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A Mathematical Model for MHD Couette flow with Transpiration Cooling

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Abstract
In this study for Magnetohydrodynamic (MHD) Couette flow incorporating the effects of transpiration cooling, which consists the problem of a Couette flow between two horizontal parallel porous flat plate of an electrically conducting viscous incompressible fluid. The stationary plate is subjected to a transverse sinusoidal injection of the fluid and its corresponding removal by constant suction through the other plate, in uniform motion and because of injection velocity the flow becomes three-dimensional. a magnetic field of uniform strength is also applied normal to the planes of the plates. the effect of injection/suction velocity and the magnetic field on the flow field, skin friction and heat transfer are reported and discussed in detail in graphically.
Keywords: *Magnetohydrodynamic (MHD), Couette flow, transpiration cooling*

I. Introduction

Transpiration cooling, achieved by the injection or suction of fluid through a porous surface, is an effective thermal control technique widely used in high-temperature environments such as gas turbines, nuclear reactors, aerospace structures, and electronic cooling systems. The interaction between transpiration effects and non-Newtonian fluid behavior significantly alters the velocity and temperature distributions, making mathematical modeling essential for accurate prediction and design.

Bansal and Jain (1973) discussed about on the plane couette flow of a viscous compressible fluid with transpiration cooling with exact solution for the plane couette flow of a viseous compressible, heat conducting, perfect gas with the same gas injection at the stationary plate and its corresponding removal at the moving plate has been studied. They found that the gas injection is very helpful in reducing the temperature recovery factor. Effects of injection on the shearing stress at the lower plate, longitudinal velocity pro¡ and the enthalpy arr shown graphically. Gersten and Gross (1974) studied on the three dimensional convective flow and heat transfer through a porous medium, while Gulab and Mishra (1977) expressed an idea through the equation of motion for MHD flow. Raptis (1983) worked on the free convective flow through a porous medium bonded by the infinite vertical plate with oscillating plate temperature and constant suction, and again Raptis and Perdikis (1985) further worked on the free convective flow through a highly porous medium bounded by the infinite vertical porous plate with constant suction. Although in above studies the investigators have restricted themselves to two-dimensional flows, but there may arise situations, where the flow field may be essentially three-dimensional. Therefore Singh (1991) worked on three dimensional MHD flow past a porous plate, and again Singh (1993) also worked in the same direction and studied the problem of three dimensional viscous flow and heat transfer along a porous plate. Again Ahmed and Sharma (1997) discussed about the three-dimensional free convective flow of an incompressible viscous fluid through a porous medium with uniform free steam velocity, while Singh (1999) again studied about a three-dimensional Couette flow with transpiration cooling by applying transverse sinusoidal injection velocity at the stationary plate velocity. Kim (2000) discussed the unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction, and Kamel (2001) discussed about the unsteady MHD convection through porous medium with combined heat and mass transfer with heat source/sink. Kumar et al (2004) discussed about the Hall current on MHD free- convection flow through porous media past a semi-infinite vertical plate with mass transfer, while Muhammad et al (2005) discussed the effects of Hall current and heat transfer on the flow due to a pull of eccentric rotating. Attia (2006) observed the unsteady MHD Couette flow and heat transfer of dusty fluid with variable physical properties..

Here our main motto flow velocity decreases with the increasing Hartmann number (M), and injection parameter (λ).The cross flow velocity component *w* due to the transverse sinusoidal injection velocity distribution applied through out the porous plate at rest and the cross flow velocity profile is shown by graphically, while increasing the Hartmann number (M) or the injection parameter (λ), the velocity component

w first decreasing up to the middle of channel and increases there after. The skin-friction components τ_x *and* τ_z in the main flow and transverse direction, respectively.

II. Basic equations

Consider a coordinate system with plate lying vertically on x-z plane such that x- axis is taken along the plate in the direction of flow and y- axis is perpendicular to the plane of the plate and direction into the fluid which is flowing with free stream velocity U and the lower plate is to have a transverse sinusoidal injection velocity of the form

$$V^{/}(z') = V\left(1 + \varepsilon \cos\frac{\pi z'}{a}\right) \qquad \ldots(1)$$

The problem is governed by the following non dimensional equations:

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \ldots(2)$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{1}{\lambda}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{M^2}{\lambda}u \qquad \ldots(3)$$

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{\lambda}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{1}{k^{/}}v \qquad \ldots(4)$$

$$v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\lambda}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{M^2}{\lambda}w \qquad \ldots(5)$$

$$v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{1}{\lambda p_r}\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right), \qquad \ldots(6)$$

Here the non-dimensional variables are:

$$y = \frac{y'}{a},\ z = \frac{z'}{a},\ u = \frac{u'}{U}, v = \frac{v'}{V}, w = \frac{w'}{V}, p = \frac{p'}{\rho V^2}, T = \frac{(T' - T_0)}{(T_1 - T_0)} \qquad \ldots(7)$$

and $k^{/} = \dfrac{a\upsilon}{Vk}$

The boundary conditions to this problem in dimensionless form are as:

$$\begin{aligned} &u = 0,\ v(z) = 1 + \varepsilon\cos\pi z,\ w = 0,\ T = 0,\ for\ \ y = 0. \\ &u = 1,\ v = 1, \qquad\qquad w = 0, \quad T = 1,\ for\ y = 1. \end{aligned} \qquad \ldots(8)$$

III. Mathematical Analysis

As we know that the amplitude of injection velocity ε is very smalls therefore we can assume the following form the solutions

$$f(y,z) = f_0(y) + \varepsilon f_1(y,z) + \varepsilon^2 f_2(y,z) + ... \quad ...(9)$$

When $\varepsilon = 0$, the problem reduces to two-dimensional then the solution of this two dimensional problem is

$$u_0(y) = \frac{e^{s_1 y} - e^{s_2 y}}{e^{s_1} - e^{s_2}}, \qquad w = 0, \quad ...(10)$$

$$v_0(y) = \frac{1}{e^{t_1} - e^{t_2}}\left[\left(e^{t_1 y} - e^{t_2 y}\right) + e^{t_1 + t_2 y} - e^{t_2 + t_1 y}\right] \quad ...(11)$$

$$T_0(y) = \frac{e^{\lambda P_r y} - 1}{e^{\lambda P_r} - 1} \quad ...(12)$$

where

$$s = \left[\lambda \pm \sqrt{\lambda^2 + 4M^2}\right], \qquad t = \frac{\lambda k' \pm \sqrt{\lambda^2 k^2 + 4k'\lambda}}{2k'}$$

when $\varepsilon \neq 0$, in equation (9) then Substituting in to the equations (2) to (6) and comparing the coefficient of identical powers of ε, and neglecting the coefficient $\varepsilon^2, \varepsilon^3$ etc. The following first order equations obtained:

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \quad ...(13)$$

$$v_1 \frac{\partial u_0}{\partial y} + \frac{\partial u_1}{\partial y} = \frac{1}{\lambda}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - \frac{M^2}{\lambda} u_1 \quad ...(14)$$

$$\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{\lambda}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - \frac{1}{k'} v_1 \quad ...(15)$$

$$\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{\lambda}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - \frac{M^2}{\lambda} w_1 \quad ...(16)$$

$$v_1 \frac{\partial T_0}{\partial y} + \frac{\partial T_1}{\partial y} = \frac{1}{\lambda P_r}\left(\frac{\partial^2 T_1}{\partial y^2} + \frac{\partial^2 T_1}{\partial z^2}\right) \quad ...(17)$$

The corresponding boundary conditions are;

$$u_1 = 0,\ v_1 = \cos \pi z,\ \ w_1 = 0,\ \ T_1 = 0,\ \ for\ y = 0.$$
$$u_1 = 0,\ v_1 = 0,\ \ \ \ w_1 = 0,\ \ T_1 = 0\ \ \ for\ y = 1. \qquad \ldots(18)$$

IV. Cross flow solution

For cross flow solution we assume the following form of v_1, w_1 and p_1 :

$$\left.\begin{aligned} v_1(y,z) &= v_*(y)\cos \pi z. \\ w_1(y,z) &= -\frac{1}{\pi} v_*'(y)\sin \pi z. \\ p_1(y,z) &= p_*(y)\cos \pi z. \end{aligned}\right\} \qquad \ldots(19)$$

where the * denote the differentiation with respect to y. Substituting equation (19) in equations (15) and (16). We get the following ordinary differential equations.

$$v_*'' - \lambda v_*' - \alpha^2 v_* = \lambda p_*'. \qquad \ldots(20)$$

$$v_*''' - \lambda v_*'' - \left(\pi^2 + M^2\right) v_*' = \lambda \pi^2 p_*. \qquad \ldots(21)$$

Now using the transformed boundary conditions the equations (20) and (21) obtained in the following form

$$v_1(y,z) = \frac{1}{D}\left(\sum_{i=1}^{4} D_i e^{r_i y}\right)\cos \alpha z. \qquad \ldots(22)$$

$$w_1(y,z) = -\frac{1}{\pi D}\left(\sum_{i=1}^{4} D_i r_i e^{r_i y}\right)\sin \pi z. \qquad \ldots(23)$$

$$p_1(y,z) = \frac{1}{\lambda \pi^2 D}\left[\sum_{i=1}^{4} D_i \left[\left\{r_i^3 - \lambda r_i^2 - \left(\pi^2 + M^2\right) r_i\right\} e^{r_i y}\right]\right]\cos \pi z \qquad \ldots(24)$$

where

$$r_1 = \frac{1}{2}\left[p_1 + \sqrt{p_1^2 + 4\pi^2}\right] \quad , \quad r_2 = \frac{1}{2}\left[p_1 - \sqrt{p_1^2 + 4\pi^2}\right],$$

$$r_3 = \frac{1}{2}\left[p_2 + \sqrt{p_2^2 + 4\pi^2}\right], \quad r_4 = \frac{1}{2}\left[p_2 - \sqrt{p_2^2 + 4\pi^2}\right],$$

$$\begin{aligned} D = &\left\{(r_2 - r_1)(r_1 - r_3)\left(e^{r_1 + r_2} + e^{r_1 + r_3}\right)\right\} \\ &+ \left\{(r_3 - r_2)(r_1 - r_4)\left(e^{r_2 + r_3} + e^{r_1 + r_4}\right)\right\} + \left\{(r_2 - r_4)(r_3 - r_1)\left(e^{r_3 + r_1} + e^{r_2 + r_4}\right)\right\}. \end{aligned}$$

$$D_1 = r_2(r_1 - r_3)e^{r_3+r_1} + r_3(r_2 - r_4)e^{r_4+r_2} + r_4(r_3 - r_1)e^{r_1+r_3}.$$
$$D_2 = r_2(r_3 - r_4)e^{r_3+r_4} + r_3(r_4 - r_1)e^{r_1+r_4} + r_4(r_1 - r_2)e^{r_1+r_2}.$$
$$D_3 = r_1(r_4 - r_2)e^{r_2+r_4} + r_2(r_1 - r_4)e^{r_1+r_4} + r_4(r_2 - r_1)e^{r_2+r_1}.,$$
$$D_4 = r_1(r_2 - r_3)e^{r_2+r_3} + r_2(r_3 - r_4)e^{r_3+r_4} + r_3(r_4 - r_2)e^{r_1+r_4}.$$

Where $\alpha^2 = \left(\pi^2 + \frac{\lambda}{k'}\right)$

V. Main flow solution

We consider the equations of the main flow component $u_1(y,z)$ and temperature field $T_1(y,z)$, in the following form:

$$u_1(y,z) = u_*(y)\cos\pi z \qquad \dots(25)$$

$$T_1(y,z) = T_*(y)\cos\pi z \qquad \dots(26)$$

Substituting these values in equations (14) and (17) respectively. We obtain the ordinary differential equations in the following form:

$$u_*'' - \lambda u_*' - (\pi^2 + M^2)u_* = \lambda v_* u_0' \qquad \dots(27)$$

$$T_*'' - \lambda P_r T_*' - \pi^2 T_{11} = \lambda P_r v_* T_0' \qquad \dots(28)$$

The corresponding boundary conditions are;

$$\begin{aligned} u_* = 0, \quad T_* = 0, \quad for \ \ y = 0. \\ u_* = 0, \quad T_* = 0, \quad for \ \ y = 1. \end{aligned} \qquad \dots(29)$$

using the boundary condition (29) and equations (25) and (26) in the equations (27) and (28), we get

$$\begin{aligned} u_1(y,z) = \Bigg[\sum_{i=1}^{2} K_i e^{n_i y} + \frac{\lambda}{D(e^{m_1} - e^{m_2})}\Bigg\{\sum_{i=1}^{2}\frac{m_1 D_i e^{(m_1+r_i)y}}{r_i(3m_1 - \lambda)} + \sum_{i=3}^{4}\frac{D_i e^{(m_1+r_i)y}}{r_i} \\ - \sum_{i=1}^{2}\frac{D_i e^{(m_2+r_{i_i})y}}{r_i} - \sum_{i=3}^{4}\frac{D_i m_2 e^{(m_2+r_i)y}}{r_i(3m_2 - \lambda)}\Bigg\}\Bigg]\cos\pi z \end{aligned} \qquad \dots(30)$$

$$T_1(y,z) = \left[\sum_{i=1}^{2} N_i e^{s_i y} + \frac{\lambda^2 P_r^2}{D(e^{\lambda P_r} - 1)}\left\{\sum_{i=1}^{2}\frac{D_i e^{(\lambda P_r + r_i)y}}{r_i(m_1 + \lambda P_r)} + \sum_{i=3}^{4}\frac{D_i e^{(\lambda P_r + r_i)y}}{r_i(m_2 + \lambda P_r)}\right\}\right].\cos\pi z$$

$$\dots(31)$$

where

$$K_1 = A\left[\sum_{i=1}^{2}\frac{D_i m_1\left(e^{n_2}-e^{(m_1+r_i)}\right)}{r_i(3m_1-\lambda)}+\sum_{i=3}^{4}\frac{D_i\left(e^{n_2}-e^{(m_1+r_i)}\right)}{r_i}-\sum_{i=1}^{2}\frac{D_i\left(e^{n_2}-e^{(m_2+r_i)}\right)}{r_i}-\sum_{i=3}^{4}\frac{D_i m_2\left(e^{n_2}-e^{(m_{21}+r_i)}\right)}{r_i(3m_2-\lambda)}\right].$$

$$K_2 = -A\left[\sum_{i=1}^{2}\frac{D_i m_1\left(e^{n_1}-e^{(m_1+r_i)}\right)}{r_i(3m_1-\lambda)}+\sum_{i=3}^{4}\frac{D_i\left(e^{n_1}-e^{(m_1+r_i)}\right)}{r_i}-\sum_{i=1}^{2}\frac{D_i\left(e^{n_1}-e^{(m_2+r_i)}\right)}{r_i}-\sum_{i=3}^{4}\frac{D_i m_2\left(e^{n_1}-e^{(m_{21}+r_i)}\right)}{r_i(3m_2-\lambda)}\right].$$

$$N_1 = B\left[\sum_{i=1}^{2}\frac{D_i\left(e^{s_2}-e^{(\lambda P_r-r_i)}\right)}{r_i(m_1+\lambda P_r)}+\sum_{i=3}^{4}\frac{D_i\left(e^{s_2}-e^{(\lambda P_r-r_i)}\right)}{r_i(m_2+\lambda P_r)}\right]$$

$$N_2 = -B\left[\sum_{i=1}^{2}\frac{D_i\left(e^{s_1}-e^{(\lambda P_r-r_i)}\right)}{r_i(m_1+\lambda P_r)}+\sum_{i=3}^{4}\frac{D_i\left(e^{s_1}-e^{(\lambda P_r-r_i)}\right)}{r_i(m_2+\lambda P_r)}\right]$$

$$A = \frac{\lambda}{D\left(e^{m_1}-e^{m_2}\right)\left(e^{n_1}-e^{n_2}\right)}, \quad B = \frac{\lambda^2 P_r^2}{D\left(e^{\lambda P_r}-1\right)\left(e^{s_1}-e^{s_2}\right)},$$

$$n_1 = \frac{1}{2}\left[\lambda+\sqrt{\lambda^2+4\left(\pi^2+M^2\right)}\right], \qquad n_2 = \frac{1}{2}\left[\lambda-\sqrt{\lambda^2+4\left(\pi^2+M^2\right)}\right],$$

$$s_1 = \frac{1}{2}\left[\lambda P_r+\sqrt{\lambda^2 P_r^2+4\pi^2}\right], \qquad s_2 = \frac{1}{2}\left[\lambda P_r-\sqrt{\lambda^2 P_r^2+4\pi^2}\right]$$

VI. Results and discussion

We may now obtain the expression for the skin-friction components τ_x *and* τ_z is the main flow and transverse direction respectively, as

$$\tau_x = \frac{\tau_x^{/} a}{\mu.U} = \left(\frac{du_0}{dy}\right)_{y=0} + \varepsilon\left(\frac{du_*}{dy}\right)\cos\pi z$$

$$= \frac{m_1-m_2}{e^{m_1}-e^{m_2}} + \varepsilon\left[\sum_{i=1}^{2}K_i n_i + \frac{\lambda}{D\left(e^{m_1}-e^{m_2}\right)}\left\{\sum_{i=1}^{2}\frac{m_1 D_i e^{(m_1+r_i)}}{r_i(3m_1-\lambda)}+\sum_{i=3}^{4}\frac{D_i e^{(m_1+r_i)}}{r_i} - \sum_{i=1}^{2}\frac{D_i e^{(m_2+r_{i_i})}}{r_i}-\sum_{i=3}^{4}\frac{D_i m_2 e^{(m_2+r_i)}}{r_i(3m_2-\lambda)}\right\}\right] \quad \ldots(32)$$

$$\tau_z = \frac{\tau_z^{/} a}{\mu.V} = \varepsilon\left(\frac{dw_1}{dy}\right)_{y=0} = -\frac{\varepsilon}{\pi D}\left(\sum_{i=1}^{4}D_I r_i^2\right)\sin\pi z. \quad \ldots(33)$$

We may calculate the heat transfer coefficient in terms of the Nusselt number

$$Nu = \frac{-q'a}{k(T_1 - T_0)} = \left(\frac{dT_0}{dy}\right)_{y=0} + \varepsilon\left(\frac{dT_*}{dy}\right)_{y=0} \cos \pi z$$

$$= \frac{\lambda P_r}{e^{P_r} - 1} + \varepsilon\left[\sum_{i=1}^{2} N_i s_i + \frac{\lambda^2 P_r^2}{D(e^{\lambda P_r} - 1)}\left\{\sum_{i=1}^{2}\frac{D_i(\lambda P_r + r_i)}{r_i(m_1 + \lambda P_r)} + \sum_{i=3}^{4}\frac{D_i(\lambda P_r + r_i)}{r_i(m_2 + \lambda P_r)}\right\}\right]\cos \pi z$$

…(34)

In the present study the behavior of the λ . main flow velocity decreases with the increasing Hartmann number (M), and injection parameter (λ).The cross flow velocity component *w* due to the transverse sinusoidal injection velocity distribution applied through out the porous plate at rest and the cross flow velocity profile is shown through the figure-2. Here it is observed that while increasing the Hartmann number (M) or the injection parameter (λ), the velocity component *w* first decreasing up to the middle of channel and increases there after. The skin-friction components τ_x *and* τ_z in the main flow and transverse direction, respectively, are presented through the figure-3. This figure shows that τ_x *and* τ_z decrease with increasing λ . It is also noticed that with increasing Hartmann number (M), the skin- friction component τ_x decrease, however, τ_z increases. The rate of heat transfer coefficient at stationary porous plate in terms of the Nusselt number is shown through the figure-4. The value of prandtl number P_r are chosen as 0.7 and 7 approximately which represent air and water respectively at 20^0c. The Nusselt number is also observed to be decreasing with the injection/section parameter

VII. Conclusion

We conclude by graphically as

- **figure-1** it is clear that the main flow velocity decreases with the increasing Hartmann number M, and injection parameter λ .The cross flow velocity component *w* due to the transverse sinusoidal injection velocity distribution applied through out the porous plate at rest.
- The cross flow velocity profile is shown through the **figure-2**. Here it is observed from this figure that while increasing the Hartmann number (M) or the injection parameter (λ), the velocity component *w* first decreasing up to the middle of channel and increases thereafter.
- The skin-friction components τ_x *and* τ_z in the main flow and transverse direction, respectively, are presented through the **figure-3**. This figure shows that τ_x *and* τ_z decrease with increasing λ . It is also noticed that with increasing Hartmann number (M), the skin- friction component τ_x decrease, however, τ_z increases.
- The rate of heat transfer coefficient at stationary porous plate in terms of the Nusselt number is shown through the **figure-4**. The value of prandtl number P_r are chosen as 0.7 and 7 approximately which represent air and water respectively at 20^0c. The Nusselt number is also observed to be decreasing with the injection/section parameter λ .

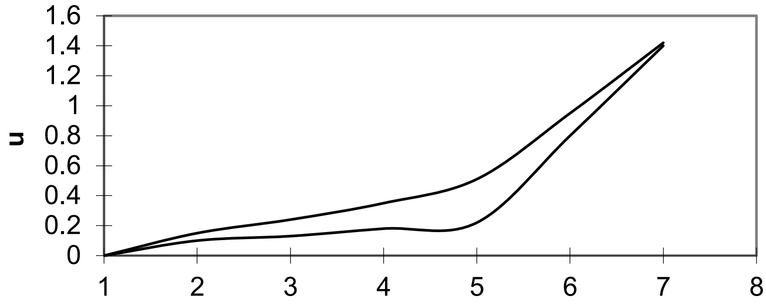

fig -1. Main flow velosity profile for z=0

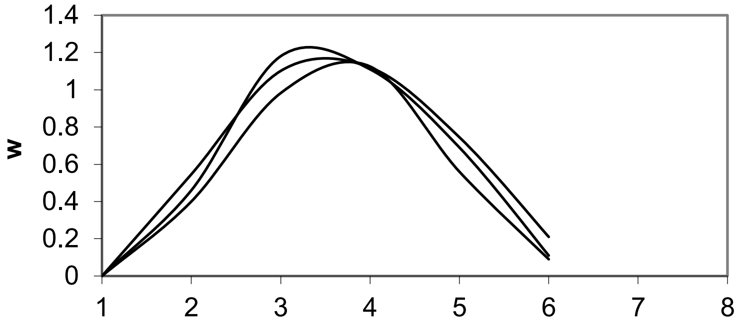

fig.-2 cross flow velosity profile for z=.5

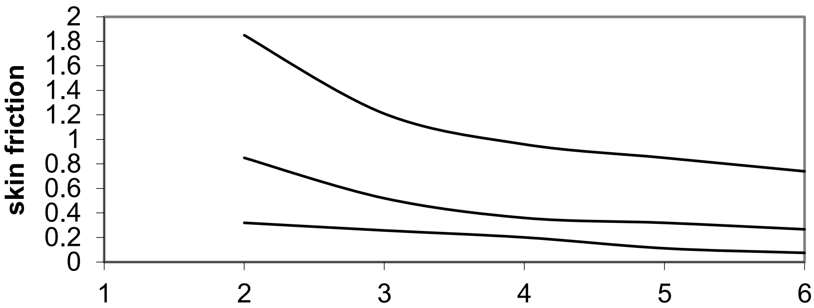

fig.-3 main flow and transvers components of skin friction for z= 0 and z= .5

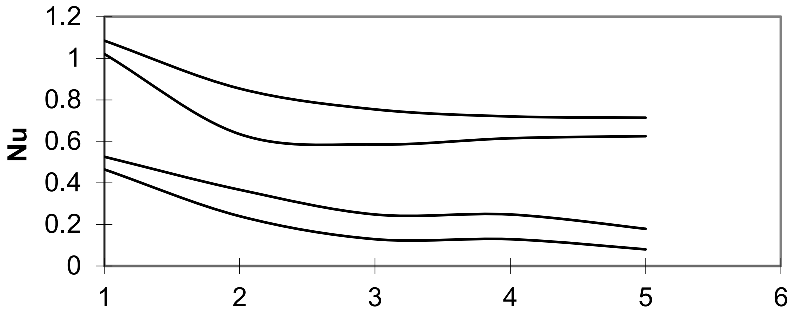

fig.-4 Nusselt number Nu for z=0 and z=.5

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