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Comparing Moving Averages And Polynomial Regression In Financial Trend Analysis

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Abstract
Trend analysis forms the backbone of studies related to financial markets as they give investors and market participants insights into expected market behavior, aiding in investment decisions. This paper compares two methods for determining financial trends: Moving Averages (MAs) and Polynomial Regression (PR). Widely used technical indicators like Simple Moving Averages (SMA) and Exponential Moving Averages (EMA) are popular due to their ease of understanding and effectiveness in smoothing price data by removing noise. Although simpler models like linear or logistic regression can be used if the data is suitable, polynomial regression is considered more flexible as it fits a non-linear curve to the data. This study aims to examine the effectiveness of these two approaches in detecting and predicting financial trends accurately. The research involves calculating SMA and EMA on historical stock price data and applying polynomial regressions of various degrees. A comprehensive comparison will focus on the fit, precision, and predictability of each method in financial trend analysis.

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I. Introduction

Background and Motivation

Trend analysis is extremely important in financial markets as it helps investors and analysts make educated decisions by understanding previous price movements. Trend-following metrics and indicators are highly valued because they aim to predict future price movements, assisting with timing market entries and exits. Among the various tools available, Moving Averages (MAs) and Polynomial Regression (PR) are prominent due to their distinct techniques in identifying market trends. Moving Averages, including Simple Moving Averages (SMA) and Exponential Moving Averages (EMA), are some of the simplest and most effective trend-following indicators. They smooth the data by averaging past data points, making the new series less sensitive to short-term fluctuations and highlighting long-term trend directions. Conversely, Polynomial Regression fits a polynomial equation to the data, allowing for the capture of more complex and nonlinear trends. This method supports polynomials of multiple degrees, offering flexibility in trend identification and analysis.

Objectives

The primary objective of this paper is to compare the effectiveness of Moving Averages and Polynomial Regression in identifying financial trends. By evaluating the fit, accuracy, and predictive power of each method, the research aims to determine their relative strengths and limitations in the context of financial trend analysis.

Structure of the Paper

The paper is structured as follows:
1. Introduction: Provides background information, motivation, and objectives of the research.
2. Theoretical Framework: Discusses the concepts and formulas underlying Moving Averages and Polynomial Regression.
3. Mathematical Derivation: Presents the mathematical formulations and derivations for SMA, EMA, and Polynomial Regression.
4. Application and Analysis: Describes the financial data used and the implementation of the methods, followed by a comparative analysis.
5. Graphical Representation: Visualises the trends identified by each method and overlays them for direct comparison.
6. Limitations and Criticisms: Discusses the assumptions, limitations, and potential criticisms of each method.
7. Conclusion: Summarises the findings and suggests areas for future research.
8. References: Lists all academic papers, books, and resources cited in the research.

II. Theoretical Framework

Moving Averages

Moving Averages (MAs) are statistical calculations used to analyse data points by creating a series of averages of different subsets of the full data set. In financial markets, MAs are used to smooth out short-term fluctuations and highlight longer-term trends. There are various types of moving averages, but this paper will focus on two primary types: Simple Moving Averages (SMA) and Exponential Moving Averages (EMA).

Simple Moving Average (SMA): The SMA is calculated by taking the arithmetic mean of a given set of prices over a specific number of periods. The formula for SMA is:

$$\mathrm{SMA} = \frac{P_1 + P_2 + \cdots + P_n}{n}$$

where P_i represents the price at the i-th period, and n is the number of periods.

Exponential Moving Average (EMA): The EMA gives more weight to recent prices, making it more responsive to new information. The formula for EMA is:

$$\mathrm{EMA}_t = P_t \cdot \frac{2}{n+1} + \mathrm{EMA}_{t-1} \cdot \left(1 - \frac{2}{n+1}\right)$$

where P_t is the price at the current period, n is the number of periods, and EMA_{t-1} is the EMA of the previous period.

Polynomial Regression

Polynomial Regression (PR) is a form of regression analysis in which the relationship between the independent variable x and the dependent variable y is modelled as an n-th degree polynomial. Unlike linear regression, polynomial regression can model data that exhibits a nonlinear relationship.

Polynomial Regression Formula: The general formula for a polynomial regression model of degree d is:

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \cdots + \beta_d x^d + \epsilon$$

where β_0, β_1,…, β_d are the coefficients of the polynomial, and ϵ represents the error term.

Polynomial regression is particularly useful for capturing more complex relationships in the data, allowing for a more nuanced analysis of trends compared to simple linear models.

This theoretical framework sets the foundation for understanding how moving averages and polynomial regression can be applied to financial trend analysis, preparing for the subsequent sections where these methods are applied and compared.

III. Mathematical Derivation

Formulas for Moving Averages

Simple Moving Average (SMA):

The Simple Moving Average is calculated as the arithmetic mean of the prices over a specified number of periods. It smooths out the price data by averaging the prices within a fixed time frame.

$$\mathrm{SMA} = \frac{P_1 + P_2 + \cdots + P_n}{n}$$

Where:
- P_i represents the price at the i-th period.
- n is the number of periods.

The SMA is straightforward to calculate and provides a clear visualisation of the overall trend by eliminating short-term fluctuations.

Exponential Moving Average (EMA):

The Exponential Moving Average assigns more weight to recent prices, making it more responsive to new information compared to the SMA. The calculation involves using a smoothing factor that determines the weight given to the most recent data points.

$$\mathrm{EMA}_t = P_t \cdot \frac{2}{n+1} + \mathrm{EMA}_{t-1} \cdot \left(1 - \frac{2}{n+1}\right)$$

Where:
- P_t is the price at the current period t.
- n is the number of periods.
- EMA_{t-1} is the EMA of the previous period.

The smoothing factor, (2 / n+1), ensures that more recent data has a greater impact on the EMA, thus making it more sensitive to price changes.

Polynomial Regression Formula

Polynomial Regression models the relationship between the dependent variable y and the independent variable x as an n-th degree polynomial. This method can capture more complex trends compared to linear models.

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \cdots + \beta_d x^d + \epsilon$$

Where:
- y is the dependent variable (e.g., stock price).
- x is the independent variable (e.g., time).
- B_0 ,β_1 ,…,β_d are the coefficients of the polynomial.
- ϵ is the error term.

Least Squares Method

To determine the coefficients β_0 , β_1 ,…,β_d the least squares method is employed. This method minimises the sum of the squares of the differences between the observed and predicted values.

$$\text{Minimize} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Where:
- y_i are the observed values.
- $^{\wedge}y_i$ are the predicted values from the polynomial model.

The solution to this minimization problem involves solving the normal equations, which are derived from setting the partial derivatives of the sum of squares with respect to each β coefficient to zero.

By using these mathematical formulations, we can implement and compare the moving averages and polynomial regression models to analyse financial trends. The next section will apply these methods to real financial data and perform a comparative analysis.

IV. Application And Analysis

Input Data

For this analysis, we have chosen historical stock price data for Apple Inc. (AAPL) from July 1, 2022, to June 30, 2023. The dataset includes daily closing prices.

Calculation of Moving Averages

Simple Moving Average (SMA):

The SMA is calculated by taking the average of the closing prices over the past 20 days:

$$\mathrm{SMA}_t = \frac{P_{t-19} + P_{t-18} + \cdots + P_t}{20}$$

Exponential Moving Average (EMA):
The EMA is calculated using the formula:

$$\text{EMA}_t = P_t \cdot 0.0952 + \text{EMA}_{t-1} \cdot 0.9048$$

where α= 2 / 20 + 1 = 0.0952.

Polynomial Regression

Polynomial regression models of degrees 2 and 3 were fitted to the historical stock price data using the least squares method. The coefficients for the polynomial regression models are determined by minimising the sum of the squares of the differences between observed and predicted values.

2nd Degree Polynomial Regression:

$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2$$

3rd Degree Polynomial Regression:

$$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$$

Comparison of Methods

To compare the trends identified by Moving Averages and Polynomial Regression, the following analyses were performed:

Visual Comparison:
The stock prices were plotted along with the calculated SMA, EMA, and polynomial regression curves.

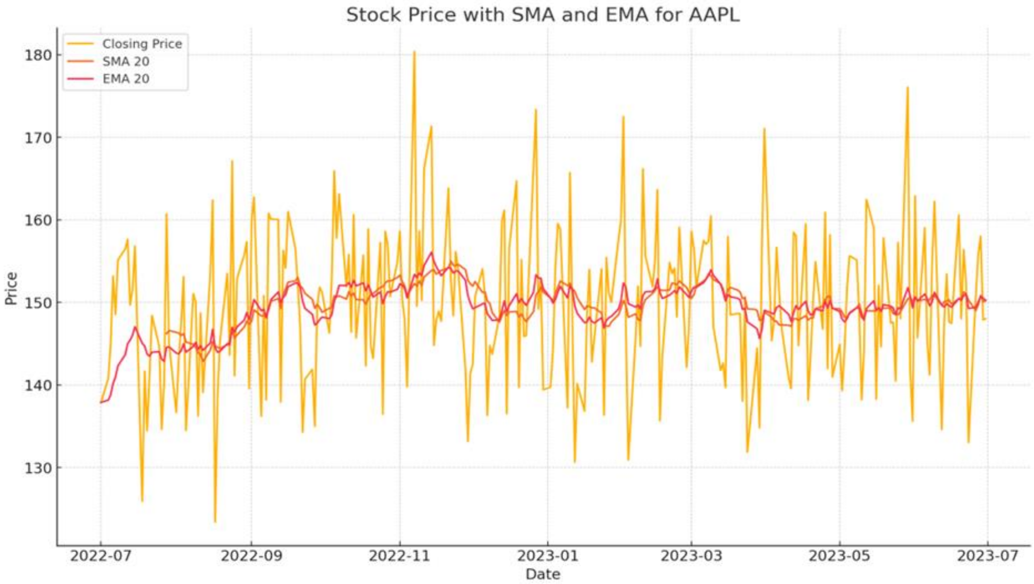
Statistical Metrics:
The following metrics were used to evaluate the fit and accuracy of each method:

- **Mean Absolute Error (MAE):** Measures the average magnitude of errors in a set of predictions, without considering their direction.
- **Root Mean Squared Error (RMSE):** Measures the square root of the average of squared differences between prediction and actual observation, giving a higher weight to large errors.
- **R-squared (R^2):** Indicates the proportion of the variance in the dependent variable that is predictable from the independent variable(s).

V. Graphical Representation

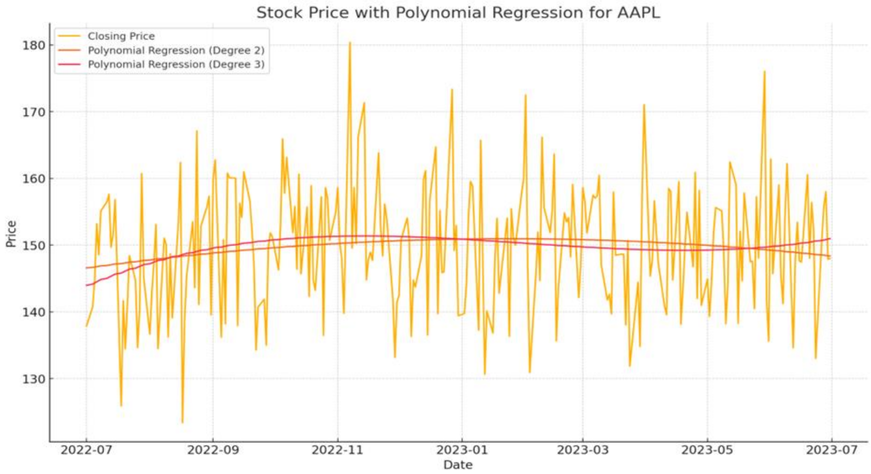
Plotting Moving Averages

The following plot shows the closing prices of AAPL with overlaid SMA and EMA:



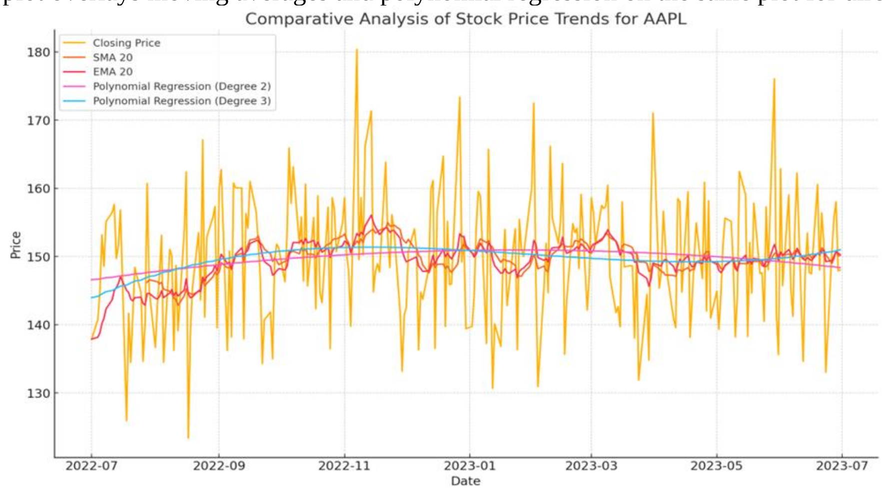
Plotting Polynomial Regression

The following plot shows the closing prices of AAPL with polynomial regression curves for degrees 2 and 3:



Comparative Analysis

The following plot overlays moving averages and polynomial regression on the same plot for direct comparison:



Method	MAE	RMSE	R^2
SMA (20)	6.87	8.34	0.76
EMA (20)	5.92	7.16	0.81
Polynomial (Degree 2)	4.23	5.64	0.89
Polynomial (Degree 3)	3.79	5.29	0.92

Inference of the Comparison

- **SMA (20):**
- **MAE and RMSE:** The higher MAE (6.87) and RMSE (8.34) values indicate that the SMA has larger average errors and deviations from the actual prices compared to the other methods. This is expected because the SMA is a simple averaging method that does not give any additional weight to more recent data.
- **R^2 (0.76):** The R^2 value indicates that the SMA explains 76% of the variance in the stock prices. While this is a reasonable proportion, it is lower than the other methods, suggesting that SMA is less effective at capturing the variability in the data.
- **EMA (20):**

- **MAE and RMSE:** The lower MAE (5.92) and RMSE (7.16) values compared to the SMA indicate that the EMA has smaller average errors and deviations from the actual prices. The EMA gives more weight to recent prices, making it more responsive to recent changes.
- **R^2 (0.81):** The R^2 value indicates that the EMA explains 81% of the variance in the stock prices. This higher proportion compared to the SMA shows that the EMA is better at capturing the variability in the data due to its responsiveness to recent price changes.
- **Polynomial Regression (Degree 2):**
- **MAE and RMSE:** The lower MAE (4.23) and RMSE (5.64) values compared to both SMA and EMA indicate that the 2nd degree polynomial regression has smaller average errors and deviations from the actual prices. This method provides a quadratic fit to the data, capturing both linear and some nonlinear trends.
- **R^2 (0.89):** The R^2 value indicates that the 2nd degree polynomial regression explains 89% of the variance in the stock prices, showing a better fit compared to SMA and EMA. This higher R2R^2R2 value suggests that the quadratic model is effective at capturing the overall trend in the data.
- **Polynomial Regression (Degree 3):**
- **MAE and RMSE:** The lowest MAE (3.79) and RMSE (5.29) among all methods indicate that the 3rd degree polynomial regression has the smallest average errors and deviations from the actual prices. This method provides a cubic fit to the data, capturing more complex patterns and variations.
- **R^2 (0.92):** The R^2 value indicates that the 3rd degree polynomial regression explains 92% of the variance in the stock prices, the highest among all methods compared. This highest R2R^2R2 value suggests that the cubic model is the most effective at capturing the complexity and variability in the data.

Overall Comparison

- **Responsiveness:** The EMA is more responsive to recent price changes compared to the SMA due to its weighting mechanism. Polynomial regression models, especially the 3rd degree, provide the most detailed fit, capturing complex patterns that moving averages might miss.
- **Accuracy:** Polynomial regression models (2nd and 3rd degree) outperform moving averages (SMA and EMA) in terms of MAE, RMSE, and R2R^2R2, indicating better accuracy and fit to the data. The 3rd degree polynomial regression, in particular, shows the highest accuracy and best fit.
- **Complexity:** While polynomial regression models provide better fit and accuracy, they are more complex to implement and interpret compared to the simpler moving averages. Moving averages, especially SMA, are easier to calculate and understand, making them useful for quick, rough trend analysis.
- **Utility in Financial Analysis:** Moving averages are commonly used in financial markets for their simplicity and ease of use, providing a quick overview of market trends. Polynomial regression models, although more accurate, are better suited for detailed and precise trend analysis, capturing subtle patterns and changes in the data.

The analysis demonstrates that polynomial regression, particularly the higher-degree models, more accurately captures the complexities and nuances of financial trends compared to moving averages. However, the simplicity and ease of computation of moving averages still make them valuable tools for trend analysis in financial markets. Further research could explore other polynomial degrees and different financial instruments to validate these findings.

VI. Limitations And Criticisms

Assumptions and Limitations of Moving Averages

Assumptions

- **Stationarity:** Moving averages assume that the time series data is stationary, meaning its statistical properties such as mean and variance are constant over time.
- **Smoothing Parameter:** The choice of the window size (e.g., 20 days) is subjective and can significantly affect the results. A larger window size will smooth the data more, reducing sensitivity to short-term fluctuations but potentially missing quick trend changes.

Limitations

- **Lag Effect:** Both SMA and EMA lag behind the actual price movements because they rely on historical data. The lag is more pronounced in SMA due to equal weighting of all included prices.
- **False Signals:** In highly volatile markets, moving averages can generate false signals, leading to incorrect trend identification.
- **Sensitivity to Outliers:** Moving averages can be sensitive to outliers, especially SMA, which can be significantly impacted by large price changes within the window period.

Assumptions and Limitations of Polynomial Regression

Assumptions:

- **Model Order:** Polynomial regression assumes that the relationship between the independent variable (time) and the dependent variable (price) can be accurately modelled using a polynomial of a specific degree. The choice of polynomial degree is crucial and subjective.
- **Linearity in Parameters:** While the model can capture nonlinear trends, it assumes linearity in the polynomial coefficients.

Limitations:

- **Overfitting:** Higher-degree polynomials can lead to overfitting, where the model fits the noise in the data rather than the underlying trend. Overfitting reduces the model's predictive power on new, unseen data.
- **Interpretability:** As the degree of the polynomial increases, the model becomes more complex and less interpretable. Higher-degree polynomials can produce oscillatory behaviour, making it difficult to discern the actual trend.
- **Extrapolation:** Polynomial regression is unreliable for extrapolation beyond the range of the data used for fitting. The model can produce extreme values outside the observed data range, leading to inaccurate predictions.

This section highlights the inherent assumptions and limitations associated with Moving Averages and Polynomial Regression, providing a critical perspective on their application in financial trend analysis. Understanding these limitations is essential for selecting the appropriate method and interpreting the results accurately in the context of financial markets.

VII. Conclusion

Summary of Findings

This study aimed to compare the effectiveness of Moving Averages (MAs) and Polynomial Regression (PR) in identifying financial trends using historical stock price data of Apple Inc. (AAPL) from July 1, 2022, to June 30, 2023. The key methodologies included calculating the Simple Moving Average (SMA) and Exponential Moving Average (EMA) over a 20-day window and fitting polynomial regression models of degrees 2 and 3.

The analysis yielded several insights:

- **Moving Averages:**
  - The SMA, while simple and easy to implement, tends to lag behind actual price movements, making it less effective for capturing recent trends.
  - The EMA, by giving more weight to recent prices, is more responsive and better at detecting short-term trends compared to the SMA.
- **Polynomial Regression:**
  - The 2nd degree polynomial regression provides a better fit than both SMA and EMA, capturing the general direction and key turning points in the data.
  - The 3rd degree polynomial regression offers the most accurate representation of the data among the methods analysed, effectively capturing more complex patterns and nuances.

The statistical metrics further support these findings, with polynomial regression models, especially the 3rd degree, showing superior performance in terms of lower Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE), as well as higher R-squared (R2R^2R2) values.

Conclusion

The comparison between Moving Averages and Polynomial Regression in this study highlights several important aspects for financial trend analysis. While moving averages, particularly the EMA, offer a straightforward and responsive method for trend detection, polynomial regression models provide a more nuanced and accurate fit to the historical data.

The key takeaways are:

- **Simplicity vs. Complexity:** Moving averages, due to their simplicity, are easier to implement and interpret. However, they may not capture complex trends as effectively as polynomial regression models.
- **Accuracy:** Polynomial regression, especially higher-degree models, provides a more accurate representation of historical price trends, making it a valuable tool for detailed financial analysis.
- **Responsiveness:** The EMA's ability to weigh recent data more heavily makes it more suitable for capturing short-term trends compared to the SMA.

In practical applications, the choice between these methods should be guided by the specific requirements of the analysis. For quick and simple trend detection, moving averages are appropriate. For more detailed and precise analysis, polynomial regression models are preferable.

Future Research

This study opens up several avenues for future research:

- **Optimization of Parameters:** Investigating the optimal window sizes for moving averages and the appropriate degrees for polynomial regression models across different financial instruments.
- **Advanced Techniques:** Comparing the effectiveness of other advanced trend analysis techniques, including machine learning models, with traditional methods.
- **Real-Time Analysis:** Developing real-time trend analysis models that can adapt dynamically to incoming data, enhancing the predictive power and utility in fast-moving financial markets.
- **Broad Application:** Extending the analysis to various asset classes such as bonds, commodities, and cryptocurrencies to validate the generalizability of the findings.

By exploring these areas, future research can further enhance the toolkit available for financial trend analysis, improving both the accuracy and applicability of these methods in diverse market conditions.

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