ respectively for the following definitions and results.

Definition 2.7. [8] A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called
i) S-continuous if $f^{-1}(V)$ is supra closed (resp. supra open) in X for every closed (resp. open) set V of Y .
ii) S^{*}-continuous if $f^{-1}(V)$ is supra closed (resp. supra open) in X for every supra closed (resp. supra open) set V of Y .

Definition 2.8. A subset A of the product space $X \times Y$ is closed in $X \times Y$ if for each $(x, y) \in (X \times Y) - A$ there exist two open neighbourhoods U and V of x and y respectively such that $(U\times V) \cap A = \emptyset$.

Definition 2.9. [5**]** A function $f : (X, \tau) \rightarrow (Y, \sigma)$ has closed graph if the graph $G(f) = \{ (x, f(x)): x \in X\}$ is closed in $X \times Y$.

Definition 2.10. [8 **]** Let (X, τ) and (Y, σ) be two supra topological spaces. A subset A of the product space $X \times Y$ is S-closed in $X \times Y$ if for each $(x, y) \in (X \times Y) - A$ there exist two supra open neighbourhoods U and V of x and y respectively such that $(U \times V) \cap A = \emptyset$.

Definition 2.11.[8**]** A function f: $(X, \tau) \rightarrow (Y, \sigma)$ has S-closed graph if the graph $G(f) = \{ (x, f(x)): x \in X \}$ is S- closed in $X \times Y$.

3. Pasting Lemma For gpr^{μ} -Continuous Maps

Definition 3.1. A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called gpr^{μ}-continuous [14] if $f^{-1}(V)$ is gpr^{μ} -closed (resp. gpr^{μ} -open) in X for every closed (resp. open) set V of Y.

Definition 3.2. A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called gpr^{μ^*} -continuous if $f^{-1}(V)$ is gpr^{μ}-closed (resp. gpr^{μ}-open) in X for every supra closed (resp. supra open) set V of Y.

Definition 3.3. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be gpr^{λ}-closed (resp. gpr^{λ}-open) if the image of every closed (resp. open) set in X is gpr^{λ}-closed (resp. gpr^{λ}-open) in Y.

Definition 3.4. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be gpr^{λ^*} -closed (resp. gpr^{λ^*} -open) if the image of each supra closed (resp. supra open) set in X is gpr^{λ} -closed (resp. gpr^{λ}-open) in Y.

Definition 3.5. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be strongly gpr^{λ^*} -closed (resp. strongly gpr^{λ^*}-open) if the image of each gpr^{μ} - closed (resp. gpr^{μ}-open) set in X is gpr^{λ} -closed (resp. gpr^{λ}-open) in Y.

Lemma 3.6. [14 **]** If $A \subset Y \subset X$ and Y be supra open in (X,μ) , then $pcl^{\mu}{}_{Y}\ A = pcl^{\mu}{}_{X}\ A \cap Y$.

Lemma 3.7. [14**]** In a supra topological space (X,μ) , let $A \subset X$. If A is supra open then $RO^{\mu}(A,\ \mu \mid A) = \{ W \cap A : W \in RO^{\mu}(X,\mu)\}$.

Lemma 3.8.[14 **]** In a supra topological space (X,μ) , let $A \subset Y \subset X$. Then if Y is supra open & supra pre-closed in X then $A \in GPRC^{\mu}(Y)$ implies $A \in GPRC^{\mu}(X,\mu)$.

Theorem 3.9. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a gpr^{μ}-continuous function and H be supra regular open, gpr^{μ}-closed subset of X. Assume that $GPRC^{\mu}(X, \mu)$ is closed under finite intersection. Then the restriction $f \mid H : (H, \tau \mid H) \rightarrow (Y, \sigma)$ is gpr^{μ} -continuous.
Proof: Let A be closed subset of Y. Since f is gpr^{μ} -continuous, $f^{-1}(A)$ is gpr^{μ} - closed in X. If $f^{-1}(A) \cap H = H_1$, then is H_1 is gpr^{μ} -closed in X by assumption. Now $(f \mid H)^{-1}(A) = H_1$, it is enough to prove that H_1 is gpr^{μ} -closed in H. Let $H_1 \subset M^*$ where M^* is a supra regular open set in H. By lemma 3.7 $M^* = M \cap H$ for some supra regular open set M in X. Then $H_1 \subset M$ implies $pcl^{\mu}{}_{X}\, H_1 \subset M$. That is $pcl^{\mu}{}_{X}\, H_1 \cap H \subset M \cap H = M^*$. Thus $pcl^{\mu}{}_{H}\, H_1 \subset M^*$ by lemma 3.6 . Hence H_1 is gpr^{μ}-closed in H thereby implying $f \mid H$ is gpr^{μ} -continuous.

Theorem 3.10. Let $X = G \cup H$ be a topological space with topology τ and Y be a topological space with σ. Let $GPRC^{\mu}(X, \tau)$ be closed under finite union and let f: $(G, \tau \mid G) \rightarrow (Y, \sigma)$ and g: $(H, \tau \mid H) \rightarrow (Y, \sigma)$ be gpr^{μ}-continuous functions such that $f(x) = g(x)$ for every $x \in G \cap H$. Suppose that both G and H are supra open & supra pre-closed sets in X. Then their combination α: $(X, \tau) \rightarrow (Y, \sigma)$ defined by $\alpha(x) = f(x)$ for $x \in G$ and $\alpha(x) = g(x)$ for $x \in H$ is gpr^{μ} -continuous.
Proof: Let F be any closed set in Y. Then $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F) = C \cup D$ where $C = f^{-1}(F)$ and $D = g^{-1}(F)$. Since C is gpr^{μ}-closed in G and G is supra open and supra pre- closed in X, by lemma 3.8, C is gpr^{μ}-closed in X. Similarly, D is gpr^{μ}-closed in X. Also $C \cup D$ is gpr^{μ}-closed in X by hypothesis. Therefore $\alpha^{-1}(F)$ gpr^{μ}-closed in X. Thus α is gpr^{μ} -continuous.

4. Supra Generalized Pre-Regular Closed Graph

Definition 4.1. A subset A of the product space $X \times Y$ is supra pre-closed (resp. supra g-closed, supra gp-closed) in $X \times Y$ if for each $(x, y) \in (X \times Y) - A$ there exist two supra pre-open (resp. supra g-open, supra gp-open) neighbourhoods U and V of x and y respectively such that $(U \times V) \cap A = \emptyset$.

Definition 4.2. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ has supra pre-closed graph (resp. supra g-closed graph, supra gp-closed graph) if the graph $G(f) = \{(x, f(x)): x \in X \}$ is supra pre-closed (resp. supra g - closed, supra gp-closed) in $X \times Y$.

Definition 4.3. A subset A of the product space $X \times Y$ is supra gpr -closed in $X \times Y$ if for each $(x, y) \in (X \times Y) - A$ there exist two supra gpr-open neighbourhoods U and V of x and y respectively such that $(U \times V) \cap A = \emptyset$.

Definition 4.4. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ has an supra gpr-closed graph, if the graph $G(f) = \{ (x, f(x)): x \in X \}$ is supra gpr-closed in $X \times Y$.

Definition 4.5. Let (X, μ) be a supra topological space . Then X is
(i) gpr^{μ}-T_0 if for every two distinct points x and y in X , there exists a gpr^{μ}-open set U containing one of them but not the other.
(ii) gpr^{μ}-T_1 if for every two distinct points x and y in X, there exists a pair of gpr^{μ}-open sets U and V such that $x \in U, y \notin U$ and $y \in V , x \notin V$.
(iii) gpr^{μ}-T_2 if for every two distinct points x and y in X there exists a pair of disjoint gpr^{μ} - open sets U and V such that $x \in U$ and $y \in V$.

Theorem 4.6. Every closed graph (resp. S-closed graph, supra pre-closed graph, supra g-closed graph , supra gp- closed graph) is a supra gpr- closed graph.

Remark 4.7. However the converse need not be true.

Example 4.8. Let $X = \{e, i, g\}$, $Y = \{0, 1, 2\}$ with topological spaces $\tau = \{ \emptyset, X, \{e, i\}\}$ and $\sigma = \{\emptyset , Y, \{0, 1\}\}$ with respect to X and Y respectively. Also let $\mu = \{\emptyset, X, \{e, i\}, \{i, g\}\}$ and $\lambda = \{\emptyset, Y, \{0, 1\}, \{1, 2\}\}$ be the supra topologies associated with τ and σ respectively. Define $f : X \to Y$ by $f(e) = 0$, $f(i) = 1$, $f(g) = 2$. Then $f : X \to Y$ is a supra generalized pre-regular closed graph, but f is not a (supra *gp*-closed graph, supra *g*-closed graph, supra pre-closed graph, *S*-closed graph, closed graph).

4.9 From the above results we have the following diagram

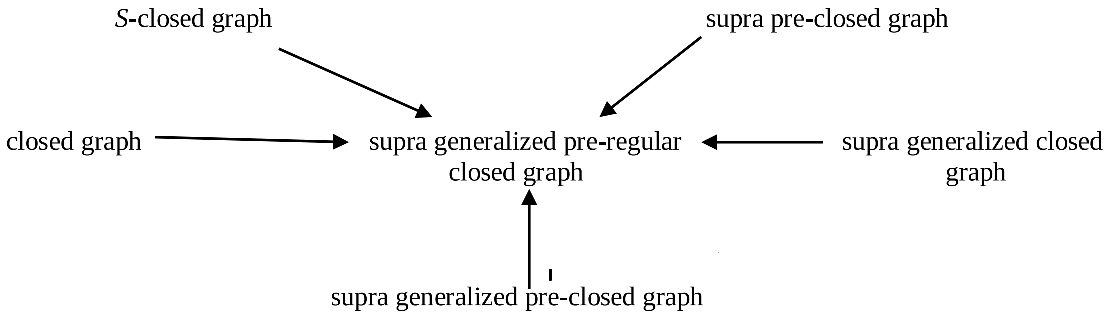

Fig: 4.9.1

Lemma 4.10. A function $f : (X, \tau) \to (Y, \sigma)$ has an supra *gpr*-closed graph iff for each $x \in X$, $y \in Y$ such that $y \neq f(x)$, there exists two supra *gpr*- open sets U and V containing x and y respectively such that $f(U) \cap V = \emptyset$.

Proof: *Necessity*. Let f has an supra *gpr* - closed graph, then for each $x \in X$, $y \in Y$ such that $y \neq f(x)$ there exist two supra *gpr* - open sets U and V containing x and y respectively such that $(U \times V) \cap G(f) = \emptyset$. This implies for every $x \in U$ and $y \in V$, $f(x) \neq y$. Therefore, $f(U) \cap V = \emptyset$.

Sufficiency. Let $(x, y) \in X \times Y - G(f)$, then there exists two supra *gpr*-open sets U and V containing x and y respectively such that $f(U) \cap V = \emptyset$. This implies that, for each $x \in U$ and $y \in V$, $f(x) \neq y$. So , $(U \times V) \cap G(f) = \emptyset$. Hence f has an supra *gpr* - closed graph.

Lemma 4.11. If $f : (X, \tau) \to (Y, \sigma)$ is $gpr^{\mu *}$-continuous then for each $x \in X$ and each supra open set $V \subset Y$ containing $f(x)$, there exist a gpr^{μ}-open set $U \subset X$ containing x such that $f(U) \subset V$.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be $gpr^{\mu *}$-continuous. Then , for any $x \in X$ and any supra open set V of Y containing $f(x)$, $U = f^{-1}(V)$ is gpr^{μ} -open in X and $f(U) = f(f^{-1}(V)) \subset V$.

Theorem 4.12 . If $f : (X,\tau) \to (Y, \sigma)$ is $gpr^{\mu *}$-continuous and Y an S-T_2 space, then f has an supra *gpr*- closed graph.

Proof: Let $(x,y) \notin G(f)$. Then $y \neq f(x)$ and since Y is S- T_2, there exist a pair of disjoint supra open sets U and V such that $f(x) \in U$, $y \in V$ and $U \cap V = \emptyset$. Now f is $gpr^{\mu *}$-continuous, there exist a gpr^{μ}-open set W of x such that $f(W) \subset U$ by lemma 4.11. Hence $f(W) \cap V = \emptyset$. This implies that $(W \times V) \cap G(f) = \emptyset$. Therefore f has an supra *gpr*- closed graph.

Theorem 4.13. If $f : (X,\tau) \to (Y, \sigma)$ is $gpr^{\mu *}$- continuous injective function with a supra *gpr*- closed graph then X is gpr^{μ}- T_2.

Proof: Let $x_1, x_2 \in X$, $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$. This shows that $(x_1 , f(x_2)) \in X \times Y - G(f)$. Since f has an supra *gpr*-closed graph there exist two supra *gpr*-open neighborhoods U and V of x_1 and $f(x_2)$ respectively such that $(U \times V) \cap G(f) = \emptyset$. This gives $f(U) \cap V = \emptyset$. Since f is $gpr^{\mu *}$-continuous there exist a gpr^{μ}-open set W containing x_2 such that $f(W) \subset V$. Hence $f(W) \cap f(U) = \emptyset$. Therefore $W \cap U = \emptyset$ and X is an gpr^{μ}- T_2 space.

Theorem 4.14. If $f : (X,\tau) \to (Y, \sigma)$ is an injective function with supra *gpr*- closed graph $G(f)$ then X is gpr^{μ}- T_1 space.

Proof: Let x_1 and x_2 be two distinct points of X, then $f(x_1) \neq f(x_2)$. Thus $(x_1, f(x_2)) \notin G(f)$. Since $G(f)$ is supra *gpr*-closed, there exists supra *gpr*-open sets U and V containing x_1 and $f(x_2)$ respectively such that $f(U) \cap V = \emptyset$. Therefore $x_2 \notin U$. Similarly there exist supra *gpr*-open sets M and N containing x_2 and $f(x_1)$ respectively such that $f(M) \cap N = \emptyset$. Therefore $x_1 \notin M$. Thus X is gpr^{μ}-T_1.

Theorem 4.15. If $f : (X, \tau) \to (Y, \sigma)$ is an surjective function with supra *gpr*-closed $G(f)$ then Y is gpr^{λ}- T_1 .
Proof: Let y and z be two distinct points of Y. Since f is surjective there exist a point x in X such that $f(x) = z$. Therefore $(x, y) \notin G(f)$. By lemma 4.10 there exist supra gpr-open sets U and V containing x and y respectively such that $f(U) \cap V = \emptyset$. It follows that $z \notin V$. Similarly there exist $w \in X$ such that $f(w) = y$. Thus $(w, z) \notin G(f)$. Similarly there exist supra gpr-open sets M and N containing w and z respectively such that $f(M) \cap N = \emptyset$. It implies that $y \notin N$. Therefore Y is gpr^{λ}- T_1.

Theorem 4.16. If $f: (X, \tau) \to (Y, \sigma)$ is a bijective function with supra gpr-closed graph $G(f)$ then X and Y are supra *gpr*-T_1 .
Proof: From theorem 4.14 & 4.15.

Theorem 4.17 . If $f : (X, \tau) \to (Y, \sigma)$ is gpr^{μ^*}-continuous injective function and Y is S-T_2 then X is gpr^{μ}-T_2.
Proof: Let $x_1, x_2 \in X$, such that $x_1 \neq x_2$. Since Y is S-T_2 , then there exist disjoint supra open sets U and V in Y such that $f(x_1) \in$ U and $f(x_2) \in$ V . Now, f is gpr^{μ^*}-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are gpr^{μ}-open in X containing x_1 and x_2 respectively. Also $f^{-1}(U) \cap f^{-1}(V) = \emptyset$.This shows that X is gpr^{μ}- T_2 .

Theorem 4.18. If $f : (X, \tau) \to (Y, \sigma)$ is a surjective strongly gpr^{λ}-open function with supra *gpr*- closed graph $G(f)$ then Y is gpr^{λ}- T_2.
Proof: Let y_1 and y_2 be distinct points of Y. Since f is surjective there exist a point x in X such that $f(x) = y_1$ and $(x, y_2) \in (X \times Y) - G(f)$. Since f is supra *gpr*-closed graph, there exist a gpr^{μ}-open set A of X and a gpr^{λ}-open set B of Y such that $(x, y_2) \in A \times B$ and $(A \times B) \cap G(f) = \emptyset$. Thus $f(A) \cap B = \emptyset$. Since f is strongly gpr^{λ}-open, $f(A)$ is gpr^{λ}- open such that $f(x) = y_1 \in f(A)$. Thus Y is gpr^{λ}- T_2.

5. Strongly supra generalized pre-regular closed graph

Definition 5.1. [5**]** A function $f : (X, \tau) \to (Y, \sigma)$ has a strongly closed graph if for each $(x, y) \notin G(f)$, there exist two open sets U and V containing x and y respectively such that $(U \times \text{cl}(V)) \cap G(f) = \emptyset$.

Definition 5.2. [8**]** A function $f : (X, \tau) \to (Y, \sigma)$ has a strongly S-closed graph if for each $(x,y) \notin G(f)$, there exist two supra open sets U and V containing x and y respectively such that $(U \times V^{sc})) \cap G(f) = \emptyset$.

Definition 5.3. A function $f : (X, \tau) \to (Y, \sigma)$ has a strongly supra *gpr*- closed graph if for each $(x,y) \notin G(f)$, there exist two supra gpr- open sets U and V containing x and y respectively such that $(U \times gpr^{\lambda}\text{-}cl(V)) \cap G(f) = \emptyset$.

Lemma 5.4. A function $f : (X, \tau) \to (Y, \sigma)$ has a strongly supra *gpr*- closed graph iff for each $(x,y) \notin G(f)$ there exist two supra gpr- open sets U and V containing x and y respectively such that $f(U) \cap gpr^{\lambda}\text{-}cl(V) = \emptyset$.

Theorem 5.5. If $f : (X, \tau) \rightarrow (Y, \sigma)$ has an strongly closed graph , then it has a strongly supra *gpr*-closed graph.
Proof: Let $x \in X$, $y \in Y$ such that $f(x) \neq y$. Now by hypothesis f has an strongly closed graph, there exist two open sets U and V containing x and y respectively such that $f(U) \cap cl(V) = \emptyset$. Since every open set is supra open which in turn is supra *gpr*-open and $gpr^{\lambda}\text{-}cl(V) \subset cl^{\lambda}(V) \subset cl(V)$, there exist two supra *gpr*-open open sets U and V containing x and y respectively such that $f(U) \cap gpr^{\lambda}\text{-}cl(V) = \emptyset$. Thus by lemma 5.4 , f has an strongly supra *gpr*-closed graph.

Theorem 5.6. If $f : (X, \tau) \rightarrow (Y, \sigma)$ be a surjective function with a strongly supra *gpr*-closed graph . Then Y is gpr^{λ}- T_2 space.
Proof: Let y_1 and y_2 be two distinct points of Y, then there exist an $x_1 \in X$ such that $f(x_1) = y_1$. Now, $(x_1, y_2) \notin G(f)$. Since f has strongly supra *gpr*-closed graph, there exist two supra *gpr*- open neighbourhods U and V of x_1 and y_2 respectively such that $f(U) \cap gpr^{\lambda}\text{-}cl(V) = \emptyset$.Thus $y_1 \notin gpr^{\lambda}\text{-}cl(V)$. This implies $y_1 \notin V$. This means that there exist a gpr^{λ}- open set W of y_1 such that $W \cap V = \emptyset$. Therefore Y is $gpr^{\lambda}\text{-}T_2$ space.

Theorem 5.7. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is $gpr^{\lambda*}$-open and has a supra closed graph $G(f)$, then $G(f)$ is strongly supra *gpr* -closed.
Proof: Let $(x,y) \in (X \times Y) - G(f)$. Since $G(f)$ is supra closed , there exist supra open sets U and V containing x and y respectively such that $f(U) \cap V = \emptyset$. Now, f is a $gpr^{\lambda*}$-open map, $f(U)$ is gpr^{λ}-open in Y. Thus we have, $V \subseteq X - f(U)$. This implies $X - f(U)$ is gpr^{λ}-closed in Y. Also $gpr^{\lambda}\text{-}cl(V) \subseteq gpr^{\lambda}\text{-}cl\,(X - f(U)) = X - f(U)$. Therefore $f(U) \cap gpr^{\lambda}\text{-}cl(V) = \emptyset$. This shows that $G(f)$ is strongly supra *gpr*-closed.

Theorem 5.8 . If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ has a strongly supra *gpr*-closed graph then for each $x \in X$, $\{f(x)\} = \cap\{ gpr^{\lambda}\text{-}cl(f(A))\,;\, A \in GPR^{\mu}O\,(X, x)\}$.
Proof: Suppose $y \neq f(x)$ and $y \in \cap\{ gpr^{\lambda}\text{-}cl(f(A))\,;\, A \in GPR^{\mu}O\,(X, x)\}$. Then $y \in gpr^{\lambda}\text{-}cl(f(A))$ for each $x \in A \in GPRO^{\mu}(X, x)$. This implies that for each gpr^{λ}- open set B containing y, $B \cap f(A) \neq \emptyset$. Thus $B \cap f(A) \subset gpr^{\lambda}\text{-}cl(B) \cap f(A) \neq \emptyset$. This shows that $f(A) \cap gpr^{\lambda}\text{-}cl(B) \neq \emptyset$. Since $(x, y) \notin G(f)$ and $G(f)$ is strongly supra *gpr*- closed graph , this is a contradiction. Therefore the result.

Theorem 5.9. Let $GPRC^{\lambda}(Y)$ be closed under arbitrary intersection. If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is supra *gpr* irresolute and Y is $gpr^{\lambda}\text{-}T_2$, then $G(f)$ is strongly supra *gpr*- closed graph.
Proof: Let $(x, y) \in (X \times Y) - G(f)$. Since Y is $gpr^{\lambda}\text{-}T_2$ there exist a gpr^{λ}-open set V such that $f(x) \notin gpr^{\lambda}\text{-}cl(V)$. Then $Y - gpr^{\lambda}\text{-}cl(V) \in GPRO^{\lambda}(Y, f(x))$. Since f is supra *gpr* -irresolute, there exist a gpr^{μ}-open set U containing x such that $f(U) \subseteq Y - gpr^{\lambda}\text{-}cl(V)$. Thus $f(U) \cap gpr^{\lambda}\text{-}cl\,(V) = \emptyset$. Therefore $G(f)$ is strongly supra gpr-closed graph.

Theorem 5.10. Let (X, τ) be the topological space with supra topology μ associated with τ and $GPRC^{\mu}(X)$ be closed under arbitrary intersection . A space X is $gpr^{\mu}\text{-}T_2$ iff the identity function $f : X \rightarrow X$ has a strongly *gpr*-closed graph.
Proof: *Necessity*. Let X be a $gpr^{\mu}\text{-}T_2$ space. Since the identity function is supra *gpr* -irresolute by theorem 5.9. $G(f)$ is strongly *gpr*-closed graph.
Sufficiency : Let $G(f)$ be strongly *gpr*-closed. Since f is surjective , by theorem 5.6 X is $gpr^{\mu}\text{-}T_2$.

Theorem 5.11. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be $gpr^{\mu\,*}$- continuous function where Y is a $gpr^{\lambda}\text{-}T_2$ space. Then f has strongly supra *gpr*-closed graph.
Proof: Let $x \in X$, $y \in Y$, $y \neq f(x)$. Then since Y is $gpr^{\lambda}\text{-}T_2$ space, there exists gpr^{λ}-open sets U and V containing $f(x)$ and y respectively such that $U \cap V = \emptyset$. This implies $U \cap gpr^{\lambda}\text{-}cl\,(V) = \emptyset$. Now f is $gpr^{\mu\,*}$-continuous, $f^{-1}(cl^{\lambda}(V))$ is gpr^{μ}- closed in X and $x \notin f^{-1}(cl^{\lambda}(V))$. Let $W = X - f^{-1}(cl^{\lambda}(V))$. Then W is gpr^{μ}- open set containing x and $f(W) \cap (cl^{\lambda}(V)) = \emptyset$. Therefore $f(W) \cap gpr^{\lambda}\text{-}cl\,(V) = \emptyset$ implying thereby that f has strongly supra *gpr*-closed graph.

Definition 5.12. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be weakly supra *gpr*- irresolute if for each point x in X and each gpr^{λ}-open set V in Y containing $f(x)$ there exist a gpr^{μ}-open set U in X containing x such that $f(U) \subseteq gpr^{\mu}\text{-}cl(V)$.

Remark 5.13. Any supra *gpr*-irresolute function is weakly supra *gpr*- irresolute. However the converse need not be true.

Example 5.14. Let $X = Y = \{0, 1, 2, 3\}$ with topological spaces $\tau = \{\emptyset, X, \{0, 1\}\}$, and $\sigma = \{\emptyset, Y, \{1\}\}$. Also let $\mu = \{\emptyset, X, \{0, 1\}, \{1,2\},\{0,2\},\{0,1,2\},\{0\}\}$ and $\lambda = \{\emptyset, Y, \{0,1\},\{1\}\}$ be the supra topologies associated with τ and σ respectively. Define $f : X \to Y$ by $f(0) = 0, f(1) = 1$, $f(2) = 2, f(3) = 3$. Here f is weakly supra *gpr*- irresolute but not supra *gpr*- irresolute.

Theorem 5.15. If $f : (X, \tau) \to (Y, \sigma)$ is weakly supra *gpr*- irresolute injective function with strongly supra *gpr* -closed graph $G(f)$ then X is gpr^{μ}-T_2.

Proof: Since f is injective for any pair of distinct points x_1,x_2 in X , $f(x_1) \neq f(x_2)$. Therefore $(x_1, f(x_2)) \notin G(f)$. Now $G(f)$ is strongly supra *gpr* -closed, there exist $U \in GPRO^{\mu}(X, x_1)$, $V \in GPRO^{\lambda}(Y, f(x_2))$ such that $f(U) \cap gpr^{\lambda}\text{-}cl(V) = \emptyset$. Thus $U \cap f^{-1}(gpr^{\mu}\text{-}cl(V)) = \emptyset$ which implies $f^{-1}(gpr^{\mu}\text{-}cl(V)) \subseteq X - U$. Since f is weakly supra *gpr*- irresolute, there exist $W \in GPRO^{\mu}(X, x_2)$ such that $f(W) \subseteq gpr^{\mu}\text{-}cl(V)$. Thus $W \subseteq f^{-1}(gpr^{\mu}\text{-}cl(V)) \subseteq X - U$ which implies that $W \cap U = \emptyset$. Therefore X is gpr^{μ}-T_2.

References

[1] K. K. Dube, J .Y. Lee And O. S. Panwar, A Note On Semi-Closed Graphs, U.I.T. Report,14(2) (1983),379-383.
[2] Jamal M. Mustafa, Hamzeh A. Qoqazeh, Supra D-Sets And Associated Separation Axioms, International Journal Of Pure And Applied Mathematics, 80(5) (2012), 657- 663.
[3] D. S. Jankovic, Beograd, On Functions With A-Closed Graphs, Glasnik Matematicki, 18(38) (1983), 141-148.
[4] S. Kasahara, Operation - Compact Spaces, Math. Japonica, 24(1) (1979), 97-105.
[5] M. Lellis Thivagar And Nirmala Rebecca Paul, On Topological Closed Graphs In The New Form, Jordan Journal Of Mathematics And Statistics, 5(4) (2012), 251-266.
[6] P.E. Long, Functions With Closed Graph, Amer. Math. Monthly, 76(8) (1969), 930-932.
[7] P.E. Long And L.L.Herrington, Functions With Strongly Closed Graphs, Boll. Un. Mat. Ital.(4)12(1975),381-384.
[8] A. S. Mashour, A. A. Allam, F.S. Mahoud And F. H. Kheder, On Supra Topological Spaces, Indian Journal Pure And Applied Mathematics, 14(4) (1983), 502-510.
[9] Miguel Caldas, Saeid Jafari, Raja M.Latif And Takashi Noiri, Characterizations Of Functions With Strongly A-Closed Graphs, 2008.
[10] Nandini Bandyopadhyay And P. Bhattacharyya, Functions With Preclosed Graphs, Bull. Malays. Math. Sci.Soc.(2) 28(1) (2005), 87-93.
[11] Nandini Bandyopadhyay And P. Bhattacharyya, On Functions With Strongly Preclosed Graphs, Soochow Journal Of Mathematics, 32(1) (2006), 77-95.
[12] T. Noiri, On Functions With Strongly Closed Graphs, Acta Math. Acad. Sci. Hungar.32(1978), 373-375.
[13] Vidhya Menon , On Generalized Pre-Regular Closed Sets In Supra Topological Spaces, International Journal Of Scientific And Engineering Research, 3(11)(2012),6-11.
[14] O. R Sayed, Gnanambal Ilango And Vidhya Menon, An Extensive Study Of Supra Generalized Pre-Regular Closed Sets, International Journal Of Pure And Applied Mathematics, 6(106) (2016), 57-73.

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