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A Survey On Image Registration Techniques Using FFT

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Abstract:
Image Registration is the process of straightening two or more images of the same scene with testimonial to a particular image. The images are captured from various sensors at different times and at multiple viewpoints. Thus, to get a better picture of any change of a scene/object over a considerable period of time image registration is important. Image registration finds application in medical sciences, remote sensing and in computer vision. This paper presents a detailed review of several approaches which are classified accordingly along with their contributions and drawbacks. The main steps of an image registration procedure are also discussed. Different performance measures are presented that determine the registration quality and accuracy. The scope for the future research are presented as well.
Applications based on Fast Fourier Transform (FFT) such as signal and image processing require high computational power, plus the ability to experiment with algorithms. Reconfigurable hardware devices in the form of Field Programmable Gate Arrays (FPGAs) have been proposed as a way of obtaining high performance at an
economical price. At present, however, users must program FPGAs at a very low level and have a detailed knowledge of the architecture of the device being used. To try to reconcile the dual requirements of high performance and ease of development, this paper reports on the design and realization of a High-Level framework for the implementation of 1-D and 2-D FFTs for real-time applications. Results show that the parallel implementation of 2-D FFT achieves virtually linear speed-up and real-time performance for large matrix sizes. Finally, an FPGA-based parametrizable environment based on the developed parallel 2-D FFT architecture is presented as a solution for frequency domain image filtering application.

Keyword*: Fast Fourier transformation, Image registration, classification, contribution, drawback, performance measures, registration quality, accuracy, future research.*

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I. Introduction

Fourier transforms play an important role in many digital signal processing applications including acoustics, optics,
telecommunications, speech, signal and image processing [1,17,18]. However, direct computation of Discrete Fourier Transform (DFT) requires on the order of N2 operations where N is the transform size. The FFT algorithm, first explained by Cooley and Tukey [1],opened a new area in digital signal processing by reducing the order of complexity of DFT from N2 to N log2N. Since the early paper by Cooley and Tukey, a large number of FFT algorithms have been developed. Among these, the radix-2, radix-4, split-radix and FHT algorithms are the ones that have been mostly used for practical applications due to their simple structure, with a constant butterfly geometry, and the possibility of performing them "in place". Most of the research to date for the implementation and benchmarking of FFT algorithms have been performed using general purpose processors [3,4], Digital Signal Processors(DSPs) [5,6,7] and dedicated FFT processor ICs [8,9]. However, as Field Programmable Gate Arrays (FPGAs) have grown in capacity, improved in performance, and decreased in cost, they have become a viable solution for performing computationally intensive tasks (i.e computation of FFT), with the ability to tackle applications for custom chips and programmable DSP devices [10-14]. Although there has been extensive research on the hardware implementation of the FFT algorithms, there are some inherent drawbacks of existing studies. They are designed and optimized for:

- fixed type of ASIC, DSP and FPGA platform;
- fixed type of FFT algorithm; and
- certain fixed design parameters such as transform size (N), input/output data wordlengths (L).

This narrows the application area of such implementations.

This paper is concerned with:
• the design and implementation of a parametrizable architecture, which provides a framework for the implementation of different types of 1-D FFT algorithms;
• the development of an FPGA-based FFT library by implementing radix-2, radix-4, split-radix and FHT algorithms under the same framework in order to provide system designers and engineers with the flexibility to meet different application requirements with given hardware resources;
• the design and implementation of a generic parallel 2-D FFT architecture for real-time image processing applications; and
• the development of an FPGA-based parametrizable system for frequency-domain filtering of large images.

The FFT architectures have been designed using Handel-C language [15]. Although the task could have been accomplished using traditional Hardware Description Languages (such as VHDL or Verilog), Handel-C has been selected as we aimed to evaluate its rapid design capabilities and suitability for the design of IP cores.

Image Registration is interpreted as the process of overlaying two or more images of the same scene with respect to a particular reference image. The images may be taken at various circumstances (time-points), from various perspectives(view-points), and additionally by various sensors. The reference image is generally one of these captured images. It geometrically transforms different sets of data into a particular reference co-ordinate system. The discrepancies among these images are interposed owing to the disparate imaging conditions. Image acquisition devices underwent rapid modifications and proliferating amount and diversity of acquired images elicited the research on automatic image registration. In image analysis ventures, one of the most significant step is Image Registration. It is a necessary step to obtain the final information from a combination of a multitude of divergent sources capturing the same information in varied circumstances and diverse manners. Essentially the objective is to detect the concealed relationship existing between the input and the reference images which is usually indicated by a coordinate transformation matrix. Accordingly, an image registration can be essentially devised as an optimization problem. Image registration plays a crucial role in many real-world applications. Image registration finds applications in remote sensing [1-3] involving multispectral classification, environmental monitoring, change detection, image mosaicking, weather forecasting, creating super-resolution images and integrating information into geographic information systems (GIS), in medicine [4-8] including fusion of computer tomography (CT) and NMR data to obtain more complete information about the patient, multi-modal analysis of different diseases like epilepsy where the protocols incorporate functional EEG/MEG data along with anatomical MRI, monitoring tumor evolution, treatment verification, juxtaposition of the patient’s data with anatomical atlases, in cartography for map updating, and in computer vision for target localization, automatic quality control and motion tracking. According to the manner of image acquisition the application of Image Registration can be segregated into the following groups.
1. Multi-view Analysis: Images of the similar object or scene are captured from multiple viewpoints to gain a better representation of the scanned object or scene. Examples include mosaicking of images and shape recovery from the stereo.
2. Multi-temporal Analysis: Images of the same object/scene are captured at various times usually under dissimilar conditions to notice changes in the object/scene which emerged between the successive image’s acquisitions. Examples include motion tracking, tracking the growth of tumors. 3. Multi-modal Analysis: Different sensors are used to acquire the images of the same object/scene to merge the information obtained from various sources to obtain the minutiae of the object/scene. Examples include integration of information from sensors with disparate characteristics providing better spatial and spectral resolutions independent of illumination-this depends upon the robustness of the registration algorithm, combination of sensors capturing the anatomical information like magnetic resonance image (MRI), ultrasound or CT with sensors acquiring functional information like positron emission tomography (PET), single photon emission computed tomography (SPECT) or magnetic resonance spectroscopy (MRS) to study and analyze seizure disorders, Alzheimer’s disease, depression and other diseases.

II. Material And Methods

1-D DFT and Its Fast Computation

The DFT of an N-point discrete-time complex sequence x(n), indexed by n = 0,1,…,N-1, is defined by

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot W_M^{kn}, \quad k = 0,1,..,N-1 \qquad (1)$$

where $W_N = e^{-j2\pi/N}$ and W_N is referred as the *twiddle factor*.

The excessively large amount of computations required to compute the DFT directly when N is large has prompted alternative methods for computing the DFT efficiently. This problem was alleviated with the development of special fast algorithms, collectively known as fast Fourier transform. Most of FFT algorithms decomposes the overall N-point DFT into successively smaller and smaller

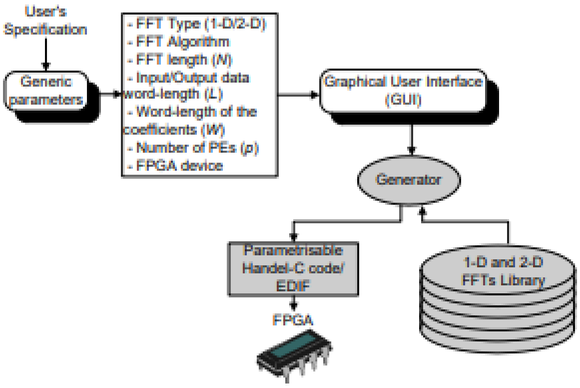

Figure 1. Proposed system for the FFTs implementation.

Transforms known as a butterfly. An overview of the most common FFT algorithms is presented in the following subsections. Again, references [1,2,17,18,19] contain a detailed development of these FFT algorithms.

Radix-2n FFT Algorithms

For Cooley-Tukey radix-2 algorithm, decimation-infrequency (DIF) decomposition, Eq. (1) is decomposed into even and odd frequency components [1]:

$$X_{2k} = \sum_{n=0}^{N/2-1} \left(x_n + x_{n+N/2}\right) \cdot W_{N/2}^{nk} \qquad (2)$$

$$X_{2k+1} = \sum_{n=0}^{N/2-1} \left(x_n - x_{n+N/2}\right) \cdot W_{N/2}^{n} \cdot W_{N/2}^{nk} \qquad (3)$$

For radix-2 FFT algorithm, the smallest transform or butterfly (basic computational unit) used is the 2-point DFT as shown in Figure 2.a.

The radix-4 algorithm can be obtained by decomposing Eq. (2) and (3) into X_{4k}, X_{4k+2}, X_{4k+1} and X_{4k+3} frequency components. The radix-4 butterfly has 4 inputs and 4 outputs, essentially combining two stages of a radix-2 algorithm in one. Figure 2.b shows the radix-4 butterfly.

The split-radix algorithm [19] decomposes the odd frequency component in Eq (3) into 4k+1 and 4k+3 frequency components as follows;

$$X_{4k+1} = \sum_{n=0}^{N/4-1} \left(x_n - j \cdot x_{n+N/4} - x_{n+N/2} + j \cdot x_{n+3N/4}\right) \cdot W_N^{3n} \cdot W_{N/4}^{nk} \quad (4)$$

$$X_{4k+3} = \sum_{n=0}^{N/4-1} \left(x_n + j \cdot x_{n+N/4} - x_{n+N/2} - j \cdot x_{n+3N/4}\right) \cdot W_N^{3n} \cdot W_{N/4}^{nk} \quad (5)$$

The L-shape butterfly element of split-radix FFT algorithm is given in Figure 2.c.

The Fast Hartley Transform

The Discrete Hartley Transform (DHT) belongs to the family of frequency transforms. The significant difference between DHT and DFT is that DHT is a real-valued transform.

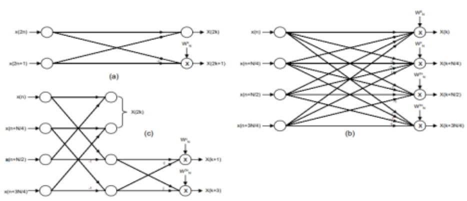

Figure 2. The "butterfly" used in (a) radix-2, (b) radix-4, (c) split-radix algorithms.

The DHT is defined for a real-valued N-point sequence $x(n)$, n=0,1,…,N-1, by the following equation [2]:

$$H_k = \sum_{n=0}^{N-1} x_n \cdot cas\left(\frac{2\pi}{N}kn\right), \quad k = 0,1,\ldots,N-1 \qquad (6)$$

where cas(x) = cos(x) + sin(x).

The FHT algorithm for the computation of DHT resembles the radix-2 FFT algorithm. The FHT algorithm is based on the decomposition of Eq (6) as follow:

$$H_{2k} = \sum_{n=0}^{N/2-1}\left[x_n + x_{n+N/2}\right]\cdot cas(4\pi nk/N)$$

$$H_{2k+1} = \sum_{n=0}^{N/2-1}\left[x_n - x_{n+N/2}\right]\cdot \begin{Bmatrix} cas(4\pi nk/N)cos(2\pi n/N) + \\ cas(4\pi(N-n)k/N)sin(2\pi n/N) \end{Bmatrix} \qquad (7)$$

Each of the (N/2)-point DHT's can be further decomposed in a similar fashion to complete the FHT DIF algorithm.

1. Extrinsic Methods

In this method artificial foreign objects which are easily detectable are attached to the patient body [46-53]. They serve as external features to be used for feature matching. The complexity is lessened and hence computational is fast and accuracy is also maintained. Examples are markers glued to patient's skin or stereo-tactic frame attached rigidly to the patient's outer skull for invasive neurosurgery related purposes.

2. Surface Methods

Surfaces or boundaries or contours are generally distinct in medical images unlike landmarks. For example, surface-based approach is employed for registering multimodality brain image. These surface matching algorithms are generally applied to rigid body registration. A collection of points, generally called a point set is extracted from the contours in an image. If two surfaces are considered for registration then there will be two such sets. The surface covering the larger volume of the patient, or that having a higher resolution if volume coverage is comparable, is generally considered for generation of the surface model. Iterative Closest Point Algorithm and Correspondence Matching Algorithm are successfully applied as registration algorithms for surface-based techniques [54-63].

Meta-heuristics and Evolutionary Optimization are also seen to solve these high dimensional optimization problems of surface registrations.

3. Moments and Principle Axes Methods

The orthogonal axes about which the moments of inertia are minimized are known as the principle axes. Two identical objects can be registered accurately by bringing their principal axes into concurrence without employing any rigid/affine transformations. If the objects are not identical but similar in appearance then they can be approximately registered by this technique [16, 64]. For moment-based methods pre-segmentation is done in many cases to engender satisfactory outcomes.

III. Result & Discussion

The frequency domain image filtering is one of the most important applications where 2-D FFT can be applied. The 2-D convolution in spatial/time domain is commonly used for image filtering. It is fast and easy if the input image and the filter kernel being used are relatively small. But, as the image or kernel grows in size the computational complexity increases geometrically. Filtering of large images can be done much faster in frequency domain using 2-D FFT, based on the convolution property of the Fourier transform as follows [27]:

Step 1. Compute 2-D FFT of input image and filter: FFT{I(x,y)} and FFT{H(x,y)}.

Step 2. Apply filter H(u,v) to the FFT of input image by point-by-point multiplication: Y(u,v) = I(u,v)*H(u,v).

Step 3. Compute the 2-D IFFT of result: IFFT{Y(u,v)} The speed-up is approximately $N^2/N \log_r N = N/\log_r N$, which is significant when dealing with large images.

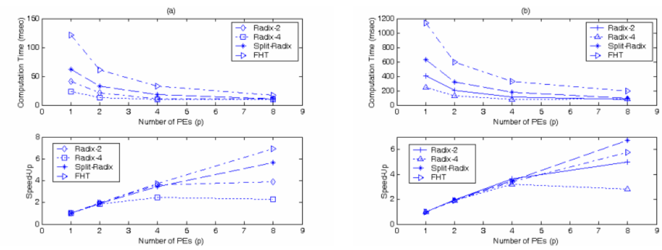

Figure 3: Computation time and speed-up versus number of PEs: (a) matrix size of 256x256. (b) matrix size of 1024x1024

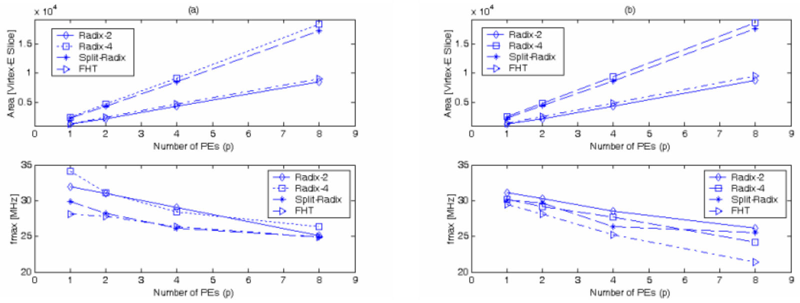

Figure 4. Area and fmax versus number of PEs: (a) matrix size of 256x256 (b) matrix size of 1024x1024.

Image Registration in Frequency Domain

Correlation theorem has one useful property. Correlation theorem states that, the Fourier transform of the correlation of two images is the product of Fourier transform of one image and complex conjugate of Fourier transform of other. The Fourier transform of an image f(x,y) is a complex function, each function value has real part R(ωx,ωy) and an imaginary part I(ωx,ωy)at each frequency (ωx,ωy) of frequency spectrum.

$$F(\omega_x\omega_y) = |F(\omega_x\omega_y)|\, e^{-\varphi(\omega_x\omega_y)}$$

Where $|F(\omega_x\omega_y)|$ is magnitude , and $\varphi(\omega_x\omega_y)$ *is* phase angle

$$|F(\omega_x\omega_y)|^2 = R^2(\omega_x\omega_y) + I^2(\omega_x\omega_y)$$

$$\varphi(\omega_x\omega_y) = \tan^{-1}\left[\frac{I(\omega_x\omega_y)}{R(\omega_x\omega_y)}\right]$$

Cross power spectrum of two images is defined as

$$F(\varphi_x\varphi_y) = \frac{F1(\varphi_x\varphi_y)F2^*(\varphi_x\varphi_y)}{|F1(\varphi_x\varphi_y)F2^*(\varphi_x\varphi_y)|}$$

IV. Conclusion

Due to the importance and use of FFT in many signal and image processing applications, a range of 1-D FFT algorithms including radix-2, radix-4, split-radix and FHT have been implemented using proposed parametrizable framework architecture. In addition, an efficient for the implementation of 2-D FFTs has also been proposed and implemented. A complete environment based on the developed parallel 2-D FFT architecture has been presented as a solution for 2-D frequency-domain image filtering application. The performances of implementations have been discussed, investigated, and compared with existing works. High speed-up and efficiency have been attained for the parallel implementation of 2-D FFT compared to existing works.

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